Large N_c baryons and Regge trajectories

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Abstract

The mixed symmetric positive and negative parity baryons are described in a similar way in the $1/N_c$ expansion method of QCD by using a procedure where the permutation symmetry is incorporated exactly. This allows to express the mass formula in terms of a small number of linearly independent operators. We show that the leading term follows a different Regge trajectory from that found for symmetric states, when plotted as a function of the band number N.

1 Introduction

The large N_c or alternatively the $1/N_c$ expansion method of QCD [1] became a valuable and systematic tool to study baryon properties in terms of the parameter $1/N_c$ where N_c is the number of colors. According to Witten's intuitive picture [2], a baryon containing N_c quarks is seen as a bound state in an average self-consistent potential of a Hartree type and the corrections to the Hartree approximation are of order $1/N_c$. Also, it has been shown that QCD has an exact contracted $\mathrm{SU}(2N_f)_c$ symmetry when $N_c \to \infty$, N_f being the number of flavors [3, 4]. For ground state baryons the $\mathrm{SU}(2N_f)$ symmetry is broken by corrections proportional to $1/N_c$ [5, 6].

For excited states the symmetry has to be extended to $\mathrm{SU}(2N_f) \times \mathrm{O}(3)$. In the spirit of the Hartree approximation a procedure for constructing large N_c baryon wave functions with mixed symmetric spin-flavor parts has been proposed [7] and an operator analysis was performed for $\ell = 1$ baryons [8]. It was proven that, for such states, the $\mathrm{SU}(2N_f)$ breaking occurs at order N_c^0, instead of $1/N_c$, as for the ground and symmetric excited states [9, 10].

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The procedure has been extended to positive parity nonstrange baryons belonging to the $[70, \ell^+]$ with $\ell = 0$ and 2 [11].

More recently the $[70, 1^-]$ multiplet was reanalyzed by using an exact wave function, instead of the Hartree-type wave function, with the Pauli principle satisfied at any stage of the calculations [12]. The novelty was that the isospin-isospin term, neglected previously [8] becomes as dominant in Δ resonances as the spin-spin term in N^* resonances.

In the present work we follow the approach of Ref. [12] both for positive and negative parity mixed symmetric states and compare the leading mass term with that of symmetric states. We show that in each case it follows a distinct Regge trajectory as a function of the band number.

Evidence for Regge trajectories in large N_c QCD is of current interest for mesons and glueballs as well, as shown, for example, in Ref. [13].

2 The mass operator

The most general form of the mass operator is [14]

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \tag{1}$$

The formula contains two types of terms. In the first category are the operators O_i, which are invariant under SU(N_f) and are defined as

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{2}$$

where $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2)-spin. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms. The rank $k = 2$ tensor operator of SO(3) is

$$L^{(2)ij} = \frac{1}{2}\left\{L^i, L^j\right\} - \frac{1}{3}\delta_{i,-j}\vec{L}\cdot\vec{L}, \tag{3}$$

which acts on the orbital wave function $|\ell m_\ell\rangle$ of the whole system of N_c quarks. The second category are the operators B_i which are SU(3) breaking and are defined to have zero expectation values for non-strange baryons.

3 Symmetric states

If an excited baryon belongs to a symmetric [**56**]-plet the three-quark system can be treated similarly to the ground state in the flavour-spin degrees of freedom, but one has to take into account the presence of an orbital excitation in the space part of the wave function [9, 10]. As an example, in Table 1 we reproduce the results of Ref. [10] for $[\mathbf{56}, 4^+]$ where $\chi^2_{\rm dof} = 0.26$. One can see that the number of dominant operators turns out to be very small.

Table 1: List of dominant operators and their coefficients in the mass formula (1) for the multiplet $[\mathbf{56}, 4^+]$ (from Ref. [10]).

