

Quark model: recent issues

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Abstract

In the first part I briefly survey recent issues in constituent quark models raised by the observation of unusual hadronic states. In particular I discuss the role of higher Fock components in the wave function of baryons and the possible interpretation of open charm and of new charmonium-type resonances as tetraquarks. In the second part I show support for the quark model dynamics obtained in a model independent way from the $1/N_c$ expansion approach of QCD which proved to be successful in describing baryon properties.

1 Introduction

The organizers asked me to talk about recent issues in the quark model (QM). This is a vast subject and I had to make a selective choice. My talk contains two distinctive parts. The first is devoted to specific issues in the QM related to the recent observation (since 2003) of unusual hadronic states. In this context I present a few aspects of the QM developments in the light of the newly found resonances. The second part is devoted to a comparison between QM results and the $1/N_c$ expansion approach of QCD, both being successful in baryon spectroscopy, the latter being closer to QCD and model independent.

2 The QM and the newly found resonances

Here I refer to constituent quark (or potential) models. The basic assumptions are that the Hamiltonian consists of a kinetic (non-relativistic or relativistic) part, a confinement part and a hyperfine interaction of a one-gluon

exchange (OGE) type, a Goldstone boson exchange (GBE) type or resulting from an instanton induced interaction (III), or a mixture of them.

The quark models are generally successful in reproducing baryon spectra. Relativistic effects turn out to be specially important in describing electromagnetic or axial form factors of light baryons. The strong decays of baryons remain problematic in all potential models. The decay widths are generally underestimated in OGE or GBE models [1] as well as in III models [2]. These are results based on the description of the baryons as a system of three valence quarks. Till recently most of mesons were well described as $q\bar{q}$ systems.

The discovery of new exotic resonances starting from 2003 brought new aspects into the standard treatment of hadrons. These are: 1) higher Fock components in the wave function of some baryons 2) additional spin-orbit term in mesons with non-identical quark masses or the interpretation of some of them as tetraquarks, for example.

2.1 Higher Fock components in baryon states

The debate on the existence of pentaquarks lead to the study of the role of five quark components ($q^4\bar{q}$) in the wave function of the nucleon whenever there is a problem in the description of a baryon as a q^3 system. For example, the implication of such components has been analyzed in connection with experiments on parity violation in electron-proton scattering which suggest that the strangeness magnetic moment μ_s of the proton is positive. So far calculations gave either positive or negative values. A positive value was obtained [3] by including in the wave function a positive parity component $uuds\bar{s}$ with one quark in the $uuds$ subsystem excited to the p -shell. The most favorable configuration is $[31]_O[22]_F[22]_S$, the same as for positive parity pentaquarks [4] with a flavor-spin dependent GBE interaction.

Also, it is known that the QM, irrespective of the hyperfine interaction included in the Hamiltonian model, cannot explain the low mass of the $\Lambda(1405)$ resonance. Recently dynamical calculations based on the $q^4\bar{q}$ configurations have been performed [5]. These studies require an embedded q^3 pole in the continuum and a coupling between q^3 and $q^4\bar{q}$ configurations which remains an open problem.

Finally, there is the suggestion that a $uuds\bar{s}$ component in the wave function of the $N^*(1535)$ resonance could lead to a larger coupling to $N\eta$ and $N\eta'$ channels [6], in agreement to experiment.

2.2 Tetraquarks

Since 2003 an important number of exotic meson-like resonances have been discovered. These are open charm resonances: $D_s(2317)$, $D_s(2460)$, $D_s(2690)$, $D_s(2860)$ and charmonium type (hidden charm) resonances: $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3930)$, $Y(4260)$, $Z^\pm(4433)$.

While for open charm resonances a canonical interpretation as $c\bar{s}$ systems is still possible [7,8] through the addition of a spin-orbit term which vanishes for equal quark and antiquark masses, in the case of hidden charm resonances one has to assume more complicated structures as: tetraquarks $(c\bar{c})(q\bar{q})$, $D\bar{D}^*$ molecules, hybrids, glueballs, etc. (for a review see for example Refs. [9,10]). The tetraquark interpretation of $X(3872)$ is quite attractive [11–13]. The quark model [13] gives twice more states than the diquark model [11]. However much work is still needed in the framework of QM or other approaches in order to understand the exotic hidden charm resonances.

