Benchmarks for the Forward Observables at RHIC, the Tevatron-run II and the LHC

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Abstract

We present predictions on the total cross sections and on the ratio of the real part to the imaginary part of the elastic amplitude (ρ parameter) for present and future pp and $\bar{p}p$ colliders, and on total cross sections for $\gamma p \to$ hadrons at cosmic-ray energies and for $\gamma\gamma \to$ hadrons up to $\sqrt{s} = 1$ TeV. These predictions are based on an extensive study of possible analytic parametrisations invoking the biggest hadronic dataset available at $t = 0$. The uncertainties on total cross sections, including the systematic errors due to contradictory data points from FNAL, can reach 1.9% at RHIC, 3.1% at the Tevatron, and 4.8% at the LHC, whereas those on the ρ parameter are respectively 5.4%, 5.2%, and 5.4%.

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In recent works [1, 2], we have performed an exhaustive study of the analytic parametrisations of soft data at $t = 0$. For this purpose, we gathered the largest available set of data at $t = 0$, which includes all measured total cross sections and ratios of the real part to the imaginary part of the elastic amplitude (ρ parameter) for the scattering of pp, $\overline{p}p$, $\pi^{\pm}p$, $K^{\pm}p$, and total cross sections for γp, $\gamma\gamma$ and $\Sigma^{-}p$ [3, a].

Several experiments are under way [4], or being planned, to measure the hadronic amplitudes at $t = 0$. Some authors [5, 6] also presented what they feel are reference values for the total γp and $\gamma\gamma \to$ hadrons cross sections. Thus it is timely and appropriate to present independently our predictions for the forward observables at RHIC, the Tevatron-run II and the LHC as well as for γp total cross section at cosmic-ray energies and for $\gamma\gamma$ total cross sections up to 1 TeV.

We can summarize the general form of the parametrisations by quoting the form of total cross sections, from which the ρ parameter is obtained via analyticity. The ingredients are the contribution Y^{ab} of the highest meson trajectories (ρ, ω, a and f) and the rising $C = +1$ term H^{ab} from the pomeron contribution to the total cross section, which can be written for the scattering of a on b:

$$\sigma_{tot}^{ab} = (Y^{ab} + H^{ab})/s \tag{1}$$

The first term is parametrised via Regge theory, and we allow the lower trajectories to be partially degenerate, *i.e.* our experience shows that it is enough to introduce one intercept for the $C = +1$ trajectories, and another one for the $C = -1$ [7]. A further lifting of the degeneracy is certainly possible, but does not seem to modify significantly the results [8]. Hence we use

$$Y^{ab} = Y_{+}^{ab}\,(s/s_1)^{\alpha_+} \pm Y_{-}^{ab}\,(s/s_1)^{\alpha_-} \tag{2}$$

with $s_1 = 1$ GeV^2. The contribution of these trajectories is represented by RR in the model abbreviations.

As for the part rising with energy, we consider here two main options: it can rise as a $\log s$, or as a $\log^2 s$, with in each case the possibility to add a constant term. We shall not consider the simple-pole parametrisation [9], not only because it is disfavored by our ranking procedure (see below), but also because we want to make predictions at very high energies, where unitarisation must set in [b]. In the following, we shall only refer explicitly to our preferred parametrisation of H^{ab}, which we note as PL2:

$$H^{ab} \;=\; s(B^{ab}\ln^2(s/s_0) + P^{ab}) \tag{3}$$

where s_0 is a universal scale parameter (to be determined by the fits) identical for all collisions.

We have considered several possible constraints on the parameters of Eqs. (2-3): degeneracy of the reggeon trajectories ($\alpha_+ = \alpha_-$); universality of rising terms (B^{ab} independent of the hadrons) [10, 11]; factorization for the residues in the case of the $\gamma\gamma$ and γp cross sections ($H_{\gamma\gamma} = \delta H_{\gamma p} = \delta^2 H_{pp}$); quark counting rules [12] (predicting the Σp cross section from pp, Kp and πp); and finally the Johnson-Treiman-Freund [13] relation for the cross section differences.

