Dynamics of pentaquarks in constituent quark models: recent developments

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Abstract. Some recent developments in the study of light and heavy pentaquarks are reviewed, mainly within constituent quark models. Emphasis is made on results obtained in the flavor-spin model where a nearly ideal octet-antidecuplet mixing is obtained. The charmed antisextet is reviewed in the context of an SU(4) classification.

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1. INTRODUCTION

The existence of pentaquarks has been discussed for more than 30 years. Light and heavy pentaquarks have alternatively been predicted and searched for. The new wave of interest in light pentaquarks was triggered by the prediction of a narrow width antidecuplet, made by Diakonov, Petrov and Polyakov [1] in the framework of the chiral soliton model, although predictions for the mass of Θ^+ have been around for nearly 20 years (see e. g. [2] and references therein).

The observation of the exotic pentaquarks Θ^+, Ξ^{--} and Θ_c^0 still remains controversial, the number of positive results being, in each case, about the same as that of null evidence. However new efforts are currently being made to confirm the previous positive results of LEPS and CLAS Collaborations for Θ^+, of NA49 Collaboration for Ξ^{--} and of H1 Collaboration for Θ_c^0 [3].

Theoretically there is a large variety of approaches to describe pentaquarks: Skyrme and chiral soliton models, large N_c studies, constituent quark models, QCD sum rules, lattice calculations, etc. (for recent reviews see for example [4, 5, 6, 7]).

Regarding the light antidecuplet the main issues are: the mass of Θ^+, the spin and parity of the antidecuplet members, the splitting between isomultiplets, the influence of the representation mixing on the masses and on the strong decay widths, etc.

After discussing the light antidecuplet main issues, the charmed antisextet is shortly reviewed in the context of an SU(4) classification.

2. CONSTITUENT QUARK MODELS

Constituent quark models describe a large variety of observables in baryon spectroscopy. It seems thus natural to inquire about their applicability to exotic hadrons. Any constituent quark model Hamiltonian has a spin-independent part (free mass term + kinetic energy + confinement) and a short-range hyperfine interaction. The most common constituent quark models used in pentaquark physics have either a color-spin (CS) or a flavor-spin (FS) interaction. There are also studies in the so-called hybrid models, which contain a superposition of CS and FS interactions [8]. Attempts to describe Θ^+ by using an instanton induced interaction have also been made [9].

In the following we shall present results in the FS model and compare them with corresponding results from other models.

3. THE LIGHT ANTIDECUPLET

In $SU(3)_F$ $q^4\bar{q}$ multiplets can be obtained from the direct product of four quarks and an antiquark irreducible representation as

$$
\begin{aligned}
3_F \times 3_F \times 3_F \times 3_F \times \bar{3}_F &= 3(1_F) + 8(8_F) + 3(27_F) + 4(10_F) \\
&+ 2(\overline{10}_F) + 35_F.
\end{aligned}
$$

which shows that the antidecuplet $\overline{10}_F$ is one of the possible multiplets. The $SU(3)_F$ breaking induces representation mixing. One expects an important mixing between octet members and antidecuplet members with the same quantum numbers. This will be discussed below.

3.1. Parity and Spin

The parity and spin can be found by looking first at a q^4 subsystem to which an antiquark is subsequently coupled.

In the FS model the lowest negative parity state of a $q^4\bar{q}$ system with total spin $S = 1/2$ results from a q^4 subsystem which has the structure $|[4]_O[211]_C[211]_{OC}; [211]_F[22]_S[31]_{FS}\rangle$, where O, C, F and S stand for orbital, color, flavor and spin degrees of freedom. The symmetry $[4]_O$ implies that there is no orbital excitation and the parity of the pentaquark is negative, i. e. the same as the intrinsic parity of the antiquark. But if one quark is excited to the p-shell the parity becomes positive and the lowest symmetry allowed for the orbital part of the wave function is $[31]_O$. Then the Pauli principle requires the q^4 subsystem to have the structure $|[31]_O[211]_C[1111]_{OC}; [22]_F[22]_S[4]_{FS}\rangle$ in its lowest state, which has $I = 0$ and $S = 0$. Although this state contains one unit of orbital excitation the attraction brought by the FS interaction is so strong that it overcomes the excess of kinetic energy and generates a positive parity state below the negative parity one [10]. After coupling q^4 to $\bar{q}$ the total angular momentum is $1/2$ or

3/2. Calculations based on the realistic FS Hamiltonian of Ref.[11], have been performed for Θ^+ in Ref. [12] and for the whole antidecuplet in Ref. [13]. Similar variational calculations were made earlier for heavy pentaquarks. The positive parity pentaquarks turned out to be lighter by several hundreds MeV [10] than the negative parity ones with the same quark content [14].

