

MR941586 (89f:90171) [90C31](#) ([00A69](#) [90A30](#) [90C50](#))

★ **Multicriteria optimization in engineering and in the sciences.**

Edited by Wolfram Stadler.

Mathematical Concepts and Methods in Science and Engineering, 37.

Plenum Press, New York, 1988. xiv+405 pp. \$55.00. ISBN 0-306-42743-5

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The book can serve as a textbook for study in all areas where optimal decision making is of primary importance; it begins with the fundamentals of multicriteria optimization (MCO) theory, the mathematical framework for decision making in the presence of conflicting criteria. These theoretical fundamentals (preferences and orderings, the optimality concept, necessary conditions for Edgeworth-Pareto optimality, scalarization and sufficient conditions, etc.) are presented in enough detail so that the reader can begin to work his own optimization problems in any area of scientific endeavor where decision can be quantified in some fashion.

Chapter 2 covers the numerical methods for the linear case. “Real” problems (such as water resources applications) are considered as well as the theoretical concepts that are essential for a complete numerical analysis and understanding of these “real” applications. Subsequent chapters, dealing with applications in diverse areas, include discussions of the modeling process within the corresponding disciplines: approximation theory, resource planning and management, welfare theory, mathematical biology, engineering, aircraft control. Chapter 3 deals with certain vector approximation problems, where a vectorial “Kantorovich” norm is used instead of a usual real-valued norm. A collection of approximation problems with multiple criteria that arise in applications is first presented. Characterizations of Pareto optima are then described in which special scalarization techniques are used. Based on these theoretical considerations, numerical results are presented for two examples outlined at the beginning of the chapter. Finally, alternation theorems for Chebyshev vector approximation problems are formulated.

Welfare economics and the attempt to analyse and to evaluate the performance of market processes formulated as a vector maximum problem are considered in Chapter 4. The basic ingredients

of the most standard formal model of an economy, the Arrow-Debreu model (ADM), are first introduced. The so-called fundamental theorems of welfare economics are then stated and proved. The ADM is generalized to infinite dimensions. The effect of deviating from the assumptions and structural elements of the ADM, on the decomposition result and its economic interpretation, is analysed. Finally, a short glimpse of certain aspects of the scalarization of the vector maximum problem and some hints on the theory of social choice are presented. The broadest area of application, resource planning and management, is described in Chapter 5; water resources, energy policy planning, facility planning and land use planning are discussed. Selected MCO techniques are briefly reviewed and categorized into multicriterion choice methods and multicriterion programming techniques. The sciences are also represented by applications in mathematical biology; renewable resource management in Chapter 6 and competition, kin selection and evolutionary stable strategies in Chapter 7. The problem considered in Chapter 6 is a multispecies ecosystem (e.g., predator-prey) that is exploited by different groups of harvesters. Each group will concentrate on a single species. The operation is to be directed by a manager who must set rules for the maximum level of harvesting by each group of harvesters. The manager must set these limits without knowing the specific details of how the harvesters may actually operate under these rules, except that it is assumed that the harvesters will not violate the maximum limits. The manager's objective is to "maximize" the harvested yield for each species without having any of them become endangered by being driven to unacceptably low population levels. Three factors are involved: stability, vulnerability and optimization; the maximum harvesting limits as set by the manager must not produce an instability in the system that would result in the extinction of a species; it should satisfy an additional vulnerability requirement that none of the species will become endangered and finally, with these requirements satisfied, the harvesting limits should provide for an opportunity to attain return of profit or yield. A general two-species model and a specific two-species predator-prey model are analysed.

Chapter 7 presents a number of models that treat "evolutionarily stable strategies" (ESS) from a dynamic viewpoint. Classical competition theory deals mostly with the condition for competitive coexistence of species without regard to how coexistence actually evolves. This chapter attempts to take competition theory one step closer to its genetic substrate by supplying each competitor with a class of strategic alternatives and then enquiring how each should behave so as to coexist stably with the other. By adding controllable parameters to competition equations, the coexistence problem is tackled from the viewpoint of differential game theory. By restricting to annually breeding organisms with bilinear population dynamics, the search for strategic equilibria is reduced to games of resource allocation and timing. The model is generalized to the case where the competitors are related to one another; it is then possible to see how kin selection modifies competitive strategies. The competition model is then extended to include the effects of group selection, so that the interaction among individual, kin and group selection can be studied.

