Identification of a Time-varying Beam Using Hilbert Vibration Decomposition

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The present research is focused on the identification of time-varying systems

$$\boldsymbol{M}(t)\,\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}(t)\,\dot{\boldsymbol{x}}(t) + \boldsymbol{K}(t)\,\boldsymbol{x}(t) = \boldsymbol{f}(t)$$

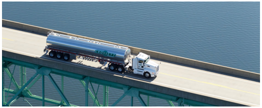
The dynamics of such systems is characterized by:

- Non-stationary time series
- Instantaneous modal properties
  - Frequencies : $\omega_r(t)$
  - Damping ratio's : $\zeta_r(t)$
  - Modal deformations : $\boldsymbol{q}_r(t)$

Why time-varying behaviour can occur ?

Several possible origins :

- Structural changes



- Operating conditions

- Damages

Existing techniques for the identification of time-varying systems

Short-time analyses

Wavelet analysis

ARMA methods

Hilbert-Huang Transform (HHT)

Hilbert Vibration Decomposition (HVD)

Outline of the presentation

Introduction to the Hilbert transform and the HVD method

Drawbacks and illustrative example

Adaptation of the initial method

Application to the case study

The Hilbert Transform

The Hilbert transform $\mathcal{H}$ of a signal $x(t)$ is the convolution product of this signal with the impulse response $h(t) = \frac{1}{\pi\, t}$

$$\begin{aligned}
\hat{x}(t) &= \mathcal{H}(x(t)) = (h(t) * x(t)) \\
&= \text{p.v.} \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau \\
&= \frac{1}{\pi}\, \text{p.v.} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}\, d\tau
\end{aligned}$$

It is a particular transform that remains in the same domain as the original signal

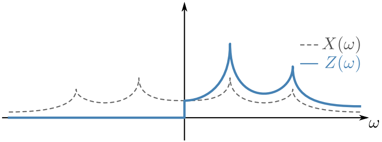
It corresponds to a phase shift of $-\frac{\pi}{2}$ of the signal

The Hilbert transform is used to build the complex analytic form of a signal

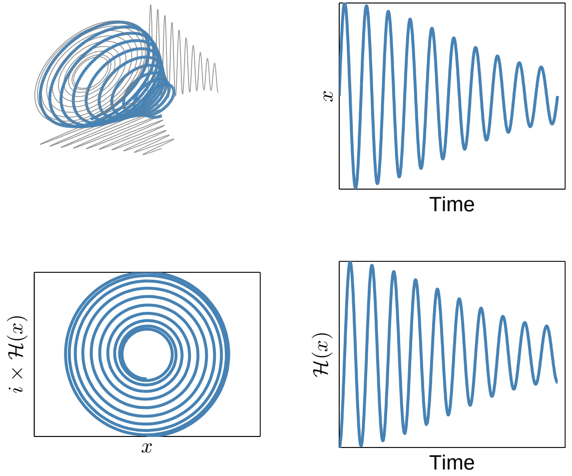
The analytic signal z is built as

$$\begin{aligned} z(t) &= x(t) + i\,\mathcal{H}(x(t)) \\ &= A(t)\,e^{i\phi(t)} \end{aligned}$$

In the frequency domain, the analytic signal becomes a one-sided signal



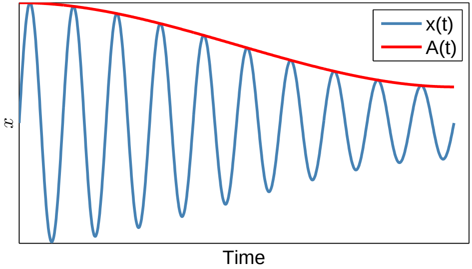
The analytic signal can be seen as a rotating phasor in the complex plane



It is suitable to find the envelope of the signal

The instantaneous envelope of the signal is given by the absolute value of the analytic signal

$$A(t) = |z(t)|$$



It also gives information about the instantaneous phase

The instantaneous phase of the signal is given by the argument of the analytic signal