Recently, more involved calculations for Θ^+, performed in the framework of a semi-relativistic version of the FS model proved once more that in the FS model the lowest state has positive parity [15].

Based on semi-schematic estimates, in Ref. [16] it was claimed that in the CS model the lowest state for Θ^+ has also positive parity. As above, this implies an excess of kinetic energy due to an extra unit of orbital angular momentum. Then, according to the Pauli principle, the lowest symmetry of the wave function in the relevant degrees of freedom is $[31]_{CS}$. This symmetry brings less attraction than $[4]_{FS}$ in the FS model, which is insufficient to overcome the excess of kinetic energy. The realistic study of Ref. [15] proves that this is the case, so that in the CS model the lowest resonant state has $J^P = 3/2^-$. The $J^P = 1/2^-$ state is even lower, but in the continuum. The same study shows that the hybrid models favor negative parity, in agreement with Ref. [8].

3.2. The Antidecuplet Mass Spectrum in the FS Model

There are several reasons to study pentaquarks in the FS model. This model reproduces the correct sequence of positive and negative parity levels in the low-energy spectra of nonstrange and strange baryons. In addition it is supported by lattice calculations [17] and the flavor-spin symmetry is consistent with the large N_c limit of QCD [18].

The results for the mass spectrum of $\overline{10}_F$ pentaquarks based on the Graz parametrization of Ref. [11] are shown in Fig. 1a. Here, as in any other model including the chiral soliton, one cannot determine the absolute mass of Θ^+. This mass has been fitted to the presently accepted experimental value of 1540 MeV. Reasons to accommodate such a value are given in Ref. [12]. The pure $\overline{10}_F$ spectrum, Fig. 1a, can approximately be described by the linear mass formula $M \simeq 1829 - 145\,Y$ where Y is the hypercharge. The FS model result is quite close to the presently estimated level spacing in the chiral soliton model [19] where the parameters were allowed to vary considerably for well justified reasons. In the CS model the level spacing is much smaller. To a good approximation one has $M \simeq M_0 - 58\,Y$ [20], where M_0 can be fixed by the mass of Θ^+.

To construct all the antidecuplet members the masses of the following systems have been calculated in the Graz parametrization [11]: $M(uudd\overline{d}) = 1452$ MeV, $M(uudd\overline{s}) \equiv M(\Theta^+) = 1540$ MeV, $M(uuds\overline{d}) = 1723$ MeV, $M(uuds\overline{s}) = 1800$ MeV, $M(ddss\overline{u}) \equiv M(\Xi^{--}) = 1962$ MeV and $M(uuss\overline{s}) = 2042$ MeV. The antidecuplet members with $Y = 1$ and $Y = 0$ were obtained according to their wave

functions (see Ref. [13]) as

$$M(N_{\overline{10}}) = \frac{1}{3}M(uudd\bar{d}) + \frac{2}{3}M(uuds\bar{s}) = 1684 \text{ MeV},$$

$$M(\Sigma_{\overline{10}}) = \frac{2}{3}M(uuds\bar{d}) + \frac{1}{3}M(uuss\bar{s}) = 1829 \text{ MeV}. \tag{1}$$

The octet members with $Y = 1$ and $Y = 0$ were obtained in a similar way

$$M(N_8) = \frac{2}{3}M(uudd\bar{d}) + \frac{1}{3}M(uuds\bar{s}) = 1568 \text{ MeV},$$

$$M(\Sigma_8) = \frac{1}{3}M(uuds\bar{d}) + \frac{2}{3}M(uuss\bar{s}) = 1936 \text{ MeV}. \tag{2}$$

3.3. Representation Mixing in the FS Model

As a consequence of the $SU(3)_F$ breaking the representations $\overline{10}_F$ and 8_F mix. The existing data require mixing.

There are some phenomenological studies where, by fitting the mass and the width of known resonances, one can obtain the mixing angle between the antidecuplet and one (or more) octets. In such studies the number of quarks and antiquarks is arbitrary in every baryon.

In phenomenological models where one assumes mixing between states having $J^P = 1/2^+$, it turns out that the selected masses require a large mixing angle [27, 28] and the widths a small mixing angle [28]. A compromise was found when the antidecuplet and octet states which mix had $J^P = 3/2^-$, in which case a large mixing angle consistent with both masses and widths was obtained [29]. As mentioned above, the $J^P = 3/2^-$ state is the lowest resonant Θ^+ state in the CS model [15]. Its negative parity corresponds to an $\ell = 2$ relative partial wave which can produce a rather large centrifugal barrier, thus a small width.

