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★**Incremental finite element modelling in nonlinear solid mechanics.**
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The objectives of this book are mainly to present and discuss the basic aspects of finite element (FE) methods applied to the nonlinear static and dynamic analysis of solid continua, excluding any thermal effects. It gives an engineer-oriented basis for the development and computer implementation of different nonlinear FE models but without giving a full account of the necessary numerical procedures. Nevertheless, the book contains some general comments on the numerical aspects of the FE method in the hope of setting up a more realistic picture of the practical usefulness of the method and in addition to bring to the reader's attention this fascinating branch of contemporary computational mechanics.

The first part of the book is devoted to the fundamentals of nonlinear solid mechanics. This part differs from the standard monographs on nonlinear mechanics by the way in which the incremental step-by-step description of finite deformation processes is introduced, together with some useful notation.

In linear elastic problems, the total (i.e. nonincremental) quantities are used as the basic unknowns. The unknowns of the kinematic type in such problems are the total displacements, while the total stress is the fundamental static-type unknown. In nonlinear problems, such an approach may prove inconvenient and for a class of inelastic materials described by some incremental constitutive laws, it is even impossible to avoid dealing with the incremental stresses and displacements. The rate (or incremental) form of the constitutive laws implies here the necessity of the step-by-step analysis of the body, subjected to gradually increasing external loads. The final state of the body, obtained as a result of the accumulation of the consecutive incremental problems, will in general differ from the hypothetical final state, obtained by a single solution of the problem with the total value of the external loads. This property of the problem is called the dependence of the response on the history of the deformation process. The analysis of such situations is the main subject of the present book.

Chapter 2 starts with the definitions of some basic kinematic variables which describe the incremental rather than the total properties of the deformation process. The expression for the incremental strain measure contains a quadratic term which will be retained in almost all the equations used in the book; this is fundamentally different from many other approaches to rate formulations in nonlinear mechanics. Instead of the usual problem of an infinitesimally small strain imposed on finite strains, the author faces here a computationally much more complex problem of imposing finite strain increments on the existing strains. Such a formulation may prove useful in working out some numerical algorithms.

In Chapter 3, the incremental stress measures (first and second Piola-Kirchhoff stress tensors) are introduced and the relationships between different stress measures are studied in the so-called convective coordinate system. Special attention is paid to the definition of the coordinate systems since the effectiveness of a numerical algorithm depends very much on the explicit forms of the equations written in the selected coordinates.

The detailed description of the solid mechanics incremental formulation follows in Chapter 4. This so-called incremental description of the deformation process is presented in order to formulate consistently the general boundary value problem of nonlinear continuum mechanics in a way oriented towards numerical methods.

Chapter 5 contains further discussion of incremental stress and strain measures; it investigates the interrelationships between the incremental stress components expressed with regard to the fixed, corotational and convective coordinates, and compares the incremental description of strains with the corresponding formula known in nonlinear continuum mechanics.

The incremental equations of motion for an infinitesimal element of the body are formulated in Chapter 6 by considering the dynamic equilibrium and the general forms of constitutive laws are discussed in Chapter 7. Only isothermal deformation of solids whose mechanical properties do not depend on time in any way are considered. A specific material law describing finite strains within the so-called J-theory of elastoplasticity is also given.

All the fundamental equations developed are rewritten once more in Chapter 8 with the use of greatly simplified notations. This is intended to facilitate the general understanding of Part II of the book without detailed knowledge of the precise formalism presented in Chapters 2–7.

Part II deals with different FE models which are all developed on a strict variational basis. Also, a discussion of basic numerical solution procedures is given. Both nonlinear static and dynamic models are dealt with.

Chapter 9 presents introductory remarks concerning the advantages of the variational approach in opposition to the classical, so-called local formulation of the equations. Variational theorems are presented in the context of linear elasticity. The possibilities of the variational alleviation of the continuity requirements imposed upon the functions appearing in the functional are illustrated. This feature is of primary significance in working out different approximate solution methods in nonlinear mechanics. It is shown that the variational approach consists in building a functional which, when subjected to variation, leads to the differential equations describing locally the considered problem. The application of these principles is fairly straightforward in many situations, but there are a number of problem areas in which it is difficult and this has provided the motivation for the development of a number of sophisticated alternative formulations. They derive from modifications of the basic variational principles which can be achieved by means of the method of Lagrange multipliers. A broad discussion of the explicit forms of these advanced methods constitutes an essential part of the remainder of this book. A typical generalization of the classical variational statements consists in the construction of a functional involving some additional independent fields which are utilized simultaneously. Such variational principles usually lose their extremal character, which means that the second variation of the corresponding functionals does not preserve the sign; however, the conditions of functional stationarity still generate the governing differential equations of the problem. Another essential feature which differs from variational formulations is the explicit inclusion (or exclusion) of the boundary conditions in the functional. The boundary conditions can be taken into account either by imposing them a priori upon the trial functions (the so-called forced conditions) or by their explicit inclusion in the functional, which has then a more complicated form (the so-called natural boundary conditions).

Chapter 10 presents a number of variational formulations describing incrementally posed problems of solid mechanics and introduces a series of variational principles in which different subsidiary conditions are introduced, thus reducing the complexity of the functionals themselves.

In Chapter 11, the basic concepts of the FE models are introduced as a method for real engineering situations to get an approximate solution in the sense that, in general, some (or all) of the differential field equations and boundary conditions are satisfied in an approximate sense only. The object is a consistent development of a variety of FE models based upon different variational formulations: the compatible displacement model, the displacement model in nonincremental formulation, the equilibrium model I and the mixed models. Some other mixed models can readily be developed by considering other continuity requirements at inter-element boundaries. However, apart from the models discussed, none of the other mixed models has found any applications yet. The functions appearing in the variational formulations in this chapter are smooth enough to ensure the existence of the functionals. The difficulty in practical generation of such shape functions is one of the major shortcomings of the conventional models. In order to circumvent this, some more general incremental variational statements allowing for discontinuities across the inter-element boundaries are constructed in Chapter 12. Such modifications are achieved by means of the method of Lagrange multipliers. The formulation forms the basis for the so-called hybrid FE models discussed in Chapter 13. Chapter 14 presents the stress models: the equilibrium model II, the hybrid stress model and the modified hybrid stress model. The hybrid stress model is derived from the modified complementary energy principle within the framework of the incremental formulation, by using independent expansions for interior and inter-element boundary incremental displacements. The modified hybrid stress model is based upon the Hellinger-Reissner variational principle, with some relaxed inter-element continuity requirements.

Chapter 15 presents incremental FE formulations in which the effects of equilibrium imbalance and compatibility mismatch are included.

Chapter 16 considers the dynamical problem without requiring the motion of the body to be quasi-static. First, linear FE models are constructed and, subsequently, the formulations are generalized to include nonlinear effects. The compatible displacement model, FEs in space and time, and the hybrid stress model based on Gurtin's functional are presented. The objective of Chapter 17 is to give a general outline of what are believed to be, at present, the most effective numerical algorithms for solving nonlinear FE equations, typical of the problems of nonlinear solid mechanics. First, the effective solution of the fundamental system of equations describing the quasi-static equilibrium of an FE nonlinear system is briefly described. Linearized stability and a nonlinear dynamics perturbation method are then presented.

Reviewed by P. Rochus (Liège)