The Macroeconomics of PAYG Pension Schemes in an Aging Society

Lionel Artige*†
HEC - University of Liège

Laurent Cavenaile
New York University

Pierre Pestieau
HEC - University of Liège

May 19, 2014

Abstract

This paper analyzes and compares the macroeconomic performance of defined-benefit and defined-contribution pay-as-you-go pension systems when population ages. When the fertility rate decreases or longevity rises, it is shown that a shift from defined benefit (defined total benefit or defined annuities) to defined contribution always results in higher per-capita income and life-cycle welfare at the steady state. All results are derived with general production and utility functions.

Keywords: Aging, defined benefit, defined contribution, fertility, longevity, PAYG pension

JEL Classification: E13, H55, J13, J26

*The authors acknowledge the financial support of BELSPO.
†Corresponding author: HEC - Université de Liège, Economics department, Boulevard du Rectorat 7, B. 31, 4000 Liège, Belgium. E-mail: lionel.artige@ulg.ac.be

1 Introduction

All advanced economies and many developing countries are experiencing population aging characterized by a decrease in the fertility rate and a rise in longevity. Since the Industrial Revolution, longevity has been increasing steadily almost everywhere in the world while fertility rates have fluctuated from generation to generation and across countries. The annual population growth rate in Western countries illustrates these generational cycles over the last 60 years (Figure 1). The strong decrease in the population growth rate from 1960 to the early 1980s (the so-called baby bust) accounts for the rapid aging of their population, which has slowed down since then thank to the slight rebound of that rate. Economies are thus subject to periodic changes in fertility rates and, which have significant but predictable macroeconomic effects. Figure 2 shows that the old-age dependency ratio increased linearly between 1950 and 2010 and, according to the UN projections, its trend should rise substantially from 2010 onwards.

There is a lot of concern about the consequences of population aging on the financial sustainability of pension and health care systems or on potential income growth.[1] In a pay-as-you-go (PAYG) pension system, aging - implying an increase in the old-age dependency ratio - requires a financial adjustment between the pension benefits and the pension contributions. On an accounting point of view, this adjustment can be realized by a reduction in the benefits, an increase in the contributions, or by a combination of the two. All these options are politically undesirable, which may explain why aging is also viewed as a worrying prospect. However, the concern should also regard the macroeconomics of aging. As Barr and Diamond (2006) put it, ”what matters is output” and one should focus on the macroeconomic effects of changes in the fertility rate and longevity. On a macroeconomic point of view, standard neoclassical growth models show

[1]See, for instance, Bös and Weiszäcker (1989), Cutler et al. (1990), Auerbach and Kotlikoff (1992), Bloom, Canning, and Fink (2010), EC (2012) and OECD (2013).

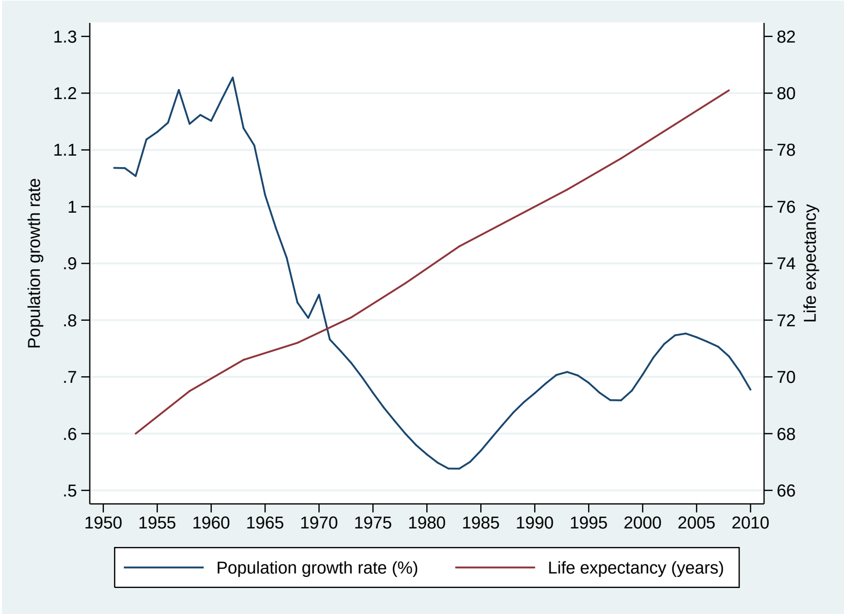
that a decline in fertility reduces the dilution of the capital stock and, therefore, always yields higher capital accumulation and per-capita income, which is empirically confirmed by Brander and Dowrick (1994), Kelley and Schmidt (1995) or more recently by Li and Zhang (2007). Nonetheless, empirical studies find a negative effect of an increase in the old-age dependency ratio on saving and investment (Li, Zhang, and Zhang 2007). Regarding longevity, a rise in life expectancy will, with certainty, increase the old-age dependency ratio and (private and public) expenditures of the elderly. In the absence of any transfer to the elderly, the macroeconomic effect depends on the reaction of the old individuals. If they do not change their retirement age, they will save more and capital accumulation will be higher. If they decide to work longer, they may save less and capital accumulation will be lower. Empirically, an increase in longevity appears to be favorable to saving and investment (Li, Zhang, and Zhang 2007). In total, one cannot be conclusive on the macroeconomics of aging. One conclusion that can be drawn, however, is that there is no parallelism between the financial accounting and macroeconomic effects of aging.

The introduction of intergenerational transfers under the form of PAYG pensions complicates further the macroeconomics of aging. If the effect of a decline in the fertility rate is clearly positive on capital accumulation when young people save for retirement in a fully funded pension scheme, the result is much less clear, as we show in this paper, if the pension system is PAYG because the dilution of the capital stock is reinforced in a defined-contribution system while it is weakened in a defined-benefit system. As for longevity, the macroeconomic effect is ambiguous if retirees are promised to receive defined annuities. All different PAYG pension schemes are not macroeconomic equivalents and their economic relevance depends strongly on demographic trends. In a number of countries over the last decades the nature of pension plans has changed dramatically as coverage has shifted from defined-benefit to defined-contribution arrangements. This shift started in funded private pension schemes in the US and in the UK. Recently it has hit the unfunded public social insurance programs. In particular a number of Continental European countries with generous pay-as-you-go public schemes have shown an interest in moving towards defined-contribution format including notional defined-contribution schemes. This evolution has been motivated by two joint concerns: a better risk-taking balance between the retirees and the workers and the financial sustainability of the plans in aging societies.[2] Is this evolution sound on a macroeconomic point of view?

