An optimization approach to the material tailoring of microstructures with damage resistant constraint

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Introduction/Goal

Material tailoring can be formulated as a structural optimization problem. The final objective of this work is to perform microstructural design under damage resistant constraint.

The work is divided in three main parts:

1. the **microstructural design problem:** maximizing the linear properties as stiffness, thermal conductivity, …
2. the **damage propagation problem:** propagating damage on fixed microstructural geometries
3. the **combination** of the two previous problems: optimizing microstructures under damage resistant constraint

The developed method will be designed to be applied to composite materials, functionally graded materials, damage materials, …

Microstructural design problem

The microstructural design is carried out through shape optimization. Shape optimization is performed using an approach that combines:

- a Level Set description of geometries
- a non-conforming analysis method (XFEM)

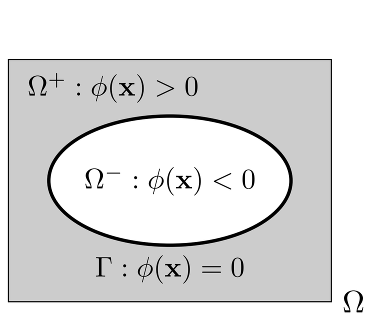
The design problem is casted in a mathematical programming approach providing a general and robust framework:

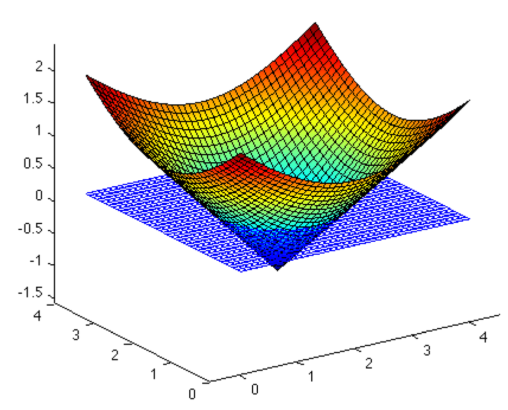
$$\begin{aligned} &\min_{\mathbf{x}} && f_0(\mathbf{x}) \\ &s.t. && f_j(\mathbf{x}) \geq \overline{f}_j \quad j = 1,\dots,m \\ & && \underline{\mathbf{x}}_i \geq \mathbf{x} \geq \overline{\mathbf{x}}_i \quad i = 1,\dots,n \end{aligned}$$

Level Set description

Basic principles of the Level Set Description:

- a function $\phi(\mathbf{x})$ is used to represent implicitly any shape Γ
- the desired shape is drawn by the iso-zero Level Set
- working on a finite mesh, $\phi(\mathbf{x})$ is discretized and interpolated





Extended Finite Element Method

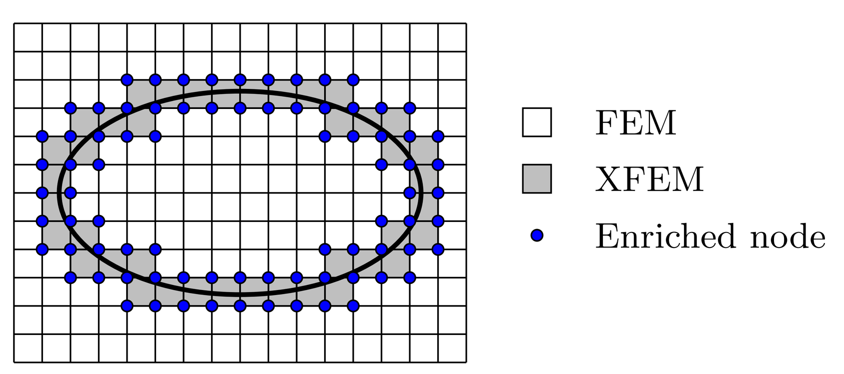
Basic principles of the eXtended Finite Element Method:

- adding special shape functions to the approximation to deal with particular behavior near an interface
- in the case of material-void interface:

$$u^h(\mathbf{x}) = \sum_i H(\mathbf{x})\, N_i(\mathbf{x})\, u_i$$

- in the case of material-material interface:

$$u^h(\mathbf{x}) = \sum_{i \in I} N_i(\mathbf{x})\, u_i + \sum_{i \in I^\star} N_i(\mathbf{x}) \left(\sum_j N_j(\mathbf{x})\, |\phi_j| - |N_j(\mathbf{x})\, \phi_j| \right) a_i$$



