Figure 1: David Brink in his office at Balliol College, Oxford, 1992

On the existence of heavy tetraquarks

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In honour of David Brink

Abstract

Previous work done in collaboration with David Brink is reviewed in the light of the recent observation of new charmonium-like resonances which can be interpreted as tetraquarks. In the framework of a schematic quark model the spectrum of $c\bar{c}q\bar{q}$ tetraquarks is presented.

1 Introduction

The discovery by Belle [1] of the anomalously narrow $X(3872)$ meson and of other hidden-charm states, in particular $X(3940)$ [2] and $Y(4260)$ [3], revitalized the experimental and theoretical interest in heavy quarkonium. The observation of $X(3872)$ has been confirmed by CDF [4], DO [5] and Babar [6]. The difficulty of interpreting these resonances as charmonium states are explained, for example, in Ref. [7]. As alternatives, these resonances might be interpreted as tetraquarks, meson-meson molecules, hybrids, glueballs, etc. Here we shall discuss the tetraquark option.

The study of multiquark systems, as tetraquarks or hexaquarks has been initiated by Jaffe [8] in the MIT bag model. Later on it has been extended to potential models and applied to light scalar mesons $a(980)$ and $f(980)$ [9, 10]. Subsequently heavy tetraquarks have been considered in search for stability [11, 12]. Flux tube models [13, 14] suggested instability.

In a previous work we studied a light tetraquark system of identical particles in a standard nonrelativistic potential model with colour confinement and hyperfine chromomagnetic interaction [15]. Our result showed explicitly the role of hidden colour states (see below).

Subsequently we have studied the stability of a system of two light quarks $q = u, d$ and two heavy antiquarks $Q = c, b$ [16]. Taking $Q = b$ we performed a simple variational calculation with results comparable to other studies [12]. We focussed on the $qq\bar{Q}\bar{Q}$ system because it has more of a chance to be bound than $q\bar{q}Q\bar{Q}$, the latter having a lower threshold. The argument was based on the following inequality [17]

$$m_{Q\overline{Q}} + m_{q\bar{q}} \leq 2m_{Q\bar{q}} , \tag{1}$$

valid for any value of the heavy-to-light mass ratio and where $m_{Q\bar{q}} = m_{q\bar{Q}}$. This means that $(Q\overline{Q}) + (q\bar{q})$ is a lower threshold than $2\,(Q\bar{q})$. In addition the $qq\bar{Q}\bar{Q}$ system is free of quark-antiquark annihilation processes which in $q\bar{q}Q\bar{Q}$ cannot be avoided.

The open charm $cc\bar{q}\bar{q}$ tetraquark with spin $S = 1$ and isospin $I = 0$ appears unbound in Ref. [12], where the four-body problem was solved by an expansion in a harmonic-oscillator basis up to $N = 8$ quanta. However, in the recent study of Janc and Rosina [18] it is bound. This work, where a more sophisticated variational basis was used, has been inspired by the considerations made in Ref. [16] to include all possible channels which accelerate the convergence and in particular the meson–meson channels.

In the above calculations the potential of Bhaduri et al. [19] has been used. The hyperfine interaction is of chromomagnetic type. The parameters include constituent quark masses, the string tension of a linear confinement, the strength of the Coulomb interaction, and the strength and size parameter of the hyperfine interaction which is a smeared contact term. They were fitted over a wide range of mesons and baryons.

In this paper we use the classification of tetraquark states as given in Refs. [15, 16] and a simple quark model to calculate the spectrum of $c\bar{c}q\bar{q}$ tetraquarks and we discuss it in the light of recent data.

2 The basis states

Here we suppose that particles 1 and 2 are quarks and particles 3 and 4 antiqurks, see Fig. 2. Below we define the basis states in the orbital, colour and spin space taking into account the Pauli principle. The flavour space is trivial if we restrict to SU(2). The total wave function of a tetraquark is a linear combination of products of orbital, spin, flavour and colour parts.