The objectives of this paper are to clarify the macroeconomic effects of changes in fertility and longevity when pension is PAYG and assess the macroeconomic relevance of the shift from defined-benefit to defined-contribution pension in an aging society. Using an OLG model *à la* Diamond (1965) and general functions for technology and preferences, we study the implications of a decline in the fertility rate and of an increase in longevity on the capital accumulation and the welfare of a society with unfunded pensions that can be either of the defined-contribution (DC) or of the defined-benefit type (defined total benefit (DB) and defined annuities (DA)). Despite ambiguous effects of declining fertility and of rising longevity on capital accumulation and welfare depending on the pension scheme, we show that the shift from a defined-benefit (DB or DA) to a defined-contribution plan is a right move in terms of both per-capita income and life-cycle welfare.

The rest of the paper is organized as follows. Section 2 presents the model with defined-benefit (DB and DA) and defined-contribution pension schemes. Sections 3 and 4 give the conditions for the existence and uniqueness of the intertemporal equilibrium and the steady state respectively. Section 5 discusses the effect of a decline in the fertility rate on capital accumulation and welfare in a DC, a DB and a DA pension system and shows that a shift from DA (or DB) to DC always results in higher steady-state per-capita income and life-cycle welfare. Section 6 studies the effect of a rise in longevity on capital accumulation and welfare in a DC, a DB and a DA pension system and shows that a shift from DA to

[2]See on this Barr and Diamond (2006) and Munnell (2006).



Source: Penn World Table 8.0 and UN Population data

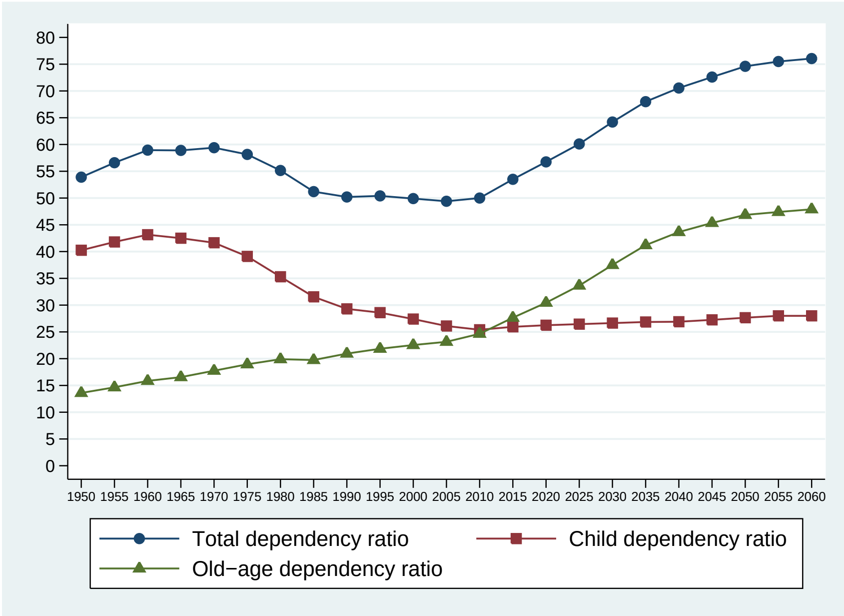
Figure 1: Annual population growth and life expectancy in Western countries (1950-2010)

Western countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

DC (or DB) always results in higher steady-state per-capita income and life-cycle welfare. Section 7 concludes.

2 The Model

We consider a discrete-time deterministic model of an economy producing a single aggregate good under perfect competition from date $t = 0$ to infinity. The economy is populated by overlapping generations living for two periods and the population size grows at a constant rate $n \in]-1, +\infty[$. The second period is of length $l \in [0, 1]$ (longevity) so that life expectancy at birth is equal to $1 + l$. When young, individuals supply inelastically one unit of labor to the firms in a perfectly competitive labor market, receive a wage



Source: UN Population data

Figure 2: Dependency ratios in Western countries (Cross-Country means in percent, 1950-2060)

and allocate this net of tax income between consumption and saving. When old, they consume the return on their saving and the pension benefit. This benefit is financed by the contribution of the workers belonging to the next generation.

2.1 Production

The representative firm produces the single good using a neoclassical technology of the form

$$Y_t = AF(K_t, L_t), \tag{1}$$

where K_t and L_t are respectively the stock of capital and the labor input at time t, and A is a technological parameter. The production function $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is assumed to be increasing in its arguments, concave and homogeneous of degree 1. We assume that physical capital fully depreciates after one period. At time t, the representative firm has an installed stock of capital K_t, chooses the labor input paid at the competitive wage w_t, equal to the marginal product of labor, and maximizes its profits

$$\pi_t = \max_{L_t} AF(K_t, L_t) - w_t L_t, \tag{2}$$

where $\pi_t = (1 + r_t)K_t$ are the profits distributed to the owners of the capital stock and r_t the real interest rate, which is equal to the marginal product of capital minus one. Since returns to scale are constant, the production function can be written in intensive form

$$y_t = Af(k_t), \tag{3}$$

where $f(k_t) = F(k_t, 1)$, $k_t \equiv K_t/L_t$ is the capital stock per worker and y_t is the income per worker.

2.2 Pension

We assume that the government introduces a pay-as-you-go (PAYG) pension system in the first period. The first old generation thus benefits from a free lunch. The other generations pay a lump-sum tax when young and receive a lump-sum pension or annuities when old. The pension financing constraint is

$$p_t = (1 + n)\tau_t, \tag{4}$$

where p_t is the individual pension received by the old individual born at time $t - 1$ and $\tau_t \geqslant 0$ is the lump-sum tax levied by the government on each young worker born at time t. The pension p_t is thus an intergenerational transfer from the young to the old individuals. In a defined-annuities pension system, $p_t = a_t l$ where a_t is the annuity paid to retired agents. We will assume throughout the paper that the pension, the tax and the annuity are constant across generations: $p_t = p$, $\tau_t = \tau$ and $a_t = a$ for any t and given n and l.

We consider three PAYG pension systems: a defined-benefit (DB) pension system, in which the pension is constant across all generations and the tax adjusts to the value of n, a defined-contribution (DC) pension system, in which the lump-sum tax is constant across all generations and the pension adjusts to the value of n and a defined-annuity (DA) pension system in which an annuity is paid to old agents while alive and the tax adjusts to the longevity (l) of old agents and to fertility (n).