The mixing of the antidecuplet with three octets with $J^P = 1/2^+$ has also recently been investigated phenomenologically in the chiral soliton model where it was assumed that the mixing is small [30]. It was found that this can reduce the size of the widths of the antidecuplet members without much affecting the masses. However, another chiral soliton study [31], based on an "exact" treatment (not only the first order) of $SU(3)_F$ breaking advocates large $8 + \overline{10}$ mixing from the mass analysis. Thus the representation mixing seems to remain a controversial problem in the chiral soliton model.

The mixing takes place between octet and antidecuplet members with the same quantum numbers, i.e. for $Y = 1$, $I = 1/2$ and $Y = 0$, $I = 1$ states. Here we suppose that there is mixing with the lowest pentaquark octet only. Then there is only one mixing angle, introduced by the physical states, defined as

$$|N^*\rangle = |N_8\rangle \cos\theta_N - |N_{\overline{10}}\rangle \sin\theta_N,$$

$$|N_5\rangle = |N_8\rangle \sin\theta_N + |N_{\overline{10}}\rangle \cos\theta_N, \tag{3}$$

| | a) pure antidecuplet | | b) mixed with the octet |

$$\Xi^{--} \quad \text{——————} \quad 1962 \qquad\qquad \Xi^{--} \quad \text{——————} \quad 1962$$

$$\Sigma_{\overline{10}} \quad \text{——————} \quad 1829$$

$$N_5 \quad \text{——————} \quad 1801$$

$$\Sigma_5 \quad \text{——————} \quad 1719$$

$$N_{\overline{10}} \quad \text{——————} \quad 1684$$

$$\Theta^+ \quad \text{——————} \quad 1540 \qquad\qquad \Theta^+ \quad \text{——————} \quad 1540$$

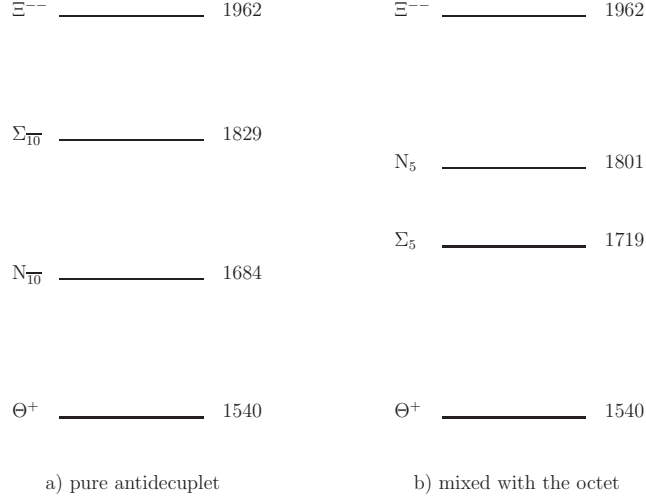a) pure antidecuplet   b) mixed with the octet

FIGURE 1. The pentaquark antidecuplet masses (MeV) in the FS model: (a) pure antidecuplet and (b) after mixing with the pentaquark octet.

for N and similarly for Σ.

In the FS model the mixing angles θ_N and θ_Σ were calculated dynamically in Ref. [13]. The states which mix are all $q^4\bar{q}$ states, i. e. the number of quarks and antiquarks is fixed, contrary to the above phenomenological studies or to the spirit of the chiral soliton model. The mixing is determined by the the coupling matrix element V of $\overline{10}_F$ and 8_F states. This has contributions from every part of the Hamiltonian which breaks $SU(3)_F$ symmetry: the free mass term, the kinetic energy and the hyperfine interaction. It reads

$$V = \begin{cases} \frac{2\sqrt{2}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3}\left[S(uuds\bar{s}) - S(uudd\bar{d})\right] = 166 \text{ MeV} & \text{for N} \\[2mm] \frac{2\sqrt{2}}{3}(m_s - m_u) + \frac{\sqrt{2}}{3}\left[S(uuss\bar{s}) - S(uuds\bar{d})\right] = 155 \text{ MeV} & \text{for } \Sigma \end{cases} \qquad (4)$$

where the first term is the free mass term which alone generates an ideal mixing and S represents the combined contribution of the kinetic energy and hyperfine interaction

$$S = \langle T \rangle + \langle V_\chi \rangle . \qquad (5)$$

The expressions (4) result from the wave functions of N and Σ respectively and reflect their quark content. One can see that the numerical values of V are similar for N and Σ.