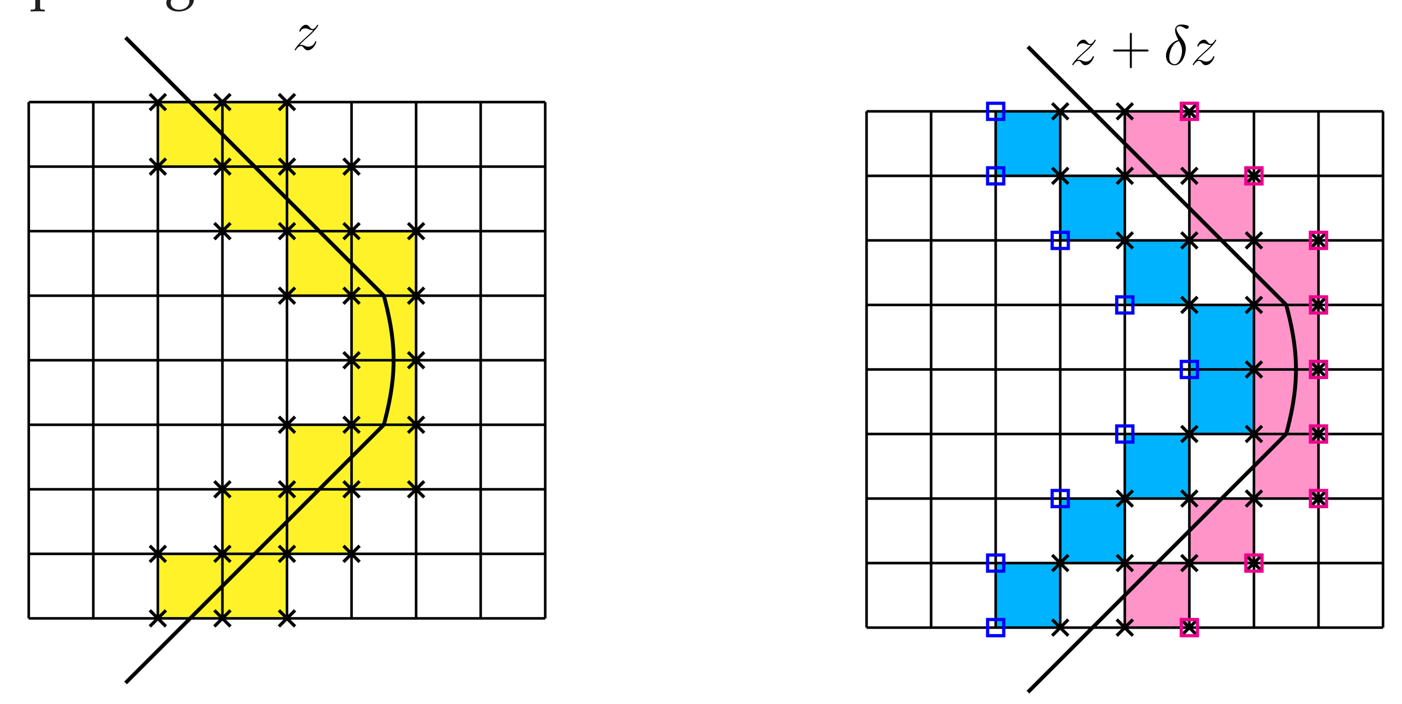
Sensitivity analysis

Van Miegroet et al. (2007) developed a semi-analytical approach to perform the sensitivity analysis in the case of material-void interface:

- Level Set parameters = design variables
- derivatives computed through forward finite difference
- derivatives are used to compute variations of design functions as compliance, displacement, stress, …

$$\frac{\partial \mathbf{K}}{\partial z} = \frac{\mathbf{K}(z+\delta z) - \mathbf{K}(z)}{\delta z} \quad \text{and} \quad \frac{\partial \mathbf{f}}{\partial z} = \frac{\mathbf{f}(z+\delta z) - \mathbf{f}(z)}{\delta z}$$

Trying to extend this approach to the material-material interface case, several additional difficulties arise. Those difficulties are highlighted by comparing the material-void and the material-material cases.



Material-void case

Initially non-included
→ cut by the interface
→ approximation ≠
→ number of dofs ↗
Finite difference ×

Initially partially filled
→ not cut anymore
→ approximation =
→ number of dofs =
Finite difference ✓

Material-material case

Initially unimaterial
→ cut by the interface
→ approximation ≠
→ number of dofs ↗
Finite difference ×

Initially bimaterial
→ not cut anymore
→ approximation ≠
→ number of dofs ↘
Finite difference ×

Damage propagation problem

Ongoing work:

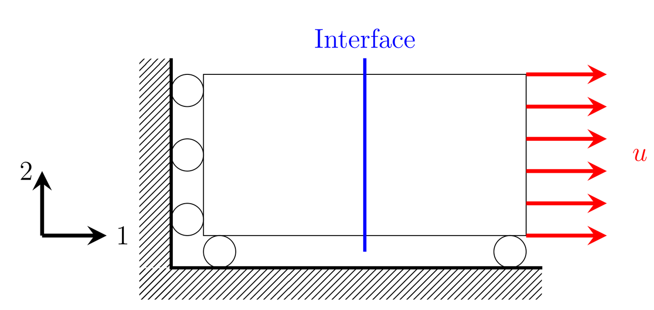
- propagation of damage through fixed geometry microstructures
- microstructural design under damage resistant constraint

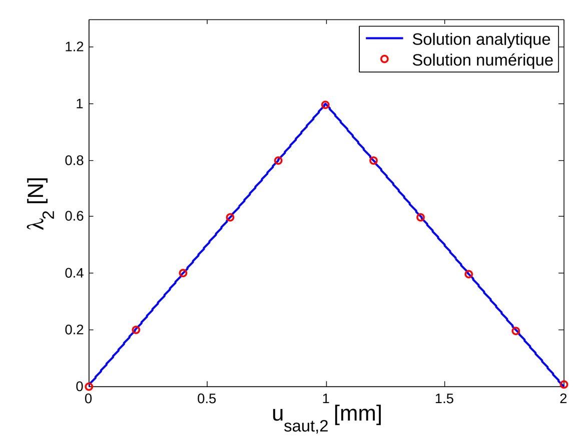
Many methods are available to simulate the propagation of damage:

- damage as an **optimal problem**: A damaged material of lower stiffness is distributed on a undamaged structure, submitted to loadings, so that the global compliance is maximized.

$$\max_z \min_d \min_u \int_\Omega \frac{1}{2}\varepsilon(u)^t D(z,d) \varepsilon(u) d\Omega - \int_\Omega f^t u d\Omega - \int_{\Gamma_\sigma} t^t u d\Gamma$$

- damage **starting at the interface**: Cohesive laws can be used to simulate a stiffness reduction of the interface as the structure undergoes different types of loadings.





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