2.3 Preferences

The consumer's preferences are represented by the following life-cycle utility function

$$U = u(c_t) + \beta l u(d_{t+1}), \tag{5}$$

where, for all $c, d > 0$, it is assumed that $u'(.) > 0$, $u''(.) < 0$ and $\lim_{c \to 0} u'(c) = +\infty$.

The consumer's problem will be studied within the three pension systems:

i) when the pension system is DC, the representative consumer maximizes (5) subject to the budget constraints

$$c_t + s_t = w_t - \tau \tag{6}$$

$$d_{t+1} = (1+n)\tau + (1 + r_{t+1})s_t \tag{7}$$

ii) when the pension system is DB, the representative consumer maximizes (5) subject to the budget constraints

$$c_t + s_t = w_t - \frac{p}{1+n} \tag{8}$$

$$d_{t+1} = p + (1 + r_{t+1})s_t \tag{9}$$

iii) when the pension system is DA, the representative consumer maximizes (5) subject to the budget constraints

$$c_t + s_t = w_t - \frac{al}{1+n} \tag{10}$$

$$d_{t+1} = al + (1 + r_{t+1})s_t \tag{11}$$

In all three systems, w_t, c_t, s_t are respectively the first-period wage, the consumption when young and the individual saving at time t. When old, the individuals consume d_{t+1} that must satisfy the budget constraint composed of the gross return on saving and the pension benefit. The parameter $\beta \in (0, 1)$ is the psychological discount factor.

2.4 Optimal Behavior

The consumer's optimization problem is well-defined if and only if

$$0 < \tau < w_t \; \forall t \tag{12}$$

The maximization of (2) with respect to L_t by the representative firm yields the wage rate and allows to calculate the profits that are distributed to the owners of the capital stock. The expressions for the wage rate and the interest rate are thus the optimal return of labor and capital respectively:

$$\begin{aligned} w_t &= w(k_t) = f(k_t) - k_t f'(k_t) && (13) \\ r_t &= r(k_t) = f'(k_t) - 1 && (14) \end{aligned}$$

The maximization of (5) with respect to the corresponding budget constraints yields the optimal level of individual saving, which is characterized by the following first-order condition in the DC system

$$u'(w_t - s_t - \tau) = \beta l(1 + r_{t+1})u'\left[(1 + n)\tau + (1 + r_{t+1})s_t\right], \quad (15)$$

in the DB system, by

$$u'\left(w_t - s_t - \frac{p}{1+n}\right) = \beta l(1 + r_{t+1})u'\left[p + (1 + r_{t+1})s_t\right], \quad (16)$$

and, in the DA system, by

$$u'\left(w_t - s_t - \frac{al}{1+n}\right) = \beta l(1 + r_{t+1})u'\left[al + (1 + r_{t+1})s_t\right], \quad (17)$$

where s_t, the solution to the maximization problem, is positive if condition (12) is satisfied. The saving function s_t is increasing in the first-period wage w_t and decreasing in the pension p (or al) or the lump-sum tax τ. In order to study the sign of the derivative of the saving function with respect to fertility and longevity, we rewrite Equation (15) as the following implicit function in the DC system

$$-u'(w_t - s_t - \tau) + \beta l(1 + r_{t+1})u'\left[(1 + n)\tau + (1 + r_{t+1})s_t\right] = 0 \quad (18)$$

and Equation (16) as the following implicit function in the DB system

$$-u'\left(w_t - s_t - \frac{p}{1+n}\right) + \beta l(1+r_{t+1})u'\left[p + (1+r_{t+1})s_t\right] = 0. \tag{19}$$

and Equation (17) as the following implicit function in the DA system

$$-u'\left(w_t - s_t - \frac{al}{1+n}\right) + \beta l(1+r_{t+1})u'\left[al + (1+r_{t+1})s_t\right] = 0. \tag{20}$$

The effect of an increase in the fertility rate on saving is unambiguously negative in the DC system

$$\left[\frac{\partial s_t}{\partial n}\right]^{DC} = -\frac{\beta(1+r_{t+1})\tau u''(d_{t+1})}{u''(c_t) + \beta l(1+r_{t+1})^2 u''(d_{t+1})} < 0 \tag{21}$$

and unambiguously positive in the DB and DA systems

$$\left[\frac{\partial s_t}{\partial n}\right]^{DB} = \left[\frac{\partial s_t}{\partial n}\right]^{DA} = \frac{\frac{p}{(1+n)^2}u''(c_t)}{u''(c_t) + \beta l(1+r_{t+1})^2 u''(d_{t+1})} > 0. \tag{22}$$

with $al = p$.

Similarly, we can determine the effect of a change in longevity on savings under any of the three pension systems:

$$\left[\frac{\partial s_t}{\partial l}\right]^{DC} = \left[\frac{\partial s_t}{\partial l}\right]^{DB} = -\frac{\beta u'(d_{t+1})(1+r_{t+1})}{u''(c_t) + \beta l(1+r_{t+1})^2 u''(d_{t+1})} > 0 \tag{23}$$

which is unambiguously positive and,

$$\left[\frac{\partial s_t}{\partial l}\right]^{DA} = -\frac{u''(c_t)\frac{a}{1+n} + \beta(1+r_{t+1})\left(u'(d_{t+1}) + alu''(d_{t+1})\right)}{u''(c_t) + \beta l(1+r_{t+1})^2 u''(d_{t+1})} \tag{24}$$

whose sign is ambiguous.

3 Intertemporal Equilibrium

The equilibrium in the labor market is given by the equality between the supply and demand for labor. The equilibrium in the capital market derives from the national income accounts identity

$$K_{t+1} = N_t s_t, \tag{25}$$

where K_{t+1}, non-negative by definition, is the aggregate capital stock at $t+1$ and $N_t s_t$ is the aggregate saving from the young generation t, which finances the capital stock at time $t+1$. Feasibility requires positive saving s_t. In intensive form, equation (25) becomes

$$(1+n)k_{t+1} = s_t, \tag{26}$$

where $k_{t+1} \equiv \frac{K_{t+1}}{L_{t+1}}$ is the capital stock per worker.