2.1 The orbital part

There are at least three possible ways to define the relative coordinates. The three relevant possibilities for our problem are shown in Fig. 2. In the cases $(a), (b)$ and (c) the internal coordinates are

$$\vec{\sigma} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}), \quad \vec{\sigma'} = \frac{1}{\sqrt{2}}(\vec{r_3} - \vec{r_4}), \quad \vec{\lambda} = \frac{1}{2}(\vec{r_1} + \vec{r_2} - \vec{r_3} - \vec{r_4}), \tag{2}$$

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_3}), \quad \vec{\rho'} = \frac{1}{\sqrt{2}}(\vec{r_2} - \vec{r_4}), \quad \vec{x} = \frac{1}{2}(\vec{r_1} - \vec{r_2} + \vec{r_3} - \vec{r_4}), \tag{3}$$

$$\vec{\alpha} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_4}), \quad \vec{\alpha'} = \frac{1}{\sqrt{2}}(\vec{r_2} - \vec{r_3}), \quad \vec{x} = \frac{1}{2}(\vec{r_1} - \vec{r_2} - \vec{r_3} + \vec{r_4}). \tag{4}$$

The first system of coordinates is convenient when the quarks or antiquarks are correlated to form diquarks, as in the diquark-antidiquark model. The coordinates (2) and (3) are useful in describing the direct and exchange meson-meson channels *i. e.* the strong decays. One should use the system which is more appropriate for a given problem. But in specific calculations one can pass from one coordinate system to the other by orthogonal transformations. In the diquark-antidiquark picture the radial wave function can be written as a function of 6 variables $R(\sigma^2, \sigma'^2, \lambda^2, \vec{\sigma} \cdot \vec{\sigma'}, \vec{\sigma} \cdot \vec{\lambda}, \vec{\sigma'} \cdot \vec{\lambda})$ but it can also be expressed in terms of coordinates (3) or (4). All channels which accelerate convergence should be included. The non-orthogonality between different R functions can be handled without problems [15, 16].

2.2 The colour part

In the colour space one can construct a colour singlet tetraquark state using intermediate couplings associated to the three coordinate systems defined above. In this way one obtains three distinct bases. These are

$$|\bar{3}_{12}3_{34}\rangle, \quad |6_{12}\bar{6}_{34}\rangle, \tag{5}$$

$$|1_{13}1_{24}\rangle, \quad |8_{13}8_{24}\rangle, \tag{6}$$

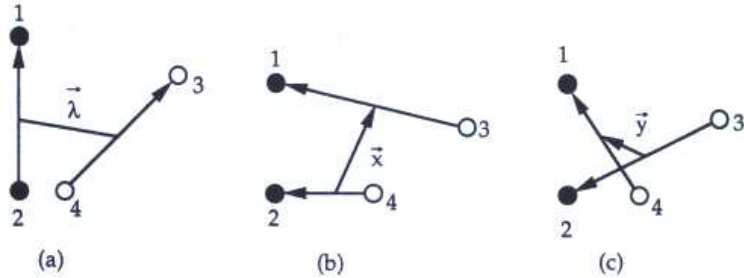$$|1_{14}1_{23}\rangle, \quad |8_{14}8_{23}\rangle. \tag{7}$$

Figure 2: Three independent relative coordinate systems. Solid and open circles represent quarks and antiquarks respectively: (a) diquark-antidiquark channel, (b) direct meson-meson channel, (c) exchange meson-meson channel.

The 3 and $\bar{3}$ are antisymmetric and 6 and $\bar{6}$ are symmetric under interchange of quarks or antiquarks. Each set, (6) or (7), contains a singlet-singlet colour and an octet-octet colour state. The amplitude of the latter vanishes asymptotically, when the mesons, into which a tetraquark decays, separate. These are called *hidden colour* states by analogy to states which appearbin the nucleon-nucleon problem, defined as a six-quark system [20]. The contribution of hidden colour states to the binding energy of light tetraquarks has been calculated explicitly in Ref. [15]. Below we shall point out their crucial role in the description of $c\bar{c}q\bar{q}$ tetraquarks.

2.3   The spin part

As the quarks and antiquarks are spin 1/2 particles the total spin of a tetraquark can be $S = 0, 1$ or 2. For $S = 0$ there are two independent basis states for each channel. The bases associated to (5), (6) and (7) are

$$|\chi_+\rangle, \quad |\chi_-\rangle, \tag{8}$$

$$|P_{13}P_{24}\rangle, \quad |(V_{13}V_{24})_0\rangle, \tag{9}$$

$$|P_{14}P_{23}\rangle, \quad |(V_{14}V_{23})_0\rangle, \tag{10}$$

where P and V stand for pseudoscalar and vector meson subsystems respectively and the lower index 0 indicates the total spin. The corresponding Young tableaux for the states (8) are shown in Fig. 3.