Out of the 256 possible variants, we showed that 24 met our criteria for applicability (an overall $\chi^2/dof \leq 1.0$ and a non-negative pomeron contribution at all energies) if one fitted only to $\sigma_{\rm tot}$ for $\sqrt{s} \geq 10$ GeV, and 5 did for $\sqrt{s} \geq 4$ GeV (see Table XI from [1]), whereas 20 (resp. 4) variants obeyed this criterion when a fit to both $\sigma_{\rm tot}$ and ρ was performed, for $\sqrt{s} \geq$ 10 (resp. 5) GeV (see Table XIV from [1]). We shall neither give here the list of models, nor spell out ranking criteria based on new indicators that quantify certain qualities of the fits, but simply mention that the triple-pole parametrisation $\mathrm{RRP}_{nf}\mathrm{L2}_u$ [10, 11] was determined to be the highest-ranking model leading to the most satisfactory description of the data (see similar conclusions in [14]). This parameterization has a universal (u) $B\log^2(s/s_0)$ term, a non-factorizing (nf) constant term and non-degenerate lower trajectories.

We start by giving the predictions of this model, adjusted for ($\sqrt{s} \geq 5$ GeV), with updated data points from ZEUS [15]. These predictions include statistical errors calculated from the full error matrix E_{ij}. We define

$$\Delta Q = \sum_{ij} E_{ij}\frac{\partial^2 Q}{\partial x_i \partial x_j} \tag{4}$$

with $Q = \sigma_{tot}$ or ρ and x_i the parameters of the model. These errors are shown in Figs. 1 and 2 by a filled band, and in Tables II, III, and IV.

In these figures and tables, we also give our estimate of the systematic uncertainty coming from the discrepancy between different FNAL measurements of σ_{tot}: we fit $\mathrm{RRP}_{nf}\mathrm{L2}_u$ either to the high data (CDF) or to the low ones (E710/E811), and get two error bands. The distances from the central value of the combined fit to the upper (resp. lower) border of

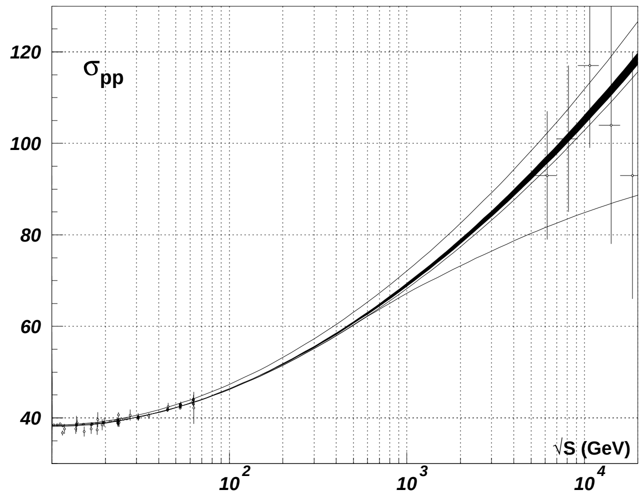

FIG. 1: Predictions for total cross sections. The black error band shows the statistical errors to the best fit, the closest curves near it give the sum of statistical and systematic errors to the best fit due to the ambiguity in Tevatron data , and the highest and lowest curves show the total errors bands from all models considered in this letter (note that the upper curve showing the systematic error is indistinguishable from the highest curve in this case).

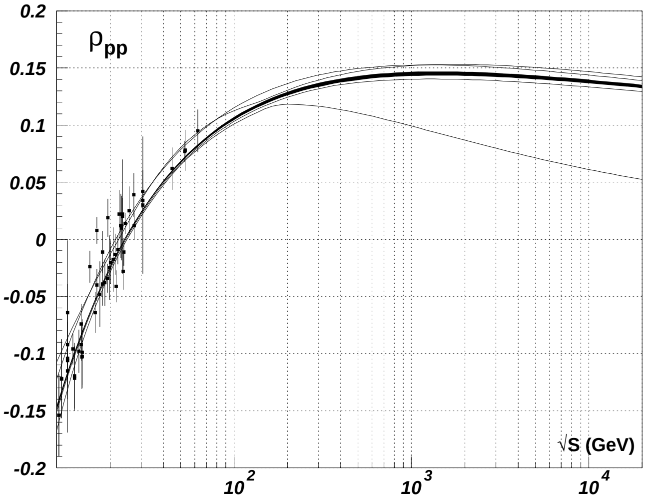

FIG. 2: Predictions for the ρ parameter. The curves and band are as in Fig. 1.

these bands give us the positive (resp. negative) systematic errors. We estimate the total errors as the sum of the systematic and of the statistical uncertainties [c].

One can see that the total errors on total cross sections are of the order of 1.9% at RHIC, of the order of 3.1% at the Tevatron and as large as 4.8% at the LHC and dominated by the systematic errors. The errors on the ρ parameter are much larger, reaching 5.4% at RHIC, 5.2% at the Tevatron and 5.4% at the LHC. This is due to the fact that experimental errors are bigger, hence less constraining, but this also stems from the incompatibility of some low-energy determinations of ρ [1]. This means that the systematic error is always bigger than the statistical one.