In order to check the existence of at least one intertemporal equilibrium, we can rewrite Equation (26) in the form of an implicit function

$$\phi(k_{t+1}, w(k_t); n) \equiv (1+n)k_{t+1} - s_t = 0, \tag{27}$$

where $s_t \equiv s[w(k_t), r(k_{t+1})]$.

Since the implicit function $\phi(k_{t+1}, w(k_t); n)$ is continuous with respect to k_{t+1} on the set $t = [0, +\infty[$, one can prove that there exists at least one k_{t+1} given k_t, i.e. there exists at least one intertemporal equilibrium, by showing that the limits of the implicit function are of opposite sign (see de la Croix and Michel (2002), p. 20). As for the uniqueness of the intertemporal equilibrium, it is sufficient to assume that the implicit function is strictly increasing in k_{t+1} (see de la Croix and Michel (2002), p. 25). This requires that

$$s_2[w(k_t), r(k_{t+1})] > \frac{1+n}{f''(k_{t+1})}, \tag{28}$$

where $s_2[w(k_t), r(k_{t+1})]$ is the first derivatives with respect to $r(k_{t+1})$. On the right-hand side of (28), the numerator is positive and the denominator is negative. Therefore, if condition (28) is verified, for any given $w_t > 0$, there exists an intertemporal equilibrium with perfect foresight and it is unique.

4 Steady State

The intertemporal equilibrium with perfect foresight is characterized by the dynamics of the capital stock per worker k_t. Equation (27) determines the saving locus implicitly. By differentiating Equation (27) with respect to k_{t+1} and k_t we obtain

$$\frac{dk_{t+1}}{dk_t} = \frac{-s_1[w(k_t), r(k_{t+1})]f''(k_t)k_t}{1+n-s_2[w(k_t), r(k_{t+1})]f''(k_{t+1})} > 0, \tag{29}$$

where $\frac{dk_{t+1}}{dk_t}$ is the slope of the saving locus and $s_1[w(k_t), r(k_{t+1})]$ is the first derivative with respect to $w(k_t)$. The numerator of (29) is positive and the denominator is positive if condition (28) is verified. Since (29) is positive, the dynamics of the capital stock k_t is monotonic. This monotonic sequence of k_t is bounded as $s_t < \omega(k_t)$ and $\lim_{k_t \to +\infty} \frac{\omega(k_t)}{k_t} = 0$ (see de la Croix and Michel (2002), p. 31). Therefore, in a model with overlapping generations living for two periods and with a PAYG system, an economy characterized by Equations (1)-(26) admits at least one steady state. The number of steady states depends on the forms of the utility and the production functions. At the steady state, Equation (27) becomes

$$\phi(\bar{k}; n) = (1+n)\bar{k} - s[w(\bar{k}), r(\bar{k})] = 0. \tag{30}$$

At the steady state, the slope of the saving locus is given by

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{k_t=\bar{k}} = \frac{-s_1[w(\bar{k}), r(\bar{k})]f''(\bar{k})\bar{k}}{1+n-s_2[w(\bar{k}), r(\bar{k})]f''(\bar{k})} \tag{31}$$

The steady state is stable if Equation (31) is less than 1. When utility is logarithmic and production is Cobb-Douglas such that $y_t = Ak_t^{\alpha}$, Equation (31) is equal to $\alpha < 1$.

5 Fertility, Pension, Capital Accumulation and welfare

Our first objective is to examine the effect of fertility on capital accumulation in this OLG model at stable steady states. The PAYG pension system can be based on defined contribution (DC), defined benefit (DB) or defined annuities (DA). Since the effects of a change in fertility are identical in the model with DA and in the model with DB, we only derive results with DA and compare them to those with DC. In the next four sections, we show that the effect of fertility on capital accumulation and welfare varies depending on the PAYG pension scheme.

5.1 Defined-Contribution Pension Scheme

In this section, we assume that the PAYG pension scheme is based on defined contribution, i.e., the lump-sum tax is constant for any value of the fertility rate n. Therefore, the value of the pension will adjust to the value of n.

Proposition 1 *In a model with overlapping generations living for two periods and with a PAYG pension system based on defined contribution, a decrease (increase) in the fertility* *rate n has a positive (negative) effect on the steady-state level of capital per worker when the economy is at a stable steady state.*

Proof: Using the implicit function (27), we can calculate $\frac{d\bar{k}}{dn}$:

$$\left[\frac{d\bar{k}}{dn}\right]^{DC} = -\frac{\bar{k} + \frac{\beta l[1+r(\bar{k})]\tau u''(\bar{d})}{u''(\bar{c})+\beta l[1+r(\bar{k})]^2 u''(\bar{d})}}{1+n+s_1[w(\bar{k}),r(\bar{k})]f''(\bar{k})\bar{k} - s_2[w(\bar{k}),r(\bar{k})]f''(\bar{k})} \tag{32}$$

which is negative since the denominator is positive when the steady state is stable (i.e. when Equation (31) is less than one). ■

Therefore, if two countries, with PAYG pension systems based on defined contribution, are identical in all respects except in the fertility rate, the country with a higher fertility rate will have a higher steady-state pension per old individual and a lower capital accumulation per worker. A rise in the fertility rate increases the value of the pension and the dilution of the capital stock. These two effects reduce capital accumulation.

5.2 Defined-Benefit and Defined-Annuity Pension Schemes

We now assume that the PAYG pension scheme is based on defined annuities (DA). As mentioned before, all the results would be identical if the pension system were based on DB. The pension is constant for any value of the fertility rate n. Therefore, the value of the lump-sum tax will adjust to the value of n. In this case, the first effect that is obtained without pension is naturally negative. Yet, the second effect that operates through the PAYG system is positive and this makes the overall outcome ambiguous.

