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★**Thermocinétique approfondie. (French) [Advanced thermokinetics]**

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This book brings together the main analytical methods used in heat conduction problems. The mathematical tools needed are recalled in the appendices of the book. Chapter 1 is devoted to the Fourier and Hankel integral transforms and to their applications to 1-D and 2-D heat conduction in finite, semi-infinite and infinite slabs, in cylindrical and spherical geometries. In Chapter 2, the Laplace transform is used for problems similar to the ones of Chapter 1, for bimaterial walls, for moving boundaries (simulation of sublimation, ablation, erosion or material deposit), for thermal control, exchangers and regenerators. The method involving the Laplace transform is rather straightforward for all types of boundary conditions but is less efficient for problems with more than one space variable.

Thermal applications of the Duhamel method are presented in Chapter 3; this method is used to determine the time evolution of temperature in a solid under the influence of transient boundary conditions, in the cases where the solutions are known for constant boundary conditions. This is based on the linearity of the transfer equation, which holds when the thermal characteristics are temperature-independent. Chapter 4 uses the methods of fixed or movable sources of thermal energy in a three-dimensional field; the sources are instantaneous, continuous or periodic in time, point-like, linear or surface-like, or even doublets. The image method, well known in electrostatics, is also developed.

Different Green functions and their applications are outlined in Chapter 5. Analytic solutions do not always exist or are difficult to put in closed form or to compute; in these cases, approximate analytical approaches must be used. Among these, the integral Rietz method for steady state problems, the Kantorovich method for transient problems and the moment integral method are presented in Chapter 6 as well as variational methods (Rietz, Kantorovich variational methods and their extensions). Thermal problems involving phase transitions are analysed in Chapter 7. For these situations, the boundary is moving with heat absorption or rejection at the interface. The solution to these problems comprises the determination of the temperature distribution in space and time for both phases and the displacement of the movable boundary. These problems are nonlinear and very few can be solved in closed form. The following problems are considered: solidification or fusion of a semi-infinite wall (Neumann problem), solidification of an overcooled liquid, solidification in cylinder form, ablation, alloys phase transition (over a temperature interval), and phase transition with density change.

Nonlinear problems are treated in Chapter 8. No general solution method exists. Two types of nonlinearities are considered: thermal conduction with temperature-dependent thermophysical properties, and nonlinear boundary conditions. The methods outlined are the following: Kirchhoff transformation (when the thermophysical properties are only temperature-dependent); similarity transformations, with the Boltzmann transformation as a particular case (which, after a change of variables and functions, transform a system of partial differential equations into a new system of differential equations with one less variable, thus easier to solve if the boundary conditions satisfy certain constraints); linearisation methods; approximate analytical methods; Kantorovich integral method; Green function method; integral equations solved by iterations.

Chapter 9 deals with anisotropic materials, their thermal properties and applications. Particular methods are introduced in Chapter 10; these methods are perturbation theory, asymptotic development and gauge function, singular and regular perturbation methods, the corrected coordinates methods or the Lighthill method with its simplified version, the Pritulo method, series transposition, the Euler method, singularities extraction, the Shanks method, and the Padé approximant.

Some topics are not covered in the book. For example, on the theoretical side, the boundary integral method, inverse methods in general, and metrological aspects concerning anisotropic solids are not covered; on the numerical side, numerical developments such as finite difference and finite element methods, asymptotic methods, and numerical inversion of the Laplace transform are not covered.

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