Extension of Davenport's Background/Resonant decomposition for the estimation of higher response moments

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1 Introduction

The non Gaussian buffeting analysis of a structure subjected to turbulent wind may be advantageously performed with a Volterra approach. Although this concept may be applicable to nonlinear structures, the scope of this paper is limited to structures with a deterministic linear behavior. In this case, the Volterra kernels take a simple explicit expression and the statistical moments of the structural response are simply recovered by a multi-dimensional integration of the corresponding spectra. Because the multi-dimensional spectra of the loading are usually determined numerically, these integrals have to be computed numerically too. The computational burden necessary for the estimation of these integrals turns out to be prohibitive as soon as high order moments are required (beyond and including 4th moment). The numerical conditioning is in fact again worse when the structural damping is low as the Volterra kernels then exhibit very large gradients in the frequency space, which are difficult to capture properly.

Because the timescales related to the dynamics of the structure and to the random wind loading are well distinct, a nowadays customary decomposition of the response into Background and Resonant contributions (dating back to A. Davenport's earliest works) provides a very good estimation of the second statistical moment. Recently the concept has been extended to the Background/biResonant decomposition for the estimation of the third statistical moment of the response. This paper presents the extension to the computation of the fourth statistical moment, with the only assumptions of slight damping and low-frequency loading such as those that made Allan Davenport's Background/Resonant (B/R) decomposition fruitful.

After some multiple scales considerations, the fourth moment of the response is approximated as the sum of a background component, a tetraresonant component and a mixed background-biresonant component. Mathematical developments will be presented in the full paper along with illustrative examples, while the current 4-page abstract just presents of a flavour of it.

2 The Background/Resonant Decomposition

The power spectral density (psd) of the structural response of a single degree-of-freedom linear system is obtained by

$$S_x(\omega) = S_f(\omega)\ K_2(\omega) = S_f(\omega)\ |H(\omega)|^2 \tag{1}$$

where S_x and S_f are respectively the psd's of the response and of the loading, and $H = (-m\omega^2 + i\omega c + k)^{-1}$ is the linear frequency response function of a dynamical system, characterized by a mass m, viscosity c and stiffness k. The kernel K_2 is equal to $|H|^2$, by definition. In this paper, we are concerned with psd's of loading that are assumed to decrease very fast in a short frequency range,

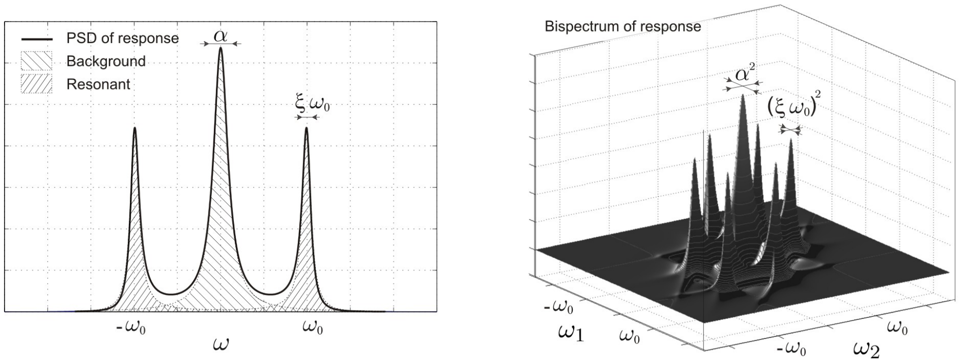

Figure 1: Typical psd (left) and bispectrum (right) of the response of a single degree-of-freedom system subjected to low frequency turbulence. The psd (left) is commonly decomposed as the sum of background and two resonant components. The bispectrum (right) is also decomposed as the sum of a background and six biresonant components.

referred to as α in the following. Formally α is assumed to be small compared to the natural circular frequency $\omega_o = \sqrt{k/m}$ of the dynamical system. The ratios

$$\varepsilon = \frac{\alpha}{\omega_o} \qquad \text{and} \qquad \xi = \frac{c}{2m\omega_o} \tag{2}$$

are actually two small parameters of the problem at hand, which makes the topology of the psd of the response rather particular. Indeed, as seen in Fig. 1, the psd of the response features three sharp and distinct peaks. For this reason, the second statistical moment of the response, the variance, may be approximated as

$$m_{2,x} = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega \simeq m_{2,b} + m_{2,r} \tag{3}$$

with the background and resonant components respectively given by

$$m_{2,b} = \frac{m_{2,f}}{k^2} \qquad \text{and} \qquad m_{2,r} = \frac{\pi\omega_o}{2\xi}\frac{S(\omega_o)}{k^2} \tag{4}$$

where $m_{2,f}$ is the variance of the loading. This decomposition is known to be due to A. Davenport (1).

In a very similar manner, the description of the response of a dynamical system subjected to a non Gaussian load is complemented by the bispectrum of the response

$$\begin{aligned} B_x(\omega_1,\omega_2) &= B_f(\omega_1,\omega_2)\, K_3(\omega_1,\omega_2) \\ &= B_f(\omega_1,\omega_2)\, H(\omega_1)\, H(\omega_2)\, \bar{H}(\omega_1+\omega_2)\,. \end{aligned} \tag{5}$$

Because of the smallness of the two parameters ε and ξ, this bispectrum features one distinctive quasi-static peak, six high biresonance peaks and, secondarily, six low biresonance peaks, see Fig. 1. Invoking the same general methodology for the estimation of the integral of B_x, the third statistical moment of the response may be expressed as

$$m_{3,x} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} B_x(\omega_1,\omega_2)\, d\omega_1 d\omega_2 \simeq m_{3,b} + m_{3,r} \tag{6}$$

see (2), with the background and biresonant components respectively given by

$$m_{3,b} = \frac{m_{3,f}}{k^3} \qquad \text{and} \qquad m_{3,r} = 6\pi \frac{\xi \omega_o^3}{k^3} \int_{-\infty}^{+\infty} \frac{B(\omega_o, \omega_2)}{(2\xi\omega_o)^2 + \omega_2^2} d\omega_2 \tag{7}$$

where $m_{3,f}$ is the third statistical moment of the loading. This approximation is valid no matter the expression of the bispectrum of the loading. It just requires the two hypotheses of smallness to be fulfilled.