3 Compatibility of the quark model and the $1/N_c$ expansion approach

The QM still remains the basic tool in hadron spectroscopy. In addition, it has played an important role in the evolution of ideas towards QCD. However there is no known way to derive the QM from QCD. Each constituent quark model is based on a given Hamiltonian which contains a number of dynamical assumptions. The results are obviously model dependent. Therefore it is very important to establish a connection between QM results and another approach, also successful in baryon spectroscopy, but model independent and much more closely related to QCD. This is the $1/N_c$ expansion method described below.

3.1 The $1/N_c$ expansion method

In 1974 't Hooft [14] extended QCD from $SU(3)$ to $SU(N_c)$, where N_c is an arbitrary number of colors and suggested a perturbative expansion in the parameter $1/N_c$, applicable to all QCD regimes. Witten applied the approach to baryons [15] and derived power counting rules which lead to a powerful $1/N_c$ expansion method to study static properties of baryons, as for example, masses, magnetic moments, axial currents, etc. The method is systematic and predictive. It is based on the discovery that, in the limit $N_c \rightarrow \infty$, QCD possesses an exact contracted $SU(2N_f)$ symmetry [16,17] where N_f is the

number of flavors. This symmetry is only approximate for finite N_c so that corrections have to be added in powers of $1/N_c$.

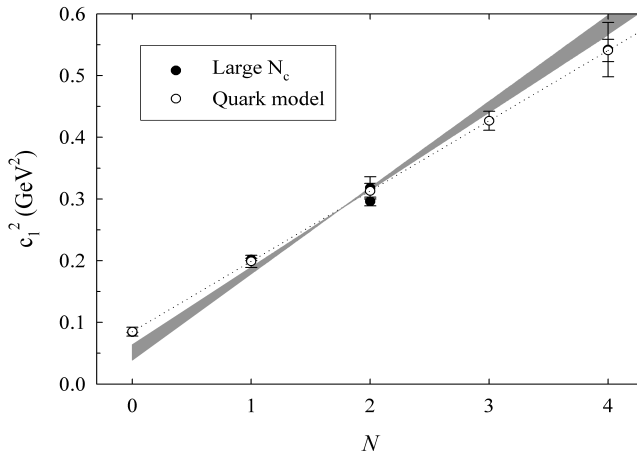


Figure 1: c_1^2 from the $1/N_c$ expansion mass formula, Eq. (1), and the QM result, Eq. (8). (for details on the large N_c points see Ref. [23]).

In the $1/N_c$ expansion approach the mass operator has the general form

$$M = \sum_i c_i O_i + \sum_i d_i B_i, \quad (1)$$

where each sum extends over a finite number of terms. The operators O_i are invariants under $SU(6)$ transformations and the operators B_i explicitly break $SU(3)$ -flavor symmetry. The coefficients c_i and d_i encode the quark dynamics and are fitted to the experimental data. In the case of nonstrange baryons, only the operators O_i contribute while B_i are defined such as their expectation values are zero. The building blocks of O_i and B_i are the $SU(6)$ generators: S_i ($i = 1, 2, 3$) acting on spin and forming an $su(2)$ subalgebra, T^a ($a = 1, \dots, 8$) acting on flavor and forming an $su(3)$ subalgebra, and G^{ia} acting both on spin and flavor subspaces. For orbitally excited states, also the components ℓ_i of the angular momentum, as generators of $SO(3)$, and the tensor operator ℓ^{ij} are necessary to build O_i and B_i . Examples of O_i and B_i can be found in Refs. [18–21]. Each operator O_i or B_i carries an explicit factor of $1/N_c^{n-1}$ resulting from the power counting rules [15], where $n - 1$ represents the minimum of gluon exchanges to generate the operator. In the matrix elements there are also compensating factors of N_c when one sums coherently over N_c quark lines. In practice it is customary to drop higher order corrections of order $1/N_c^2$.

The discussion below concerns the coefficients c_1 , c_2 and c_4 in Eq. (1) related to the following operators

$$O_1 = N_c, \quad O_2 = \ell_i S_i, \quad O_4 = \frac{1}{N_c} S_i S_i. \quad (2)$$

These are the spin-isospin independent, the spin-orbit and the spin-spin operators respectively. The analysis is straightforward for resonances described by symmetric states [56, ℓ]. For mixed symmetric states [70, ℓ] the procedure is more complicated due to the separation of the system into a symmetric core and an excited quark. In principle one can use a simpler approach in order to avoid such separation [22].