For $S = 1$ there are three independent states in each channel, to be identified by three distinct Young tableaux. As an example we give the basis for the direct meson-meson channel [16]

$$|(P_{13}V_{24})_1\rangle, \quad |(V_{13}P_{24})_1\rangle, \quad |(V_{13}V_{24})_1\rangle. \tag{11}$$

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Figure 3: The Young tableaux corresponding to the states (8).

The lower index indicates the total spin 1. The permutation property under transposition (13) manifestly is

$$(13)|P_{13}\rangle = -|P_{13}\rangle, \quad (13)|V_{13}\rangle = +|V_{13}\rangle, \tag{12}$$

and similarly for (24)

$$(24)|P_{24}\rangle = -|P_{24}\rangle, \quad (24)|V_{24}\rangle = +|V_{24}\rangle. \tag{13}$$

With the identification $1 = c$, $2 = q$, $3 = \bar{c}$ and $4 = \bar{q}$ the permutation (13)(24) is equivalent to charge conjugation, reason for which the basis vectors $|\alpha_3\rangle$ and $|\alpha_6\rangle$ defined below have charge conjugation $C = 1$ and $|\alpha_1\rangle$, $|\alpha_2\rangle$, $|\alpha_4\rangle$, $|\alpha_5\rangle$ have charge conjugation $C = -1$.

The case $S = 2$ is trivial. There is a single basis state

$$\chi^S = |(V_{13}V_{24})_2\rangle, \tag{14}$$

which is symmetric under any permutation of quarks.

3   Models for X(3872)

Below we shall first briefly describe the diquark-antidiquark model and next we shall calculate the full spectrum of tetraquarks $c\bar{c}q\bar{q}$ in the framework of the simple model of Ref. [23].

3.1   The diquark-antidiquark model

The diquark model for tetraquarks, named also exotic mesons, has been attractive for some time already. One obstacle is to estimate the mass of the diquark. A simple consistent picture has been given by Lichtenberg et al. [21]. In that context the mass of $qq\bar{c}$ with diquarks of spin 0 and 1 was 3910 MeV and that of $qc\bar{q}\bar{c}$ with diquarks of spin zero 3920 MeV.

The heavy-light diquark picture of Maiani et al. [22] is more sophisticated. It contains explicitly a hyperfine interaction between quarks (antiquarks) and the diquarks have spin zero or one in all cases. However the colour space is truncated to $\bar{3}3$ so that the number of basis vectors is twice less than that presented in the following subsection [1]. Accordingly

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[1]The truncation of the colour space is questionable for tetraquarks involving charmed quarks in view of the results of Refs. [11]. For tetraquarks involving b quarks the approximation is better.

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there are only two basis vectors for $J^{PC} = 0^{++}$, one for $J^{PC} = 1^{++}$, two for $J^{PC} = 1^{+-}$ and one for $J^{PC} = 2^{++}$, so that in the tetraquark spectrum there are twice less states than in Fig. 4 below. The narrow decay width is explained by a rearrangement and by a "bold guess" of the coupling constant $g_{XJ/\psi V}$.

3.2   The role of octet-octet channels

Here we use the simple model of Ref. [23] in order to point out the important role of hidden colour channels introduced above. Accordingly the mass of a tetraquark is given by

$$\mathcal{M} = \sum_i m_i + \langle H_{\mathrm{CM}} \rangle, \tag{15}$$

where

$$H_{\mathrm{CM}} = -\sum_{i,j} C_{ij} \; \lambda_i^c \cdot \lambda_j^c \, \vec{\sigma}_i \cdot \vec{\sigma}_j \; . \tag{16}$$

The first term in Eq. (15) contains the effective masses m_i which incorporate the conventional constituent mass plus the kinetic energy and the confinement potential contributions. The constants C_{ij} represent the integral in the coordinate space of some unspecified analytic form of the one gluon-exchange potential and of spatial wave functions. A fit within ± 10 MeV to properties of charmed baryons gave the following parameters

$$\begin{aligned} C_{qq} &= 20 \, \mathrm{MeV}, \quad C_{qc} = 5 \, \mathrm{MeV}, \quad C_{qs} = 15 \, \mathrm{MeV}, \\ C_{ss} &= 10 \, \mathrm{MeV}, \quad C_{cs} = 4 \, \mathrm{MeV}, \quad C_{c\bar{c}} = 4 \, \mathrm{MeV}. \end{aligned} \tag{17}$$

For a tetraquark the possible states are $J^{PC} = 0^{++}, 1^{++}, 1^{+-}$ and $J^{PC} = 2^{++}$. In each case a basis can be built with (1,3) and (2,4) as quark-antiquark subsystems where each subsystem has a well defined colour, singlet or octet. This arrangement corresponds to J/ψ + *light meson* channel. Other intermediate couplings can also be defined, as for example, (1,4) and (2,3) which can correspond to the $D + \overline{D}^*$ channel, when the total spin allows. One can pass from one coupling to the other, depending on the problem one looks at and also, for convenience in the calculations.

