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★**The finite element method.**
Linear static and dynamic finite element analysis.
With the collaboration of Robert M. Ferencz and Arthur M. Raefsky.
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The first chapter of this excellent book introduces the finite element method in the context of simple one-dimensional model problems. The different steps in the approximation and discretization of the problem are clearly presented, in particular the Galerkin approximation method, starting from the strong form of the problem, then its weak form, then the Galerkin approximation and finally the matrix equation, equivalent to the Galerkin approximation, to be numerically solved in the F.E. approximation. Several illustrative examples are developed and some general properties are demonstrated: properties of the stiffness matrix, properties of the approximate F.E. solution, the superconvergence phenomena, Gaussian elimination, ...Chapter 2 deals with variational formulations of two- and three-dimensional boundary value problems in heat conduction and elastostatics theory (in fact, all the problems governed by Laplace/Poisson equations, such as electrostatics, potential flow, elastic membranes and flow in porous media). These serve as the basis for finite element discretization and the techniques developed, and illustrate the relationship between "strong" or classical statements of boundary value problems and their "weak" or variational counterparts. The definition of element arrays and pertinent data processing concepts are also discussed. The Galerkin method of approximate solution is emphasized in Chapter 2 and throughout the book, rather than "variational principles" due to the significantly greater generality of the former approach.

In Chapter 3, the shape functions are defined in such a way that, as the finite element mesh is refined, the approximate Galerkin solution converges to the exact solution; sufficient conditions for convergence (the shape functions should be smooth on each element interior, continuous across each element boundary and complete) are stated and a variety of finite element interpolatory schemes are developed. These apply to triangles, quadrilaterals, tetrahedra, hexahedra, wedges, etc. The isoparametric concept is emphasized and special-purpose interpolatory strategies are also developed (e.g. "singular element"). Three of the most important implementational styles of stiffness formulation are described. Programming techniques are introduced for numerically integrated finite elements.

Chapter 4 deals with basic error estimates for standard "displacement" finite element methods and introduces mixed and penalty methods for constrained media applications such as incompressible elasticity and Stokes flow. It is shown how an arbitrary combination of displacement and pressure interpolations may prove ineffective in incompressible cases; the Babuška-Brezzi stability condition is presented. A heuristic approach for determining the ability of an element to perform well in incompressible and nearly incompressible applications is explained. A variety of "variational crimes" (as termed by Strang) are described: for example, incompatible elements, reduced and selective integration, strain projection (i.e., B-) methods, etc. (Most of these have been "decriminalized" in recent years.) The mathematical analysis of finite element methods for incompressible media is rather complex. David Malkus, an authority on this subject, explains some of the subtleties in an appendix to Chapter 4.

Chapter 5 is concerned with finite element methods for Reissner-Mindlin plates and elastic frame structures composed of straight beam elements accounting for axial, torsional, bending and transverse shear deformations (i.e. Timoshenko beams). This chapter discusses the basic techniques and considerations involved and summarizes recent developments in this area.

Chapter 6 deals with three-dimensional curved shell elements and two-dimensional special cases such as rings, tubes, and shells of revolution. A general formulation for curved structural elements is first presented. Throughout, transverse shear deformations are accounted for. This makes possible the use of C^0 interpolation as in the plate and beam theories of Chapter 5. Many different approaches have been developed in finite element shell analysis and an enormous literature now exists. No attempt to review the literature is made in this brief chapter, for a literature review in finite element shell analysis would entail in itself a major work. The approach presented herein is quite general and the one currently gaining favor. As for beams and plates in Chapter 5, the shell theory is derived directly from three-dimensional elasticity theory with certain kinematic and mechanical assumptions built in. The reduction to practically important two-dimensional cases is then treated.

The problem classes discussed in Chapters 1–6 give rise to associated time-dependent, transient, initial value problems (the parabolic heat equation and the hyperbolic elastodynamics and structural dynamics) and the associated eigenvalue problems (frequency analysis and buckling). The following chapters present a comprehensive presentation and analysis of algorithms for time-dependent phenomena. The formulations of problems of these types are the subjects of Chapter 7. Standard error estimates, eigenvalue estimates, error estimates for semidiscrete Galerkin approximation, alternative definitions of the mass matrix and the absence of a general theory for obtaining higher-order accurate mass matrices are also discussed among other subjects.

Chapter 8 presents time-stepping algorithms for first-order ordinary differential equation systems such as those arising from unsteady heat conduction ("parabolic case"). The classical family of generalized trapezoidal methods (forward and backward Euler, Crank-Nicolson, . . .) is presented in a consistent way as well as the different possible implementations, the stability, the consistency, the convergence and the accuracy of these algorithms, the von Neumann stability analysis applied to elementary finite difference equations for the 1-dimensional heat equation. The implicit methods such as the trapezoidal rule, which are unconditionally stable, second-order accurate, perform very well in heat conduction analysis. The drawbacks are the storage and equation-solving burden engendered by the coefficient matrix. Recently, using the element-by-element concept, the author and his collaborators developed methods which possess the desirable properties of implicit methods but in a simple computational setting. A product approximation of the element assembly is made so that the inversion of the coefficient matrix is replaced by sequential inversions of element matrices. Two methods of that type are outlined: one-pass and two-pass EBE (element-by-element) algorithms and the second method which solves the equations of the generalized trapezoidal algorithm by preconditioned conjugate gradients with an EBE approximate factorization preconditioner. The potential of this EBE method is greatest in three-dimensional applications where the bandwidth of the coefficient matrix is large, or especially in nonlinear applications where frequent refactorizations are typically necessary. The different steps of the modal analysis, which is an alternative method to the step-by-step integration, are explained.

Chapter 9 deals with algorithms for second-order ordinary differential equation systems such as those emanating from elastodynamics and structural dynamics ("hyperbolic and parabolic-hyperbolic case").

Chapter 10 presents basic algorithmic strategies for symmetric elliptic eigenvalue problems such as those encountered in free vibration and structural stability. A very efficient major software package for matrix eigenvalue and eigenvector calculations based on the Lanczos method is also presented in Chapter 10. The documentation of the Lanczos algorithm and software were written by Bahram Nour-Omid, an expert on procedures of this type. Chapter 11 presents an extensive linear static and dynamic finite element analysis computer program DLEARN, specially prepared and based on the methods developed in the book. It contains a very complete library of finite element software tools. This program is suitable for homework assignments, projects (e.g., programming additional elements), and research studies. DLEARN is highly structured for readability, maintainability and extendability and has been written specifically to complement and enhance the procedures described in the remainder of the book.

It should be stressed that some sections touch upon the frontiers of research and that many of the procedures described in this book are presented in book form for the first time, for example: strain projection (i.e. B-) methods, implicit-explicit finite element mesh partitions in transient analysis, element-by-element iterative solvers, complete computer implementation of predictor/multicorrector implicit/explicit algorithms based upon the Hilber-Hughes-Taylor alpha method, etc.

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