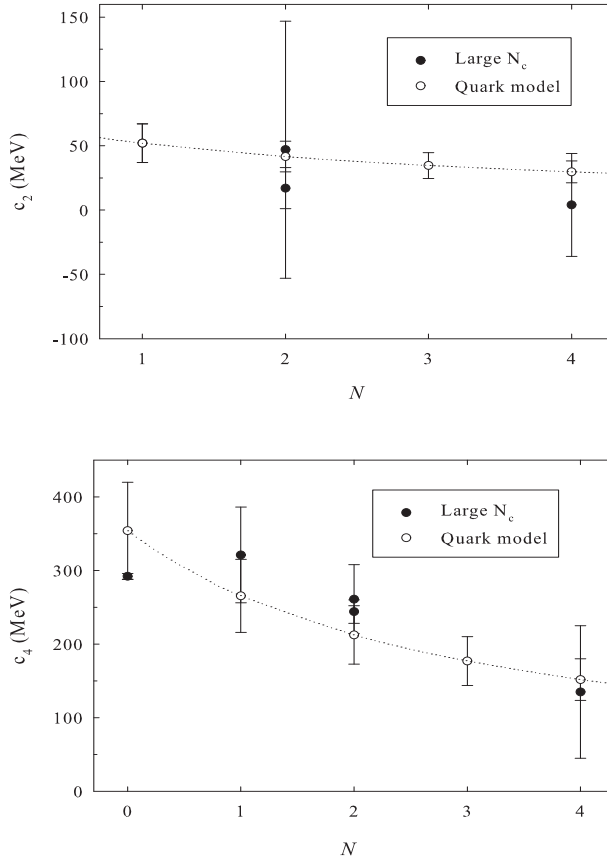


Figure 2: The coefficients c_2 and c_4 in the large N_c and QM approaches (for details see Ref. [23]). The dotted line passes through QM results.

For strange baryons one has to include both O_i and B_i operators in Eq.

(1). The contribution to each strange quark to the mass, denoted by ΔM_s , is given by

$$n_s \Delta M_s = \sum_i d_i B_i \quad (3)$$

where n_s is the number of strange quarks in a baryon.

3.2 The quark model

We follow the approach of Ref. [23] and start from the spinless Salpeter Hamiltonian

$$H = \sum_{i=1}^3 \sqrt{p_i^2 + m_i^2} + V_Y; \quad V_Y = a \sum_{i=1}^3 |\vec{x}_i - \vec{x}_T| \quad (4)$$

where m_i is the current mass, a the string tension and \vec{x}_T the Toricelli point. Our purpose is to obtain an approximate analytical form of the eigenvalues of the Hamiltonian (4). To a good approximation for the Y-junction [24] one can replace H by

$$H_0 = \sum_{i=1}^3 \sqrt{p_i^2 + m_i^2} + \frac{a}{2} \left[\sum_{i=1}^3 |\vec{x}_i - \vec{R}| + \frac{1}{2} \sum_{i<j}^3 |\vec{x}_i - \vec{x}_j| \right] \quad (5)$$

where \vec{R} is the position of the center of mass. The next step is to use the auxiliary field formalism [25] which allows to replace a semirelativistic by a nonrelativistic kinetic energy and a linear by a quadratic confinement. The eigenvalue problem becomes exactly solvable. By minimizing with respect to the auxiliary fields one obtains a good approximation to the exact mass. This is [26]

$$M_0 = 6\mu_0. \quad (6)$$

where

$$\mu_0 = \left[\frac{a}{3} Q(N+3) \right]^{1/2}, \quad Q = 1/2 + \sqrt{3}/4. \quad (7)$$

Here $N = 2n + \ell$, as in a harmonic oscillator potential, and represents the band number used in phenomenology. Adding perturbatively Coulomb-type and self-energy corrections to the squared mass of Eq. (6) one obtains

$$M_0^2 = 2\pi\sigma(N+3) - \frac{4}{\sqrt{3}}\pi\sigma\alpha_s - \frac{12}{(2+\sqrt{3})}f\sigma. \quad (8)$$

provided one makes the scaling $12aQ = 2\pi\sigma$ where σ is the standard strength tension, α_s is the strong coupling constant and f a parameter varying between 3 and 4.