For the colour-spin basis states we shall use the notation introduced in the previous section, i. e. 1_{mn} for colour singlet and 8_{mn} for colour octet subsystems and P_{mn} and V_{mn} for spin 0 and 1 respectively.

For $J^{PC} = 0^{++}$ the basis constructed from products of states (6) and (9) is

$$\begin{aligned} \gamma_1 &= |1_{13}1_{24}P_{13}P_{24}\rangle, \quad \gamma_2 = |1_{13}1_{24}(V_{13}V_{24})_0\rangle, \\ \gamma_3 &= |8_{13}8_{24}P_{13}P_{24}\rangle, \quad \gamma_4 = |8_{13}8_{24}(V_{13}V_{24})_0\rangle. \end{aligned} \tag{18}$$

The chromomagnetic interaction Hamiltonian with minus sign, $-H_{\mathrm{CM}}$, acting on this basis leads to the following symmetric matrix

$$\begin{bmatrix} 16(C_{13}+C_{24}) & 0 & 0 & 8\sqrt{\frac{2}{3}}(C_{12}+C_{23}) \\ & -\frac{16}{3}C_{13}+C_{24}) & -8\sqrt{\frac{2}{3}}(C_{12}+C_{23}) & -\frac{16\sqrt{2}}{3}(C_{12}-C_{23}) \\ & & -2(C_{13}+C_{24}) & \frac{4}{\sqrt{3}}(2C_{12}-7C_{23}) \\ & & & \frac{16}{3}C_{12}+\frac{56}{3}C_{23}+\frac{2}{3}(C_{13}+C_{24}) \end{bmatrix}$$

For $J^P = 1^+$ there are six linearly independent basis vectors built as products of colour (6) and spin (11) states.

$$\begin{aligned} \alpha_1 &= |1_{13}1_{24}(P_{13}V_{24})_1\rangle, \quad \alpha_2 = |1_{13}1_{24}(V_{13}P_{24})_1\rangle, \\ \alpha_3 &= |1_{13}1_{24}\ (V_{13}V_{24})_1\rangle, \quad \alpha_4 = |8_{13}8_{24}(P_{13}V_{24})_1\rangle, \\ \alpha_5 &= |8_{13}8_{24}(V_{13}P_{24})_1\rangle, \quad \alpha_6 = |8_{13}8_{24}\ (V_{13}V_{24})_1\rangle. \end{aligned} \tag{19}$$

from which α_3 and α_6 have charge conjugation $C = 1$ and the others $C = -1$. The 6×6 matrix of -H_{CM} can be found in Ref. [23]. Note that all matrix elements between states with opposite charge conjugation are naturally zero which means that the 6×6 matrix of Ref. [23] has a block-diagonal form. One block is the 2×2 submatrix for states of charge conjugation $C = 1$. This is

$$\begin{bmatrix} -\frac{16}{3}(C_{13}+C_{24}) & -\frac{8\sqrt{2}}{3}(C_{12}-C_{23}) \\ & \frac{2}{3}(4C_{12}+14C_{23}+C_{13}+C_{24}) \end{bmatrix}$$

which can be related to X(3872). The other block is a 4×4 submatrix for states of charge conjugation $C = -1$, not written explicitly here, but which can be easily identified from Ref. [23].