Proposition 2 *In a model with overlapping generations living for two periods and with a PAYG pension system based on defined annuities (or on defined benefit), a change in the fertility rate n has an ambiguous effect on the steady-state level of capital per worker.*

Parameter	α	βl	A	p
Value	$\frac{1}{3}$	0.75	10	3

Table 1: Parameter values used in Figure 3

Proof: Using the implicit function (27), we can calculate $\frac{d\bar{k}}{dn}$:

$$\left[\frac{d\bar{k}}{dn}\right]^{DA} = -\frac{\bar{k} - \frac{\frac{al}{(1+n)^2}u''(\bar{c})}{u''(\bar{c})+\beta l[1+r(\bar{k})]^2 u''(\bar{d})}}{1+n+s_1[w(\bar{k}),r(\bar{k})]f''(\bar{k})\bar{k} - s_2[w(\bar{k}),r(\bar{k})]f''(\bar{k})} \tag{33}$$

For low values of n, it is well possible that the effect of an increase in the fertility rate n is positive on capital accumulation per worker. When the fertility rate is high, the effect becomes negative as in the model with a defined-contribution pension scheme. A rise in the fertility rate decreases the lump-sum tax and, hence, increases saving. This positive effect on capital accumulation may offset, for low values of n, the negative effect of the higher dilution of the capital stock. ■

In order to illustrate the ambiguity of the effect of an increase in the fertility rate on the capital stock per worker when pensions are characterized by defined annuities (or defined benefit), we propose a numerical exercise based on a logarithmic utility function and a Cobb-Douglas production function. The parameter values of this numerical exercise are given in Table 1. We run simulations for the model at the steady state for values of the fertility rate n ranging from 0 to 50%. Figure 1 represents the steady-state values of the capital stock per worker $\bar{k}$, which follow an inverted U-shape. With defined contribution, the profile of $\bar{k}$ would be monotonically declining.

5.3 Macroeconomic Effects of a Decline in Fertility (n)

In this section, we determine which pension system leads to the highest steady-state level of income per capita when the fertility rate decreases in two economies which are initially identical in all respects except their pension scheme.

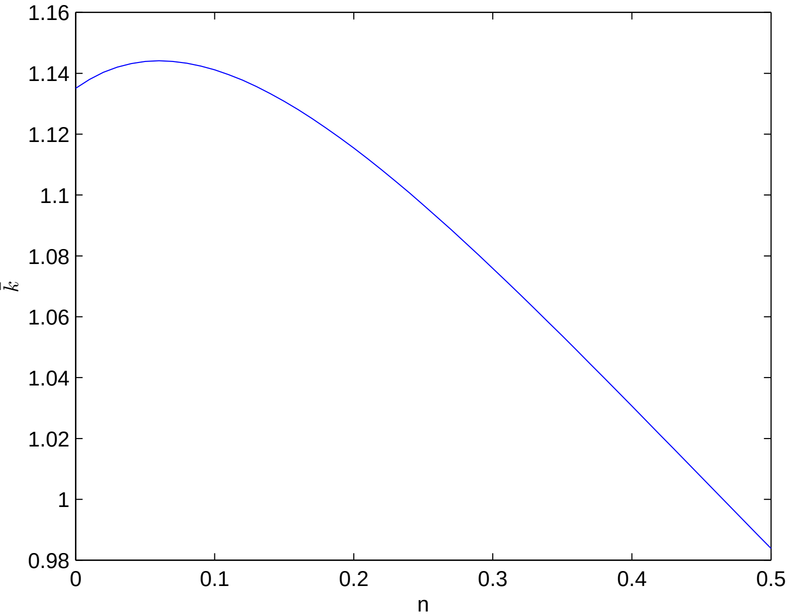


Figure 3: Steady-state capital stock per worker as a function of the fertility rate (defined benefit or defined annuities)

Proposition 3 *If two economies with a stable steady state are initially identical in all respects (in particular, $al = p = (1+n)\tau$) except their pension scheme, a decline in the fertility rate (n) always results in a higher steady state per-capita income in the economy with a defined-contribution scheme than in the economy with a defined-annuity (or defined benefit) scheme.*

Proof: Using Equations (32) and (33), we obtain:

$$\begin{aligned}
\left[\frac{d\bar{k}}{dn}\right]^{DA} - \left[\frac{d\bar{k}}{dn}\right]^{DC} &= -\frac{\bar{k} - \frac{\frac{al}{(1+n)^2}u''(\bar{c})}{u''(\bar{c})+\beta l[1+r(\bar{k})]^2u''(\bar{d})}}{1+n+s_1[w(\bar{k}),r(\bar{k})]f''(\bar{k})\bar{k} - s_2[w(\bar{k}),r(\bar{k})]f''(\bar{k})} \\
&+ \frac{\bar{k} + \frac{\beta l[1+r(\bar{k})]\tau u''(\bar{d})}{u''(\bar{c})+\beta l[1+r(\bar{k})]^2u''(\bar{d})}}{1+n+s_1[w(\bar{k}),r(\bar{k})]f''(\bar{k})\bar{k} - s_2[w(\bar{k}),r(\bar{k})]f''(\bar{k})} \\
&= \frac{\overbrace{\frac{al}{(1+n)^2}u''(\bar{c}) + \beta l[1+r(\bar{k})]\tau u''(\bar{d})}^{<0}}{\underbrace{\big(1+n+s_1(w(\bar{k}),r(\bar{k}))f''(\bar{k})\bar{k} - s_2(w(\bar{k}),r(\bar{k}))f''(\bar{k})\big)}_{>0}} \\
&\times \frac{1}{\underbrace{\big(u''(\bar{c})+\beta l[1+r(\bar{k})]^2u''(\bar{d})\big)}_{<0}} > 0 \qquad (34)
\end{aligned}$$

This proves Proposition 3. ■

This shows that if an economy is facing a decrease in its fertility rate (n), it will end up with a higher stock of capital and hence income par capita under a defined-contribution pension scheme than under a defined-annuity (or defined-benefit) scheme.

5.4 Welfare Effect of a Decline in Fertility (n)

We now study the effect of a decrease in the fertility rate n on the individual steady-state life-cycle welfare in the two pension systems. We focus on economies which are initially dynamically efficient (i.e. $r(\bar{k}) \geq n$) and that have the same initial $\bar{k}$. The change in steady-state life-cycle utility following a decline in the fertility rate is potentially ambiguous under each system. However, we prove that the relative effect on welfare is unambiguous when we are comparing economies which only differ in their pension system.