The mixing angle derived from the definitions (3) is

$$\tan 2\theta_N = \frac{2V}{M(N_{\overline{10}}) - M(N_8)} \qquad (6)$$

and similarly for Σ. The resulting numerical values are $\theta_N = 35.34^0$ and $\theta_\Sigma = -35.48^0$. Each value is very close to the ideal mixing angle $\theta_N^{ideal} = 35.26^0$ and

$\theta_\Sigma^{ideal} = -35.26^0$ respectively. This implies that the "mainly antidecuplet" state N_5 is 67 % antidecuplet and 33 % octet, which represents a large mixture. The content of the "mainly octet" state N^* is the other way round, i. e. 67 % octet and 33 % antidecuplet. Then, for example, for positive charge pentaquarks with $Y = 1, I = 1/2$ one has

$$\begin{aligned} |N^*\rangle &\simeq \frac{1}{2} \mid (ud - du)(ud - du)\bar{d}\,\rangle, \\ |N_5\rangle &\simeq \frac{1}{2\sqrt{2}} \mid [(ud - du)(us - su) + (us - su)(ud - du)]\bar{s}\,\rangle, \end{aligned} \tag{7}$$

i. e. the "mainly octet" state has no strangeness and the "mainly antidecuplet" state contains the whole available amount of (hidden) strangeness. The physical masses, obtained from the diagonalization of a 2×2 matrix, are

$$\begin{aligned} M(N_5) &= M(N_{\overline{10}}) + V\tan\theta_N = 1801 \text{ MeV}, \\ M(N^*) &= M(N_8) - V\tan\theta_N = 1451 \text{ MeV}. \end{aligned} \tag{8}$$

In the Σ sector the situation is opposite. The mixing angle, $\theta_\Sigma = -35.48^0$ ($\sin\theta_\Sigma \simeq -1/\sqrt{3}, \quad \cos\theta_\Sigma \simeq \sqrt{2/3}$) minimizes the number of strange quarks (antiquarks) in Σ_5 and maximizes it in Σ^*. This can be readily seen from the analogues of Eqs. 3 which give

$$\begin{aligned} |\Sigma_5\rangle &\simeq \frac{1}{2\sqrt{2}} \mid [(ud - du)(us - su) + (us - su)(ud - du)]\bar{d}\,\rangle, \\ |\Sigma^*\rangle &\simeq \frac{1}{2} \mid (us - su)(us - su)\bar{s}\,\rangle, \end{aligned} \tag{9}$$

so that

$$\begin{aligned} M(\Sigma_5) &= M(\Sigma_{\overline{10}}) + V\tan\theta_\Sigma = 1719 \text{ MeV}, \\ M(\Sigma^*) &= M(\Sigma_8) - V\tan\theta_\Sigma = 2046 \text{ MeV}. \end{aligned} \tag{10}$$

i. e. $M(\Sigma_5) < M(\Sigma^*)$. As a consequence, the order of N and Σ is interchanged by the mixing, as illustrated in Fig. 1. The N_5 state is 70 Mev higher than the option for a 1730 MeV resonance in the new analysis of Ref. [21]. The Σ_5 is about 30 MeV far off the upper end of the experimental mass range 1630 - 1690 MeV of the three star $\Sigma(1660)$ resonance and 10 MeV below the lowest experimental edge of the one star $\Sigma(1770)$ resonance (see the PDG [22] full listings).

The N^* state is located in the Roper resonance mass region 1430 - 1470 MeV. However this is a $q^4\bar{q}$ state, i. e. different from the q^3 radially excited state obtained in Ref. [11] at 1493 MeV. A mixing of the q^3 and the $q^4\bar{q}$ states could possibly be a better description of the reality.

3.4. The Decay Width

If the pentaquark Θ^+ exists, its width is expected to be small [1]. The positive experiments have reported upper limits. In particular the LEPS Collaboration at

SPring-8, which reported the first observation of Θ^+ [23], gives $\Gamma < 25$ MeV. Some recent analysis of the K^+N scattering gives an even smaller limit $\Gamma < 1$ MeV [24].

In quark models an option to reduce the width is to introduce rearrangements, as for example, diquark correlations in the orbital and/or color-flavor-spin spaces.

In Ref. [25] Θ^+ was described as a bound state $J^P = 1/2^+$ of two extended ud diquarks and an antiquark $\bar{s}$. The size parameters of the wave function were varied and a decay width of about 1 MeV was obtained for an asymmetric "peanut" structure with $\bar{s}$ in the center and the diquarks rotating around it. However, there is no dynamics justifying the range of values of the size parameters.