Concerning the contradictory data, we are forced to use them in our fits until the discrepancy is resolved by further experiments. In the case of the Tevatron data, one can see that the discrepancy results in a big shift (of more than 1σ) in the central value of the coefficient B of the $\log^2 s$ term, which controls the asymptotic behavior, and hence that asymptotic predictions are appreciably weakened by the present situation. The opportunities to measure σ_{tot} and ρ will be scarce in the future, hence any new measurement at RHIC, the Tevatron run II and the LHC should not be missed. Unfortunately, the recent publication of E-811 [16] does not clear the problem as their value for ρ is fully compatible with our preferred model, whereas their number for σ_{tot} (which is highly correlated with ρ) has hardly changed.

It is interesting to note that the choice of one FNAL result or the others leads to a variation of the overall fit quality, as shown in Table I (last two columns) [d]: the variant with CDF data has slightly better overall χ^2/dof and better χ^2/nop distribution over subsamples. We can consider this as an indication that the global picture emerging from fits to all data on forward observables supports the CDF data and disfavors the E710/E811 data at $\sqrt{s} = 1.8$ TeV (see also an analogous conclusion based on the other arguments in [17]).

Finally, we also present in Figs. 1 and 2 our estimate of the region where new physics would be discovered. For each of the 20 parametrisations which satisfy our criteria for applicability [1] for $\sqrt{s} \geq 10$ GeV, and which obey the Froissart-Martin bound [18, b], we construct errors bands according to Eq. (4). This gives us 20 1σ-error bands. Their union represents the “allowed region” where analytic models built according to (1) can reproduce the data. A measurement outside of this region would imply that new physics ingredients are needed.

To conclude, we believe that we have given here the best possible estimates for present and future pp and $\bar{p}p$ facilities. Although one might be tempted to use only data in an energy range close to the one measured, one must realize that analytic parametrisations are constrained both by lower-energy data, and by their asymptotic regime. Because the pomeron mixes (physically and numerically) with the f trajectory, fits to all data help to disentangle the two contributions.

TABLE I: Summary of the quality of the fits at different stages of the Review of Particle Physics (RPP) database (DB): DB02 – The 2002 RPP DB; DB02Z – the 2002 RPP DB with new ZEUS data, DB02Z-CDF – with the CDF point removed; DB02Z-E710/E811 with E710/E811 points removed. The first line gives the overall χ^2/dof for the global fits, the other lines give the χ^2/nop for data sub-samples, the last line gives in each case the parameter controlling the asymptotic form of cross sections.

Sample	DB02	DB02Z	DB02Z −CDF	DB02Z −E710/E811
total	0.968	0.966	0.964	0.951
total cross sections				
$\overline{p}p$	1.15	1.15	1.12	1.05
pp	0.84	0.84	0.84	0.84
$\pi^- p$	0.96	0.96	0.96	0.96
$\pi^+ p$	0.71	0.71	0.71	0.71
$K^- p$	0.62	0.62	0.62	0.61
$K^+ p$	0.71	0.71	0.71	0.71
$\Sigma^- p$	0.38	0.38	0.38	0.38
γp	0.61	0.58	0.58	0.58
$\gamma\gamma$	0.65	0.64	0.64	0.63
elastic forward Re/Im				
$\overline{p}p$	0.52	0.52	0.52	0.53
pp	1.83	1.83	1.83	1.80
$\pi^- p$	1.10	1.10	1.09	1.14
$\pi^+ p$	1.50	1.50	1.52	1.46
$K^- p$	1.00	0.99	1.01	0.96
$K^+ p$	1.07	1.07	1.10	0.98
values of the parameter B				
	0.307(10)	0.307(10)	0.301(10)	0.327(10)

A sharpening of our error bars would enable one to decide if the unitarisation plays an essential role and what form it takes. This in turn can have an impact on the determination of the survival of probability gaps in hard scattering, and on the usefulness of pomeron exchange as a detection tool.

Any significant deviation from the predictions based on model $\mathrm{RRP}_{nf}\mathrm{L2}_u$ will lead to a re-evaluation of the hierarchy of models and presumably change the preferred parametrisation to another one. A deviation from the "allowed region" would be an indication that strong interactions demand a generalization of the analytic models discussed so far, *e.g.* by adding odderon terms, or new pomeron terms, as suggested by QCD.