Proposition 4 *Assume an economy with a stable steady state which is initially dynamically efficient. A decline in the fertility rate (n) has an ambiguous effect on the steady-state life-cycle utility for the defined-contribution and the defined-annuity (or defined-benefit) pension systems. At the golden rule, a decline in the fertility rate has an unambiguously negative impact on steady-state life-cycle utility under the two pension systems.*

Proof: Let us define $\bar{U}_j$ as the steady-state life-cycle utility under pension scheme $j \in \{DC, DA\}$:

$$\begin{aligned} \bar{U}_{DC} &= u\left[f(\bar{k}) - \bar{k}f'(\bar{k}) - \tau - (1+n)\bar{k}\right] + \beta l u\left[(1+n)\tau + f'(\bar{k})k(1+n)\right] \quad (35) \\ \bar{U}_{DA} &= u\left[f(\bar{k}) - \bar{k}f'(\bar{k}) - \frac{al}{1+n} - (1+n)\bar{k}\right] + \beta l u\left[al + f'(\bar{k})k(1+n)\right] \quad (36) \end{aligned}$$

We can determine the effect of a change in the fertility rate (n) on the steady-state life-cycle utility of an agent under the two pension systems as:

$$\frac{d\bar{U}_{DC}}{dn} = \frac{\partial \bar{U}_{DC}}{\partial n} + \frac{\partial \bar{U}_{DC}}{\partial \bar{k}} \left[\frac{d\bar{k}}{dn}\right]^{DC} \tag{37}$$

$$\frac{d\bar{U}_{DA}}{dn} = \frac{\partial \bar{U}_{DA}}{\partial n} + \frac{\partial \bar{U}_{DA}}{\partial \bar{k}} \left[\frac{d\bar{k}}{dn}\right]^{DA} \tag{38}$$

which gives:

$$\frac{d\bar{U}_{DC}}{dn} = \beta l \tau u'(\bar{d}) + \underbrace{\bar{k} f''(\bar{k})}_{<0} \left[\underbrace{-u'(\bar{c}) + \beta l(1+n)u'(\bar{d})}_{\leq 0 \ by \ dyn. \ eff.}\right] \underbrace{\left[\frac{d\bar{k}}{dn}\right]^{DC}}_{<0} \tag{39}$$

$$\frac{d\bar{U}_{DA}}{dn} = \frac{al}{(1+n)^2} u'(\bar{c}) + \underbrace{\bar{k} f''(\bar{k})}_{<0} \left[\underbrace{-u'(\bar{c}) + \beta l(1+n)u'(\bar{d})}_{\leq 0 \ by \ dyn. \ eff.}\right] \underbrace{\left[\frac{d\bar{k}}{dn}\right]^{DA}}_{?} \tag{40}$$

The first term on the RHS of Equations (39) and (40) is positive while the second term is potentially negative in both cases. This implies that the total effect of a decline of the fertility rate on the life-cycle utility is potentially positive under any pension system. At the golden rule, however, it is unambiguously negative under any pension system as the second term on the RHS of Equations (39) and (40) is equal to zero in this case. ■

Even though, the effect of a decline of the fertility rate is ambiguous under any pension system, we can show that the steady-state life-cycle utility of two economies initially differing only in their pension systems is always higher in the economy with a defined-contribution pension after a decline in the fertility rate:

Proposition 5 *If two economies with a stable steady state are initially dynamically efficient and identical in all respects except their pension scheme (in particular, $al = p = (1+n)\tau$), a decline in the fertility rate (n) always results in a higher (or equal) steady-*

state life-cycle utility in the economy with a defined-contribution system than than in the economy with a defined-annuity (or defined benefit) scheme.

Proof: We can determine the effect of a change in the fertility rate (n) on the relative steady-state life-cycle utility of an agent under DA and DC as:

$$\frac{d(\bar{U}_{DC}-\bar{U}_{DA})}{dn} = \frac{\partial(\bar{U}_{DC}-\bar{U}_{DA})}{\partial n} + \frac{\partial \bar{U}_{DC}}{\partial \bar{k}}\left[\frac{d\bar{k}}{dn}\right]^{DC} - \frac{\partial \bar{U}_{DA}}{\partial \bar{k}}\left[\frac{d\bar{k}}{dn}\right]^{DA} \tag{41}$$

Focusing on the first element of the RHS of Equation (41) at a dynamically efficient steady state, we find:

$$\frac{\partial(\bar{U}_{DC}-\bar{U}_{DA})}{\partial n} = \frac{\tau}{1+n}\left(\beta l(1+n)u'(\bar{d}) - u'(\bar{c})\right) \leq 0 \tag{42}$$

as $r \geqslant n$. We now turn to the second part of the RHS of (41).

Noting that $\frac{\partial(\bar{U}_{DC})}{\partial k} = \frac{\partial(\bar{U}_{DA})}{\partial k}$ when the two economies are initially similar in all respects except their pension system, we can rewrite Equation (41) as:

$$\frac{d(\bar{U}_{DC}-\bar{U}_{DA})}{dn} = \frac{\partial(\bar{U}_{DC}-\bar{U}_{DA})}{\partial n} + \frac{\partial \bar{U}_{DC}}{\partial k}\left(\left[\frac{d\bar{k}}{dn}\right]^{DC} - \left[\frac{d\bar{k}}{dn}\right]^{DA}\right) \tag{43}$$

where

$$\frac{\partial \bar{U}_{DC}}{\partial k} = kf''(k)[-u'(c) + \beta l(1+n)u'(d)] \geq 0 \tag{44}$$

All this proves that, at a dynamically efficient steady state:

$$\frac{d(\bar{U}_{DC}-\bar{U}_{DA})}{dn}=\underbrace{\frac{\partial(\bar{U}_{DC}-\bar{U}_{DA})}{\partial n}}_{\leq 0}+\underbrace{\frac{\partial\bar{U}_{DC}}{\partial k}}_{\geq 0}\left(\underbrace{\left[\frac{d\bar{k}}{dn}\right]^{DC}-\left[\frac{d\bar{k}}{dn}\right]^{DA}}_{<0}\right)\leq 0 \qquad (45)$$

■

Combined with the results in the previous section, this shows that an economy facing a decrease in its fertility rate should adopt a defined-contribution pension scheme if its goal is to maximize its steady-state income per capita and life-cycle utility *ceteris paribus*. Indeed, its income per capita would be strictly higher and its life-cycle utility at least as high as under DA (or DB).

6 Longevity, Pension, Capital Accumulation and Welfare

The previous section has shown that a defined-contribution pension system leads to both higher steady-state per-capita income and life-cycle utility than any of the other two systems after a decline in the fertility rate. We now study the consequences of another cause of aging i.e. an increase in longevity. As regards longevity, the model with DC is identical to the model with DB. Therefore, we only derive results for DA and DC, knowing that, this time, DC is equivalent to DB. First, we show that the effect of longevity on capital accumulation and welfare can vary depending on the PAYG pension scheme. Second, when longevity rises, we show that a defined-contribution pension system always leads to higher steady-state per-capita income and life-cycle utility than a defined-annuity system.