In Ref. [26] both parities were considered. But the width of the lowest positive parity state was expected to be smaller than that of the lowest negative parity state by a factor of 50 due to the centrifugal barrier. It was also suggested that the width can be lowered down by adequately reducing the coupling constant $g_{KN\Theta}$ as compared to $g_{\pi NN}$, due to large N_c arguments. However the estimates of the widths have been made in a limit where the emitted meson is a point-like particle, like for ordinary baryon decay. It is hoped that more dynamical studies will better settle the width issue in the future.

4. HEAVY PENTAQURKS

In most models which accommodate Θ^+ and its antidecuplet partners, heavy pentaquarks $q^4\bar{Q}$ ($\bar{Q} = \bar{c}$ or $\bar{b}$) can be accommodated as well. From general arguments they are expected to be more stable against strong decays than the light pentaquarks [32]. In an SU(4) classification, including the charm C, in Ref. [33] it has been shown that $\overline{\mathbf{10}} + \mathbf{8}$ discussed above and having $C = 0$ and a charm antisextet $\bar{\mathbf{6}}$ with $C = $ - 1, belong to the same SU(4) irreducible representation of dimension **60**. This implies that Θ_c^0, the $I = 0$ member of this antisextet, is obtained from Θ^+ by replacing $\bar{s}$ by $\bar{c}$. The other SU(3) representations belonging to **60** of SU(4) are $\overline{\mathbf{15}} + \mathbf{6}$ with $C = 1$ and **15** with $C = 2$. Although SU(4) is badly broken, such a classification may be as useful as that of ordinary baryons [34].

TABLE 1. Masses (MeV) of the positive parity antisextet charmed and beauty pentaquarks.

Pentaquark	I	Content	FS model Ref. [10]	CS model Ref. [35]
Θ_c^0	0	u u d d $\bar{c}$	2902	2835 ± 30
N_c^0	1/2	u u d s $\bar{c}$	3161	
Ξ_c^0	1	u u s s $\bar{c}$	3403	
Θ_b^+	0	u u d d $\bar{b}$	6176	6180 ± 30
N_b^+	1/2	u u d s $\bar{b}$	6442	
Ξ_b^+	1	u u s s $\bar{b}$	6683	

The masses of the charmed antisextet calculated in the FS model [10] are presented in Table 1. [1]

For completeness, in Table 1, the masses of a beauty antisextet, calculated in the FS model [10], are presented as well. The results are consistent with the heavy quark limit. They are compared to the available estimates of Ref. [35] in the CS model. Close similarity is observed.

The experimental observation of charmed pentaquarks is contradictory so far. While the H1 collaboration [36] confirmed evidence for a narrow resonance at about 3100 MeV [3], there is still null evidence from the CDF collaboration [3].

For an orientation, it is interesting to calculate excited charmed pentaquarks Θ_c^{0*}. In the FS model the first excited state having $I = 1$ and $S = 1/2$ is located 200 Mev above Θ_c^0 which has $I = 0$. This value is close to the mass observed by the H1 collaboration [36]. In addition it supports the large spacing result obtained approximately in Ref. [35] in the FS model.

5. CONCLUSIONS

The recent research activity on pentaquarks could bring a substantial progress in understanding the baryon structure. It shows that the N and Σ partners of Θ^+ lie in the midst of low-lying positive parity baryonic states. Thus it suggests that the simple description of a baryon resonance as a q^3 configuration is insufficient. The addition of higher Fock components to the nucleon wave function may perhaps help to improve the description of strong decays of baryons.

Furthermore, in light hadrons it is necessary to clarify the role of the spontaneous breaking of chiral symmetry, the basic feature of the chiral soliton model [1] which motivated this new wave of interest in pentaquarks. Recent lattice calculations [17] suggest that the order reversal of the Roper and the negative parity $S_{11}(1535)$ resonance, compared to heavy quark systems, is caused by the flavor-spin interaction and conclude that the spontaneous breaking of chiral symmetry dictates the dynamics of light quarks.

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[1] Actually in Ref. [10], instead of masses, binding energies were presented as $\Delta E = M(pentaquark) - E_T$ where $E_T = M_{baryon} + \overline{M}_{meson}$ is the threshold energy involving the average mass $\overline{M}_{meson} = (M + 3M^*)/4$, with M the pseudoscalar and M^* the vector meson mass respectively. In Table 1 the absolute value $M(pentaquark)$ is indicated. The lowest physical threshold is $N + D = 2808$ MeV for Θ_c^0 and $N + B = 6219$ MeV for Θ_b^+.

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