For $J^{PC} = 2^{++}$ the basis vectors are

$$\delta_1 = |1_{13}1_{24}\chi^S\rangle, \quad \delta_2 = |8_{13}8_{24}\chi^S\rangle, \tag{20}$$

where χ^S is the $S = 2$ spin state (14). The corresponding 2×2 matrix is

$$\begin{bmatrix} -\frac{16}{3}(C_{13}+C_{24}) & \frac{8\sqrt{2}}{3}(C_{12}-C_{23}) \\ & -\frac{2}{3}(4C_{12}+14C_{23}-C_{13}-C_{24}) \end{bmatrix}$$

In the calculation of the matrix elements we have used the equalities

$$C_{14} = C_{23}, \quad C_{12} = C_{34}, \tag{21}$$
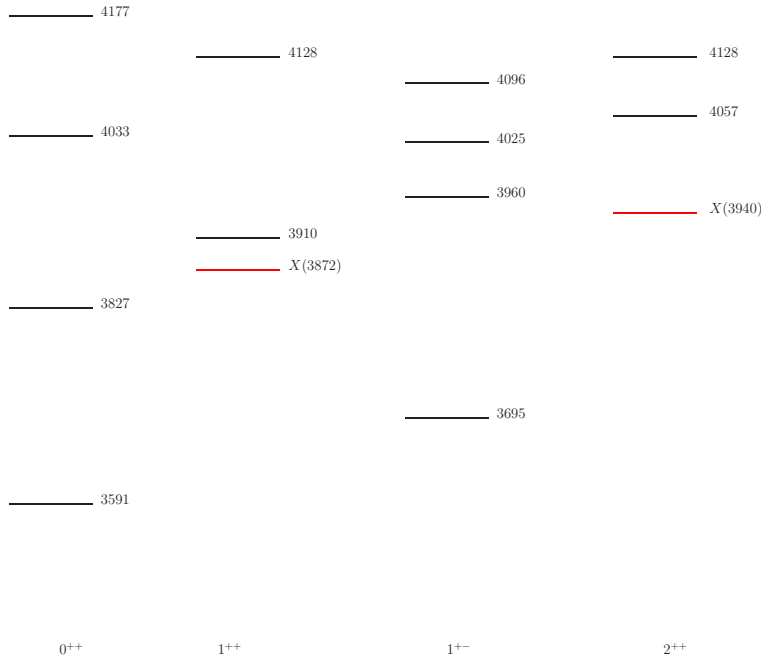
due to charge conjugation.

Figure 4: The spectrum of $c\bar{c}q\bar{q}$ tetraquarks

One can extend the observation made in Ref. [23], for 1^{++} to 2^{++} states as well, namely the matrices of both 1^{++} and 2^{++} are diagonal provided the chromomagnetic interaction is the same for a quark-quark pair as for a quark-antiquark pair, $i.e.$ $C_{12} = C_{23}$. This implies that the eigenvectors are either a pure colour singlet-singlet or a pure colour octet-octet state. In each case the pure octet-octet state is the lowest and obviously cannot dissociate into a charmonium state and a vector meson. But if one allows for a difference between C_{12} and C_{23}, see parameters (17), then the lowest state receives a small singlet-singlet component which can then decay into $J/\psi + \rho$ or $J/\psi + \omega$ with a small width. Such a description is very suitable for $X(3872)$ which has a width $\Gamma < 2.3$ MeV (95 % C.L.) [1].

The states with $J^P = 1^{+-}$ can decay into $J/\psi + pseudoscalar\ meson$ as required by charge conjugation. There is no observation which can be associated to these states.

In Fig. 4 we show the calculated spectrum with the parameters (17). These parameters were obtained from a fit to charmed baryons, but determined within ± 10 MeV. This means that the levels in Fig. 4 should be submitted to some incertaities, not easy to be estimated. One can see that the 3910 MeV 1^{++} level is close to $X(3872)$ but the 4057 MeV 2^{++} level is about 100 MeV above the observed $X(3940)$ resonance.

4 Conclusions

The work of Ref. [23] and its present extension suggests that the resonance X(3872) is an interesting candidate for the tetraquark $c\bar{c}q\bar{q}$. But the resonance X(3940) does not fit so well

Figure 5: David Brink, vice-director of ECT* ,Trento, 1993

into this scheme as a $J^P = 2^{++}$ state, its mass being 100 MeV below the calculated value. For X(3872) further work is necessary. One has to quantitatively estimate the contribution of annihilation channels. Also estimates of the strong decay widths are necessary before any definite conclusion.

Besides the chromomagnetic interaction, other effects may be important to the description of tetraquarks, as for example a long-range strong attraction between light quarks, generated by one-meson exchange. Actually this is the basic mechanism of the molecular picture [24, 25]. A speculative discussion on an additional effect of the one-meson exchange interaction can be found in Ref. [26]

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