6.1 Defined-Contribution and Defined-Benefit Pension Schemes

In this section, we study the effect of an increase in longevity on capital accumulation in DC (or DB) pension system.

Proposition 6 *In a model with overlapping generations living for two periods and with a PAYG pension system based on defined contribution (or defined benefit), an increase (decrease) in longevity (l) has a positive (negative) effect on the steady-state level of capital per worker when the economy is at a stable steady state.*

Proof: Using the implicit function (27), we can calculate $\frac{d\bar{k}}{dl}$ as:

$$\left[\frac{d\bar{k}}{dl}\right]^{DC} = -\frac{\beta u'(\bar{d})f'(\bar{k})}{\left(\underbrace{1+n+s_1(\bar{w},\bar{r})f''(\bar{k})\bar{k}-s_2(\bar{w},\bar{r})f''(\bar{k})}_{<0}\right)\left(\underbrace{u''(\bar{c})+\beta l[f'(\bar{k})]^2u''(\bar{d})}_{<0}\right)} \quad (46)$$

which is strictly positive. ■

6.2 Defined-Annuity Pension Scheme

The effect of an increase in longevity on steady-state per-capita income in a defined-annuity pension scheme is different from that in any of the other two pension systems as a change in l leads to a modification of the total pension paid to retired agents. As retirees are promised a fixed annuity, an increase in their life expectancy requires to be financed by a higher tax. In this case, the effect of an increase in longevity on steady-state per-capita income is ambiguous.

Parameter	α	β	A	a	n
Value	$\frac{1}{3}$	0.75	10	3	0.2

Table 2: Parameter values used in Figure 4

Proposition 7 *In a model with overlapping generations living for two periods and with a PAYG pension system based on defined annuities, a change in longevity l has an ambiguous effect on the steady-state level of capital per worker.*

Proof: Using the implicit function (27), we can calculate $\frac{d\bar{k}}{dl}$ as:

$$\left[\frac{d\bar{k}}{dl}\right]^{DA} = \frac{\frac{a}{1+n}u''(\bar{c}) + \beta f'(\bar{k})\left[\overbrace{u'(\bar{d})}^{>0} + alu''(\bar{d})\right]}{\left(\underbrace{1+n+s_1(\bar{w},\bar{r})f''(\bar{k})\bar{k} - s_2(\bar{w},\bar{r})f''(\bar{k})}_{<0}\right)\left(\underbrace{u''(\bar{c}) + \beta l[f'(\bar{k})]^2 u''(\bar{d})}_{<0}\right)} \tag{47}$$

The sign of Equation (47) is ambiguous as the numerator can either be positive or negative. ■

Figure 4 illustrates the ambiguity of the effect a change in l on steady-state capital and income per capita when the pension system is based on defined annuities. This example is based on a Cobb-Douglas production function and a logarithmic utility with the parameter values reported in Table 6.2.

6.3 Macroeconomic Effects of an Increase in Longevity (l)

In this section, we compare the effect on steady-state per-capita income of an increase in longevity l for economies with DC and DA pension systems.

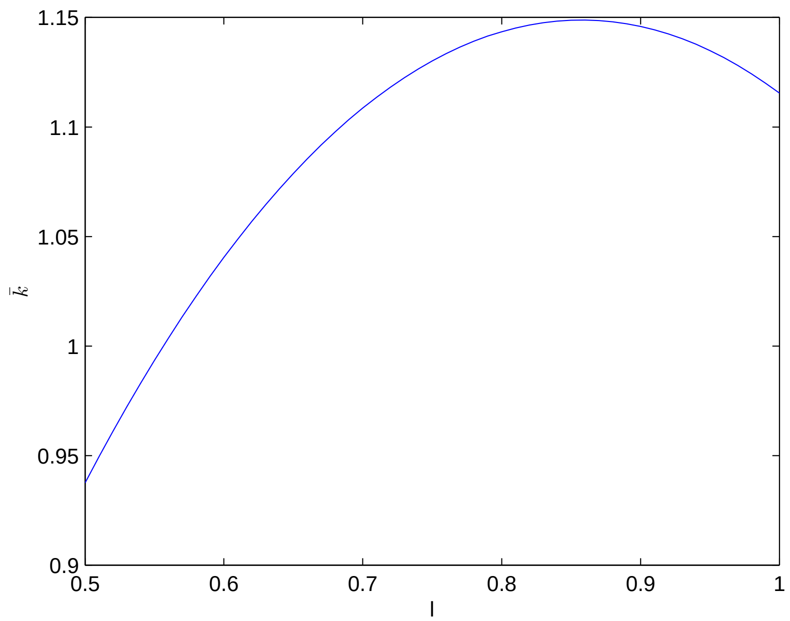


Figure 4: Steady-state capital stock per worker as a function of longevity (defined annuities)

Proposition 8 *If two economies with a stable steady state are initially identical in all respects (in particular, $al = p = (1+n)\tau$) except their pension scheme, an increase in longevity (l) always results in a lower steady-state per-capita income in the economy with a defined-annuity scheme than in an economy with a defined-contribution (or defined-benefit) scheme.*

Proof: We need to compare $\left[\frac{d\bar{k}}{dl}\right]^{DA}$ and $\left[\frac{d\bar{k}}{dl}\right]^{DC}$:

$$\left[\frac{d\bar{k}}{dl}\right]^{DA} - \left[\frac{d\bar{k}}{dl}\right]^{DC} = \frac{-\frac{a}{1+n}u''(\bar{c}) - \beta a l f'(\bar{k})u''(\bar{d})}{\left(\underbrace{1+n+s_1(\bar{w},\bar{r})f''(\bar{k})\bar{k} - s_2(\bar{w},\bar{r})f''(\bar{k})}_{>0}\right)\left(\underbrace{u''(\bar{c}) + \beta l[f'(\bar{k})]^2 u''(\bar{d})}_{<0}\right)} < 0 \tag{48}$$

■

A change in longevity does not directly alter the tax in the DC system (nor the pension in the DB system). The impact of a rise in longevity on income per capita comes from increased saving of young agents who are experiencing a longer retirement period. This in turn affects the stock of capital and hence income per capita. Equation (23) shows that the effect of increased longevity on savings is the same under DC and DB pension systems. Under defined annuities, however, the tax paid when young as well as the pension received when old are modified. Higher longevity results in higher taxes and pension benefits. This tends to decrease savings (see Equation (24)) and hence capital accumulation. As a result, an economy facing an increase in longevity ends up with a lower steady-state stock of capital and income per capita under a defined-annuity pension scheme than under a defined-contribution (or defined-benefit) pension system.