$$\phi(t) = \angle z(t)$$

The time derivative of the phase angle gives the instantaneous frequency

$$\omega(t) = \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

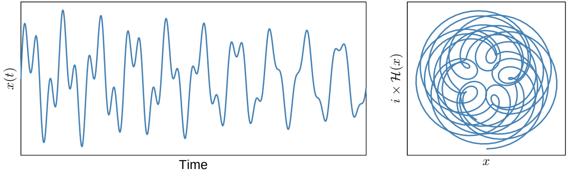
Study of multicomponent signals

Multicomponent signals as a sum of modulated harmonic components

$$x(t) = \sum_{r=1}^{n} A_r(t) \sin(\phi_r(t) + \theta_r)$$

In its analytical form, it can be seen as a phasor superposition

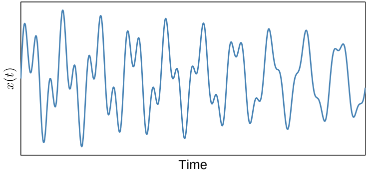
$$z(t) = \sum_{r=1}^{n} A_r(t) e^{i\,\phi_r(t)}$$

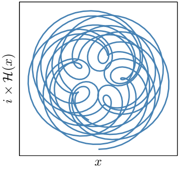


The Hilbert Vibration Decomposition (HVD) sifting process

Key steps of the method:

- Analytic signal computation
- Phase extraction and smoothing
- Synchronous demodulation
- Component subtraction and iteration





The HVD method in that scheme encounters some drawbacks

It is applicable to single channel measurement. The application on multiple channels has to be done in parallel

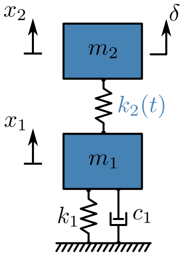
In a multivariate case, all the modes have to be excited at each time instants on all the channels

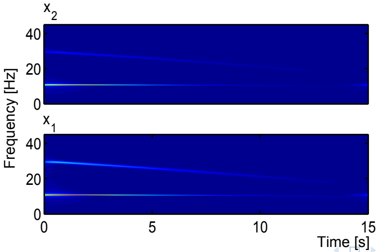
The method will always follow the instantaneous dominant mode

Example: a simple 2–DoF time–variant system

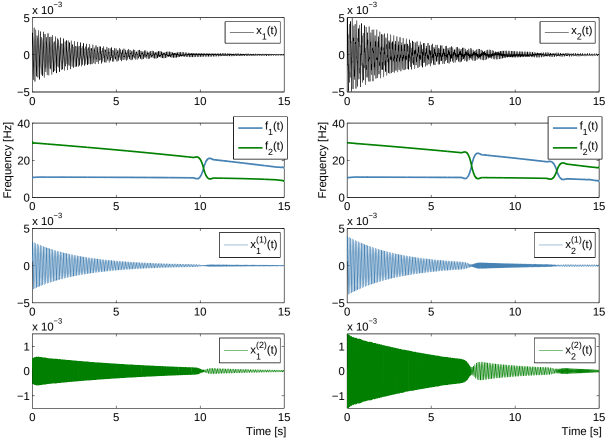
System properties:

- $m_1 = 3$ [kg]
- $m_2 = 1$ [kg]
- $k_1 = 20000$ [N/m]
- $c_1 = 3$ [N.s/m]
- $k_2 = 25000 \searrow 5000$ [N/m]

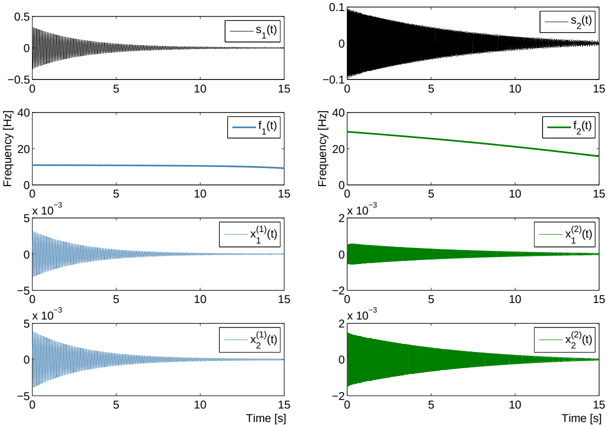




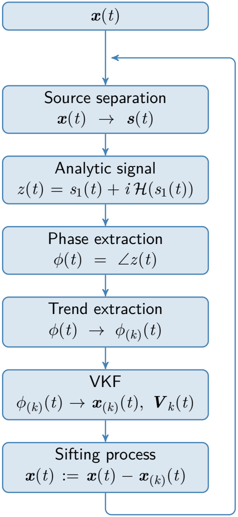
The application of the HVD method on each channel leads to mode switching