The remaining applications are in engineering: aircraft control (in Chapter 8), highly focused systems design (in Chapter 10) and structural optimization (in Chapters 9 and 11). In the optimal design of aircraft control systems, many disparate objectives must be considered; the pilot desires rapid, precise and decoupled response to his control inputs, so that the natural objective functions for computer-aided design are computable functions that are useful measures of the speed, stability,

and coupling of the responses. Additional design objective functions have been developed to avoid control limiting. Another important property of a good design is that it be “robust”; that is, the design objectives should be insensitive to significant uncertainties in system parameters. Therefore, a vector of “stochastic sensitivity” functions is defined as the vector of probabilities that each “deterministic” objective violate specified requirement limits, and decreasing the sensitivity is considered as a design objective. If both the deterministic objectives (the nominal and expected values) and their sensitivities are considered in the design process, then the number of design objective functions is doubled. The complexity of the design problem is characterized by variability and uncertainty in the objectives. Four design methods of increasing sophistication are discussed. The methods do not enable the computer to converge on an optimal design in any conventional sense of that term, but rather permit the designer to control a search for well-balanced solutions on the nondominated portion of the boundary of achievable solutions. This permits him to examine the achievable tradeoffs between the various objectives very efficiently and to make the final design choice based on his experience and judgement. The methods described in this chapter are applicable to a broad class of system design problems.

Chapter 9 briefly presents multicriteria design theory developed for optimizing elastic trusses, where the material volume and some chosen nodal displacements of a truss are chosen as criteria and where the basic configuration is specified (i.e. the support conditions as well as the number and the location of joints are fixed). The presentation includes several elements common to diverse applications in structural optimization and comprises three major topics: problem formulation, computation of the Pareto optimal set and an interactive design method. The theory is supplemented by several illustrative truss examples.

Chapter 10 is a presentation of extensive investigations on the theory of vector optimization and its application in the development and layout of components for highly accurate focusing systems (large parabolic antennas or radiotelescopes, new-technology optical telescopes as well as solar-energy collectors). Their manufacture not only requires minimizing costs but also observes such objectives as shape accuracy and reliability, since the efficiency of these instruments depends substantially on the surface accuracy. Here, the mostly competing and nonlinear objectives do not lead to one or several solution points for the optimum but rather lead to a “functional-efficient” solution set; i.e. the decision-maker selects the most efficient compromise solution out of such a set. The use of preference functions or quality criteria transforms the vector optimization problem into a scalar substitute problem. This so-called optimization strategy is a basic part of the modeling. For the transformation, a number of preference functions such as objective weighting, distance functions, constraint-oriented transformation (trade-off method) and min-max formulation are analysed and tested. It is shown that the efficiency of the single preference function depends both on the problem and on the adaptation to certain optimization algorithms (mathematical programming). A software package SAPOP has been developed by the author.

Chapter 11 deals with natural shapes, a unified optimal design philosophy. Good design is based on a thorough understanding of the limitations imposed by natural law as well as the existent technology. A clear understanding of natural phenomena can overcome perceived limitations of false theories. Therefore, in order to free ourselves from the shackles of such false limitations, our primary efforts must be directed toward an understanding of natural law. Our designs then will

reflect this understanding. The primary aim here is to match geometric shapes in nature. There are a number of obvious examples of shapes that have survived for millenia, such as seashells, stalactites, the bases of trees, the effects of erosion and so on. This survival should be an excellent indicator that they must indeed be optimal for the purpose that they serve. In order to formulate a corresponding optimal design problem, it remains to discover and quantify the purpose as well as the optimum. For example, the author explains the shape of a *Sequoia gigantea* tree up to a height of about 10 feet. If optimal design methods are to become generally accepted in industry, it is necessary that they not be limited to optimality as implied by the criteria. It should also be known what other desirable or undesirable aspects these designs exhibit from an engineering (rather than an economic) point of view. Some extremely undesirable stability aspects of minimum weight design are discussed as well as the manner in which most of these may be alleviated by inclusion of strain energy as an additional criterion. Indeed, the implications for natural shapes are deeper in the sense that these in fact exhibit extremely desirable stability aspects. It is shown through design examples in shallow arches that stability considerations are an essential aspect of optimal structural design. A structure certainly cannot realistically be considered optimal if it collapses prior to reaching its design load or if it is on the verge of collapse at the design load. The use of the natural structural shapes concept eliminates the minimum mass solution with its undesirable stability aspects as well as the existence questions encountered with the minimum strain energy problem. Aside from eliminating these negative aspects, the far-reaching stability implications for natural shapes become apparent. They concern the equivalence of stability and optimality conditions. In every natural process, there are many purposes that must be taken into simultaneous yet optimal consideration; hence, the use of the mass and strain energy in structural design.

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