6.4 Welfare Effect of an Increase in Longevity (l)

This section studies the effect of an increase in longevity on the steady-state life-cycle utility under the DC and DA pension systems. The effect of a change in longevity on welfare in any of the two systems is ambiguous.

Proposition 9 *Assume an economy with a stable steady state which is initially dynamically efficient. A increase in longevity (l) has an ambiguous effect on the steady-state life-cycle utility for the defined-contribution (or defined-benefit) and the defined-annuity pension systems.*

Proof:

$$\frac{d\bar{U}_{DC}}{dl} = \frac{d\bar{U}_{DB}}{dl} = \beta u(\bar{d}) + \bar{k}f''(\bar{k})\left(\beta l(1+n)u'(\bar{d}) - u'(\bar{c})\right)\left[\frac{d\bar{k}}{dl}\right]^{DC} \tag{49}$$

$$\frac{d\bar{U}_{DA}}{dl} = -\frac{a}{1+n}u'(\bar{c}) + \beta\left(u(\bar{d}) + alu'(\bar{d})\right) + \bar{k}f''(\bar{k})\left(\beta l(1+n)u'(\bar{d}) - u'(\bar{c})\right)\left[\frac{d\bar{k}}{dl}\right]^{DA} \tag{50}$$

where the signs of (49) and (50) are ambiguous. ■

Even though the effect of an increase in longevity has an ambiguous effect on steady-state life-cycle utility under any of the two pension systems, we can derive an unambiguous ranking across systems in terms of steady-state welfare.

Proposition 10 *If two economies with a stable steady state are initially dynamically efficient and identical in all respects except their pension scheme (in particular, $al = p = (1+n)\tau$), an increase in longevity (l) always results in a lower (or equal) steady-state life-cycle utility in the economy with a defined-annuity pension system than in an economy with a defined-contribution (or defined-benefit) pension system.*

Proof: It is enough to compare the derivatives obtained in the proof of Proposition 9.

$$\begin{aligned}\frac{d(\bar{U}_{DA}-\bar{U}_{DC})}{dl} &= \frac{a}{1+n}\left(\overbrace{-u'(\bar{c})+\beta l(1+n)u'(\bar{d})}^{\leq 0}\right) \\ &+ \underbrace{\bar{k}f''(\bar{k})}_{<0}\left(\underbrace{-u'(\bar{c})+\beta l(1+n)u'(\bar{d})}_{\leq 0}\right)\left(\underbrace{\left[\frac{d\bar{k}}{dl}\right]^{DA}-\left[\frac{d\bar{k}}{dl}\right]^{DC}}_{<0}\right) \leq 0 \quad (51)\end{aligned}$$

■

Propositions 8 and 10 show that an economy facing an increase in longevity should never choose a defined-annuity pension system if its objective is to maximize steady-state income per capita and life-cycle utility. Indeed, defined annuities result in both a lower income per capita and at most as high a life-cycle utility as defined-benefit and defined-contribution systems. DB and DC do equally well in this case.

7 Conclusion

Declining fertility rates and rising longevity are the two causes of aging. When the pension system is PAYG there are two options: either an increase in the contribution to maintain the benefit as in a defined-annuity (or defined-benefit) scheme or a decrease in the benefit to maintain the contribution as in a defined-contribution scheme. This paper shows that the two options are not equivalent regarding their macroeconomic effects. When fertility declines and longevity increases, it is always better to shift from defined annuities to defined contribution.

In this paper, we show that the defined-contribution pension system is the system which leads to the best outcome in terms of both steady-state income per capita and welfare

in terms of	Best pension system(s) after $n \downarrow$	$l \uparrow$
$\bar{k}$ and $\bar{y}$	DC	DC and DB
$\bar{U}$	DC	DC and DB

Table 3: Ranking of pension systems

when confronted with an aging population. This is true whether aging is due to a decrease in the fertility rate and/or increased longevity (see Table 3).

References

Auerbach, Alan J., and Laurence J. Kotlikoff. 1992. "The Impact of the Demographic Transition." *Scandinavian Journal of Economics* 94 (2): 281–295.

Barr, Nicholas, and Peter Diamond. 2006. "The Economics of Pensions." *Oxford Review of Economic Policy* 22 (1): 15–39.

Bloom, David E., David Canning, and Günther Fink. 2010. "Implications of Population Ageing for Economic Growth." *Oxford Review of Economic Policy* 26 (4): 583–612.

Bös, Dieter, and Robert K. Weiszäcker. 1989. "Economic consequences of an aging population." *European Economic Review* 33 (2-3): 345–354.

Brander, James A., and Steve Dowrick. 1994. "The Role of Fertility and Population in Economic Growth." *Journal of Population Economics* 7 (1): 1–25.

Cutler, David M., James M. Poterba, Louise M. Sheiner, and Lawrence H. Summers. 1990. "An Aging Society: Opportunity or Challenge?" *Brookings Papers on Economic Activity* 1990 (1): 1–73.

de la Croix, David, and Philippe Michel. 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations.* Cambridge: Cambridge University Press.

Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55 (5): 1126–1150 (December).

EC. 2012. *2012 Ageing report.* Brussels: European Commission.

Kelley, Allen C., and Robert M. Schmidt. 1995. "Aggregate population and economic growth correlations: The role of the components of demographic change." *Demography* 32 (4): 543–555.

Li, Hongbin, Jie Zhang, and Junsen Zhang. 2007. "Effects of Longevity and Dependency

Rates on Saving and Growth: Evidence from a Panel of Cross Countries." *Journal of Development Economics* 84 (1): 138–154.

Li, Hongbin, and Junsen Zhang. 2007. "Do High Birth Rates Hamper Economic Growth?" *Review of Economics and Statistics* 89 (1): 110–117.

Munnell, Alicia H. 2006. "Employer-Sponsored Plans: The Shift from Defined Benefit to Defined Contribution." Chapter 18 of *The Oxford Handbook of Pensions and Retirement Income*, edited by Gordon L. Clark, Alicia H. Munnell, and J. Michael Orszag. Oxford University Press.

OECD. 2013. "Pensions at a Glance 2013: OECD and G20 Indicators." Technical Report, OECD Publishing.