In the auxiliary field formalism, one expects that $c_2 \propto \mu_0^{-2}$ and $c_4 \propto \mu_0^{-2}$. Thus, using Eq. (7), one obtains

$$c_2 = \frac{c_2^0}{N+3}, \quad c_4 = \frac{c_4^0}{N+3}. \quad (9)$$

where the coefficients c_2^0 and c_4^0 have to be fitted.

As a matter of fact, the proof that the band number N can be considered a good quantum number for baryons including both strange and nonstrange quarks is more involved (for details see Ref. [27]).

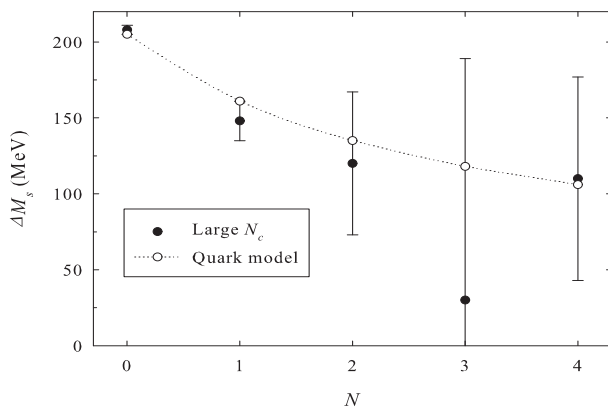


Figure 3: ΔM_s from the $1/N_c$ expansion mass formula, Eq. (1) and the QM mass result (for details see Ref. [27]).

3.3 Comparison of the two approaches

In the real world we have $N_c = 3$. Thus we have to compare the coefficient c_1^2 of Eq. (1) with $M_0^2/9$ where M_0^2 comes from Eq. (8). This comparison is made in Fig. 1 for the bands $N = 0, 1, 2, 3, 4$ studied within the $1/N_c$ approach. From the best fit one has $\sigma = 0.163 \pm 0.004 \text{ GeV}^2$, $\alpha_s = 0.4$, and $f = 3.5$, as very standard values. One can see a remarkable agreement between the two approaches. In both cases the points follow the same straight line (Regge trajectory). If the value of σ is chosen in the common phenomenological interval $\sigma = 0.17$ (0.20) GeV^2 one obtains the shaded area.

For the upper part of Fig. 2 we chose $c_2^0 = 208 \pm 60 \text{ MeV}$ so that the large N_c point at $N = 1$, for which the uncertainty is minimal, is exactly reproduced. This coefficient is related to the contribution of the spin-orbit

operator O_2 which turns out to be very small in both approaches. For the lower part of Fig. 2 a good fit to all points gave $c_4^0 = 1062 \pm 198$ MeV. Note that $c_4^0 \gg c_2^0$ which indicates that the spin-spin contribution dominates over the spin-orbit term. This justifies the quark model assumption that the spin-spin is the dominant contribution to the hyperfine interaction. Details of this comparison can be found in Ref. [23].

The comparison between the QM and large N_c results for ΔM_s is shown in Fig. 3. The point at $N = 2$ is from Ref. [27]. The points corresponding to $N = 0, 1$ and 3 are taken from Ref. [28]. Except for $N = 3$, the central values of ΔM_s in the large N_c approach are close to the quark model results. The QM results show a smooth behavior. This suggests that the $N = 3$ point in the $1/N_c$ expansion must be re-analyzed. The accuracy of the $1/N_c$ expansion results depends on the quality and quantity of experimental data on strange baryons, which is very scarce for various reasons [29]. More data are highly desired.

4 Conclusions

The key tool in the comparison of QM and large N_c approaches is the band number N which turns out to be a good and relevant quantum number in the classification of baryons. It leads to Regge trajectories where $M^2 \propto N$. The basic conclusion is that the large N_c approach supports the quark model assumptions as the relativistic kinetic energy, Y-junction confinement, dominant spin-spin interaction, vanishing spin-orbit contribution, etc. At the same time, the QM can give some physical insight into the coefficients c_i and d_i which encode the QCD dynamics. Similar studies are needed for heavy baryons.

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