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TABLE II: Predictions for σ_{tot} and ρ, for $\bar{p}p$ (at $\sqrt{s} = 1960$ GeV) and for pp (all other energies). The central values and statistical errors correspond to the preferred model $\mathrm{RRP}_{nf}\mathrm{L2}_u$, and the systematic errors come from the consideration of two choices between CDF and E-710/E-811 $\bar{p}p$ data in the simultaneous global fits.

$\sqrt{s}$ (GeV)	σ (mb)	ρ
100	$46.37 \pm 0.06 {}^{+0.17}_{-0.09}$	$0.1058 \pm 0.0012 {}^{+0.0040}_{-0.0021}$
200	$51.76 \pm 0.12 {}^{+0.39}_{-0.21}$	$0.1275 \pm 0.0015 {}^{+0.0051}_{-0.0026}$
300	$55.50 \pm 0.17 {}^{+0.57}_{-0.30}$	$0.1352 \pm 0.0016 {}^{+0.0055}_{-0.0028}$
400	$58.41 \pm 0.21 {}^{+0.71}_{-0.36}$	$0.1391 \pm 0.0017 {}^{+0.0056}_{-0.0030}$
500	$60.82 \pm 0.25 {}^{+0.82}_{-0.45}$	$0.1413 \pm 0.0017 {}^{+0.0057}_{-0.0030}$
600	$62.87 \pm 0.28 {}^{+0.94}_{-0.48}$	$0.1416 \pm 0.0018 {}^{+0.0058}_{-0.0031}$
1960	$78.27 \pm 0.55 {}^{+1.85}_{-0.96}$	$0.1450 \pm 0.0018 {}^{+0.0057}_{-0.0030}$
10000	$105.1 \pm 1.1 {}^{+3.6}_{-1.9}$	$0.1382 \pm 0.0016 {}^{+0.0047}_{-0.0027}$
12000	$108.5 \pm 1.2 {}^{+3.8}_{-2.0}$	$0.1371 \pm 0.0015 {}^{+0.0046}_{-0.0026}$
14000	$111.5 \pm 1.2 {}^{+4.1}_{-2.1}$	$0.1361 \pm 0.0015 {}^{+0.0058}_{-0.0025}$

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TABLE III: Predictions for σ_{tot} for $\gamma p \to hadrons$ for cosmic ray photons. The central values, the statistical errors and the systematic errors are as in Table II.

p_{lab}^{γ} (GeV)	σ (mb)
$0.5 \cdot 10^6$	$0.243 \pm 0.009\,^{+0.011}_{-0.010}$
$1.0 \cdot 10^6$	$0.262 \pm 0.010\,^{+0.013}_{-0.011}$
$0.5 \cdot 10^7$	$0.311 \pm 0.014\,^{+0.019}_{-0.015}$
$1.0 \cdot 10^7$	$0.333 \pm 0.016\,^{+0.021}_{-0.017}$
$1.0 \cdot 10^8$	$0.418 \pm 0.022\,^{+0.030}_{-0.024}$
$1.0 \cdot 10^9$	$0.516 \pm 0.029\,^{+0.042}_{-0.032}$

TABLE IV: Predictions for σ_{tot} for $\gamma\gamma \to hadrons$. The central values, the statistical errors and the systematic errors are as in Table II.

$\sqrt{s}$ (GeV)	σ (μ b)
200	$0.546 \pm 0.027\,^{+0.027}_{-0.027}$
300	$0.610 \pm 0.035\,^{+0.037}_{-0.035}$
400	$0.659 \pm 0.042\,^{+0.044}_{-0.042}$
500	$0.700 \pm 0.047\,^{+0.050}_{-0.048}$
1000	$0.840 \pm 0.067\,^{+0.073}_{-0.069}$

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[a] The latest ZEUS point [15] on $\sigma_{tot}(\gamma p \to hadrons)$, that supersedes the previous two ZEUS points, is included in the present study for $\mathrm{RRP}_{nf}\mathrm{L2}_u$. Also the whole database was changed slightly since the last session of the cross assessment [1, 3] due to misprints and errata corrections. This improves slightly the global fit and central values are shifted noticeably within the error bars when one compares with results calculated from the parametrizations of [1, 2].

[b] Note that we checked [2] that the predictions of simple-pole models nevertheless fall into our allowed region.

[c] Note that, apart from the tiny statistical error, none of the errors or bands given in this work has a probabilistic interpretation.

[d] We have constructed a preliminary version of a Web-predictor, allowing one to obtain the central values and fit errors without systematics from our highest rank models at any collision energies $\sqrt{s} \geq 5$ GeV, see http://www.ihep.su/~tka4ehko/CS/MODELS/.