Introducing a source separation method can help to avoid this phenomenon



In the case of multivariate measurements, a source separation step is introduced in the algorithm



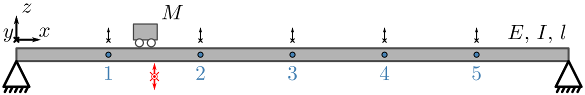
Sources are used as references to get the instantaneous frequencies

Trend extraction computes the phase of the dominant mode

Vold-Kalman filter (VKF) is used for component extraction

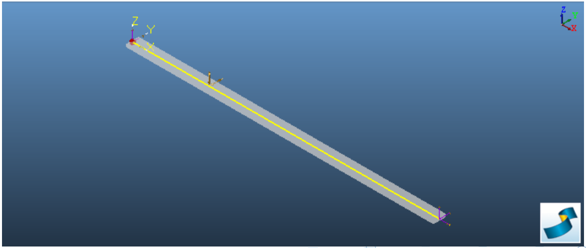
Example of application: an aluminium beam on which a mass is travelling

- Beam geometry: $2000 \times 80 \times 20$ mm
- $M = 3$ kg ($\approx$ 35% of the mass of the beam)
- Pinned connections:
  $x(0) = y(0) = z(0) = x(l) = y(l) = z(l) = 0$
- Five measurement points in two directions
- One excitation point in two directions



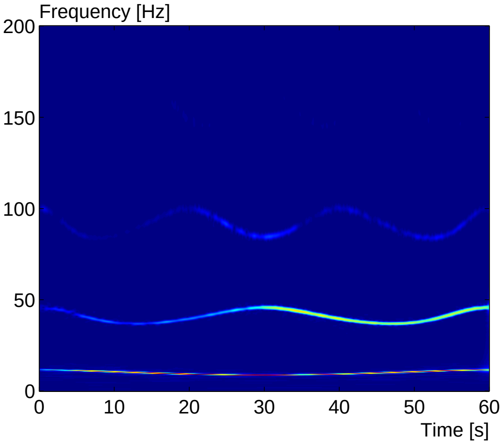
The system is simulated using LMS–Samcef Mecano

- Newmark integration over 60 seconds
- The mass travels the whole beam during the integration time at a constant speed
- Random forces excite all the modes of the structure
- Gaussian noise is added to the numerical responses (1% noise to signal ratio)

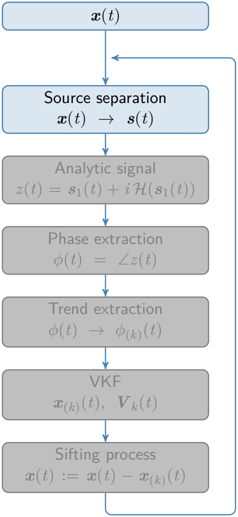


The time–variant characteristics of the system are easily visible

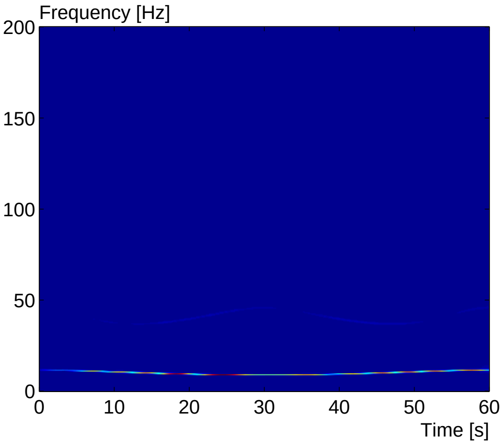
Resonance frequencies oscillate between top and bottom values depending on the position of the mass



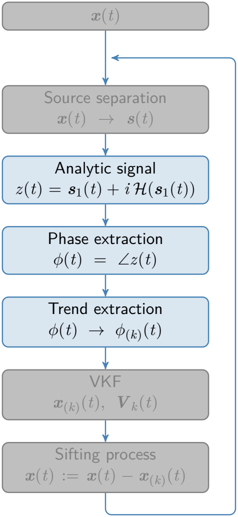
All the channels are then decomposed in sources



The smooth orthogonal decomposition (SOD) technique is used to try to highlight one mode in each source



Source $s_1(t)$ is kept and its analytic signal is calculated to extract its phase