Operator	Fitted coef. (MeV)				
$O_1 = N_c \mathbb{1}$	$c_1 =$	736	$\pm$	30
$O_2 = \frac{1}{N_c} L^i S^i$	$c_2 =$	4	$\pm$	40
$O_3 = \frac{1}{N_c} S^i S^i$	$c_3 =$	135	$\pm$	90
$B_1 = -\mathcal{S}$	$d_1 =$	110	$\pm$	67

The first operator is a spin-flavor singlet of order $\mathcal{O}(N_c)$. This is the leading operator in the mass formula, needed for obtaining the Regge trajectories below. As compared to the ground state, there is one more operator needed for excited symmetric states. This is the spin-orbit operator O_2. Note that in the case of symmetric states this is order $\mathcal{O}(1/N_c)$. For a symmetric spin-flavor state the matrix elements of the spin operator O_3 are identical to those of the flavor operator defined as $\frac{1}{N_c} T^a T^a$. As we shall see below, this is not the case for mixed symmetric states. The operator B_1 is defined as the negative of the strangeness $\mathcal{S}$.

4 Mixed symmetric states

There are two ways of studying mixed symmetric $[\mathbf{70}]$-plets. The standard one is inspired by the Hartree approximation [7] where an excited baryon is described by a symmetric core plus an excited quark, see *e.g.* [8, 11, 15, 16].

As an alternative, in Ref. [12] we have proposed a method where all identical quarks are treated on the same footing and we deal with an exact wave function in the orbital-flavor-spin space. The procedure has been successfully applied to the $N = 1$, 2 and 3 bands [17, 18, 19, 20]. In Table 2 we illustrate it by the results obtained in Ref. [19] for the mixed symmetric states $[\mathbf{70}, \ell^+]$ with $\ell = 0$, 2 of the $N = 2$ band. The leading operator O_1 is the same as above. On the other hand we identify the spin-orbit operator O_2 with the the single-particle operator

$$\ell \cdot s = \sum_{i=1}^{N_c} \ell(i) \cdot s(i), \tag{4}$$

the matrix elements of which are of order N_c^0. The analytic expression of the matrix elements of O_2 can be found in the Appendix A of Ref. [8]. Similarly, we ignore the two-body part of the spin-orbit operator as being of a lower order. The spin operator O_3 and the flavor operator O_4 are two-body and linearly independent. The expectation value of O_3 is $\frac{1}{N_c} S(S+1)$ where S is the spin of the entire system of N_c quarks. The expression of the operator O_4 given in Table 2 is consistent with the usual $1/N_c(T^a T^a)$ definition in

SU(4). In extending it to SU(6) we had to subtract the quantity $(N_c+6)/12$ as explained in Ref. [17].

By construction, the operators O_5 and O_6 have non-vanishing contributions for orbitally excited states only. They are also two-body, which means that they carry a factor $1/N_c$ in the definition. The operator O_6 contains the irreducible spherical tensor (3) and the SU(6) generator G^{ja} both acting on the whole system. The latter is a coherent operator which introduces an extra power N_c so that the order of the matrix elements of O_6 is $\mathcal{O}(1)$.

Table 2 gives three distinct numerical fits which suggest that O_5 is not so important but O_6 is crucial in obtaining a satisfactory $\chi^2_{\rm dof}$. The Fit 2 is used in Fig. 1.

Table 2: List of dominant operators and the corresponding coefficients, c_i or d_i, in the mass formula (1) obtained in three distinct numerical fits for $[\mathbf{70}, \ell^+]$ with $\ell = 0$, 2 [19].

Operator	Fit 1	Fit 2	Fit 3
$O_1 = N_c \mathbb{1}$	616 ± 11	616 ± 11	616 ± 11
$O_2 = \ell^i s^i$	150 ± 239	52 ± 44	243 ± 237
$O_3 = \frac{1}{N_c} S^i S^i$	149 ± 30	152 ± 29	136 ± 29
$O_4 = \frac{1}{N_c}\left[T^a T^a - \frac{1}{12} N_c(N_c+6)\right]$	66 ± 55	57 ± 51	86 ± 55
$O_5 = \frac{3}{N_c} L^i T^a G^i$	-22 ± 5		-25 ± 52
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	14 ± 5	14 ± 5	
$B_1 = -\mathcal{S}$	23 ± 38	24 ± 38	-22 ± 35
$\chi^2_{\rm dof}$	0.61	0.52	2.27