3 The Background/TetraResonant Decomposition

The trispectrum of the response of a single degree-of-freedom linear system is expressed as

$$\begin{aligned} T_x(\omega_1, \omega_2, \omega_3) &= T_f(\omega_1, \omega_2, \omega_3) K_4(\omega_1, \omega_2, \omega_3) \\ &= T_f(\omega_1, \omega_2, \omega_3) H(\omega_1) H(\omega_2) H(\omega_3) \bar{H}(\omega_1 + \omega_2 + \omega_3) \end{aligned} \tag{8}$$

and the corresponding moment is obtained as

$$m_{4,x} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} T_x(\omega_1, \omega_2, \omega_3) d\omega_1 d\omega_2 d\omega_3. \tag{9}$$

Under the same hypotheses as before, we may demonstrate that this integral may be approximated as

$$m_{4,x} \simeq m_{4,B} + m_{4,tBR} + m_{4,tR} \tag{10}$$

with the background, the mixed background-biresonant and the tetraresonant components respectively given by

$$m_{4,B} = \frac{m_{4,f}}{k^4}, \quad m_{4,tBR} = 5\pi \frac{\omega_0^3 \xi}{k^4} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{T_f(\omega_1, \omega_2, \omega_2 - 2\omega_o) d\omega_1 d\omega_2}{(\xi\,\omega_o)^2 + (\omega_2 - \omega_o)^2} \text{ and}$$

$$m_{4,tR} = 9\pi \frac{\omega_0^3}{k^4 \xi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{T_f(\omega_1, \omega_2, \omega_1 + \omega_2 - 3\omega_o) (\xi\,\omega_o)^2 d\omega_1 d\omega_2}{\left[(2\xi\,\omega_o)^2 + (2\omega_1 + \omega_2 - 3\omega_o)^2\right]\left[(2\xi\,\omega_o)^2 + (\omega_1 + 2\omega_2 - 3\omega_o)^2\right]} \tag{11}$$

4 Illustration

As an illustration, we assume here that a single degree-of-freedom system is subjected to a quadratic loading

$$f = \gamma (1 + u)^2 = \gamma + 2\gamma u + \gamma u^2 \tag{12}$$

where the low-frequency turbulence process u is modeled as a dimensionless Ornstein-Uhlenbeck Gaussian process with zero mean and standard deviation σ_u, and γ is a characteristic aerodynamic force.

An illustrative way to represent the statistical moments of the response is to normalize them by the corresponding background contribution. In this perspective, the dynamic amplification coefficients at the second and fourth orders respectively are defined as

$$\mathcal{A}_2 = \frac{m_{2,x}}{m_{2,b}} = 1 + \frac{m_{2,r}}{m_{2,b}} \qquad \text{and} \qquad \mathcal{A}_4 = \frac{m_{4,x}}{m_{4,b}} = 1 + \frac{m_{4,r}}{m_{4,b}}. \tag{13}$$

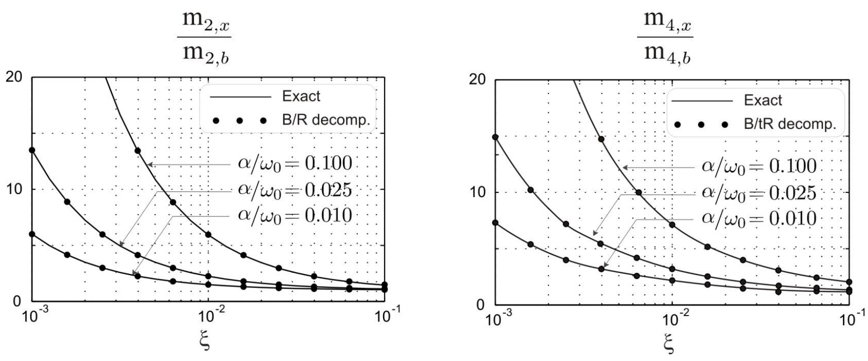

Figure 2: Dynamic amplification factors: ratio of the statistical moment of the response to the statistical moment that would be obtained if the response was quasi-static.

They are represented in Fig. 2 along with the exact results obtained through an analytical integration of the corresponding spectra.

At the second order, the perfect match between the analytical solution (labeled "Exact") and Davenport's B/R decomposition is just another illustration of the good performance of Davenport's approximation. The right-hand side of the Figure illustrates the good performance of the proposed approximation. A more detailled analysis of the contributions of the different terms is given in the full length paper.

5 Conclusions

Based on the assumption of small structural damping and the consideration of separated timescales for the wind loading and the structural vibrations, we have extended the B/R decomposition to the computation of the fourth order moment, which is not obtained as a 3-D integral as in the formal development, but is rather approximated with a simpler 2-D integral. This translates into a drastic reduction of the computational costs related to the estimation of the kurtosis of the response. Indeed, on account that the number of integration points in each direction amounts to more than one hundred, the proposed method offers therefore a reduction of the computational cost of two orders of magnitude.

References

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[2] Denoël, V., On the background and biresonant components of the random response of single degree-of-freedom systems under non-Gaussian random loading. Engineering Structures, 2011. 33(8): p. 2271-2283.