The Hodrick–Prescott (HP) filter is used as trend detection technique

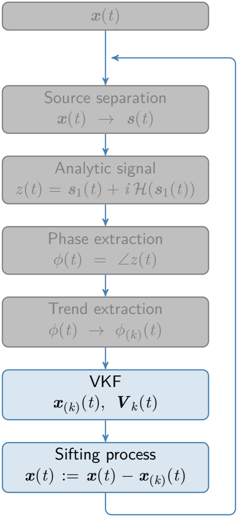
It is an optimisation problem trying to find the trend that minimises

$$\min_{[t_n]_{n=1}^{N}} \left\{ \underbrace{\sum_{n=1}^{N} (\phi_n - t_n)^2}_{(1)} + \lambda \underbrace{\sum_{n=2}^{N-1} \left[(t_{n+1} - t_n) - (t_n - t_{n-1}) \right]^2}_{(2)} \right\}$$

(1) penalises large discrepancies between the trend and the signal

(2) penalises fast variations of the trend

Once the phase of the dominant mode is known, its corresponding components are extracted



A Vold–Kalman filter (VKF) is used to this aim

The method retrieves signal components based on their phase

$$x(t) = \sum_k \underbrace{a_k(t)\, e^{i\,\phi_k(t)}}_{x_{(k)}(t)} + \nu(t)$$

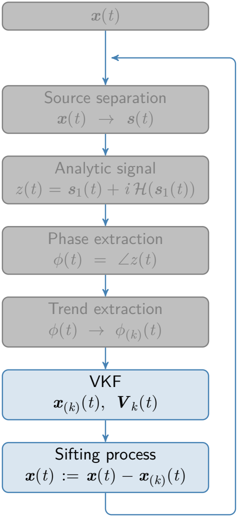
The complex amplitudes of the components minimise the data equation

$$x(t) - \sum_k a_k(t)\, e^{i\,\phi_k(t)} = \delta(t)$$

and the structural equations

$$a_k(t-1) - 2a_k(t) + a_k(t+1) = \varepsilon_k(t)$$

Analogy between VKF and modal expansion



By analogy with the modal expansion the complex amplitudes can be seen as unscaled instantaneous mode shapes

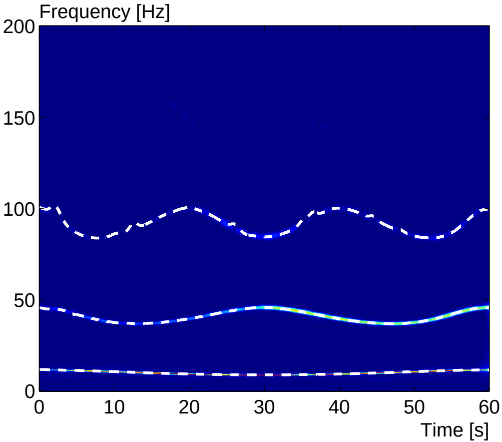
VK filter

$$\boldsymbol{x}(t) = \sum_k \boldsymbol{a}_k(t)\, e^{i\,\phi_k(t)}$$

Modal expansion

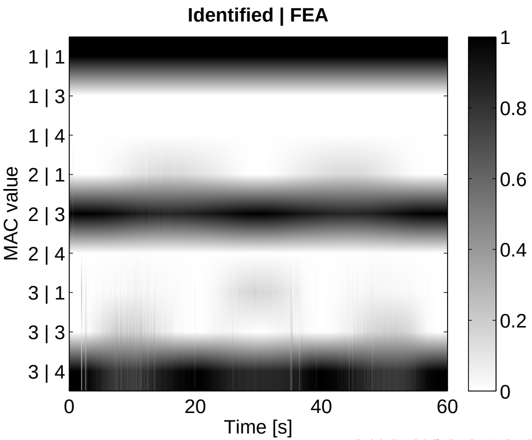
$$\boldsymbol{x}(t) = \sum_k \boldsymbol{V}_k(t)\, \eta(t)$$

All the modes are extracted by successive iterations



The Modal Assurance Criterion is adapted to time–varying mode shapes

At each time instant the MAC matrix is calculated and reshaped in a column vector



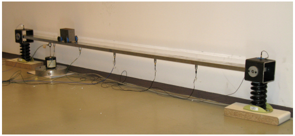
The mode shapes are perturbed by the presence of the mass on the beam

Conclusion and future work

The HVD method was presented with its strengths and weaknesses

Modifications were added to treat an MDOF system

The method was applied to identify the numerically simulated system



Application to the identification of actual sytems

Increase the robustness of the method

Thank you for your attention