5 Regge trajectories

The linear Regge trajectories are a manifestation of the nonperturbative aspect of QCD dynamics, which at long distance becomes dominated by the confinement [21]. In our previous studies we have tried to establish a connection between the $1/N_c$ method and a simple semi-relativistic quark model with a Y-junction confinement potential plus a hyperfine interaction generated by one gluon exchange [22, 23]. We showed that the band number N emerged naturally from both approaches so that one can plot the coefficients c_i as a function of N. Also we found that c_1 contains the effect of kinetic energy and the confinement.

Presently, we have a consistent description of mixed symmetric positive and negative parity states corresponding to $N = 1$, 2 and 3 bands. It is interesting to revisit the Regge trajectory problem [22, 23]. In Fig. 1 we plot c_1^2 as a function of the band number N for $N \leq 4$. One can see that two distinct trajectories emerge from this new picture, one for symmetric [**56**]-plets, the other for mixed symmetric [**70**]-plets. This behavior is different

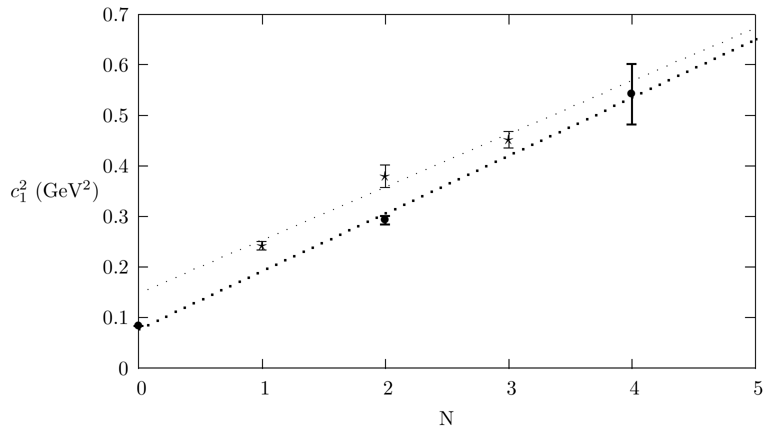

Figure 1: The coefficient c_1^2 (GeV^2) as a function of the band number N. The numerical values of c_1 were taken from Ref. [22] for $N = 0$, from Ref. [18] Fit 3 for $N = 1$, from Ref. [9] for $N = 2$ $[\mathbf{56}, 2^+]$, from Ref. [19] Fit 2 for $N = 2$ $[\mathbf{70}, \ell^+]$ ($\ell = 0$,2), from Ref. [20] Fit 3 for $N = 3$ $[\mathbf{70}, \ell^-]$ ($\ell = 1$,2,3), from Ref. [10] for $N = 4$ $[\mathbf{56}, 4^+]$. The heavy dots refer to [56]-plets and the stars to [70]-plets. The best fit of these data was obtained with two distinct linear trajectories.

from that found in Refs. [22, 23] but reminds that of Ref. [24] where the symmetric and mixed symmetric states have distinct trajectories for $(N_c c_1)^2$ as a function of the angular momentum $\ell \leq 6$ (Chew-Frautschi plots). Note that in Ref. [24] the mixed symmetric states were described within the ground state core + excited quark approach. The mass operator was reduced to the $\mathcal{O}(N_c)$ spin-flavor singlet, the $\mathcal{O}(1/N_c)$ hyperfine spin-spin interaction, acting between core quarks only, and SU(3) breaking terms. There are no $\mathcal{O}(N_c^0)$ contributions. For a consistent treatment, in Ref. [24] the hyperfine interaction was restricted to core quarks in symmetric states as well.

In our case, the symmetric and mixed symmetric states are treated on an equal basis: there is no distinction between the core and an excited quark (the core may be excited as well), and the Pauli principle is always fulfilled. The existence of two distinct Regge trajectories, one for symmetric, another for mixed symmetric states, may be due to their distinct structure in the orbital-spin-flavor space.

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