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Pion-nucleon vertex function with one nucleon off shell

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The pion-nucleon vertex function with an off-mass-shell nucleon is obtained through sideways dispersion relations with the P_{11} and S_{11} pion-nucleon phase shifts as only input. Contrary to the recent calculation of Nutt and Shakin, we find that the proper and improper vertex functions behave quite differently, indicating the importance of the nucleon propagator dressing. In particular the proper vertex function is found to have two poles in the unphysical region.

[NUCLEAR REACTIONS pion-nucleon vertex functions, sideways dispersion relations, nucleon self-energy corrections.]

Recently, with applications to pion-nucleus physics in mind, Nutt and Shakin[1] (hereafter referred as NS) performed a model calculation of the πNN vertex function in which one of the nucleon legs is put off the mass shell. Starting with a nonlinear integral equation for a completely off-shell vertex function and after numerous approximations (neglect of negative energy nucleon spinors, use of a separable form for the input πN amplitude, etc.), their solution was found to be dominated by the P_{11} Roper (1470 MeV) resonance. Their result also indicated the relative unimportance of the correction due to the nucleon self-energy, which they termed Γ_2.

In view of the nature of the approximations made in the NS model, we think it worthwhile to reexamine the problem from a different methodological point of view. We make use of sideways dispersion relations together with a spectral representation of the single nucleon propagator, as in the work of Ida.[2] This makes approximations of the NS type unnecessary.

To start with, we introduce the *improper* πNN vertex function of Bincer,[3] $K(W)$, by[4]

$$S_F^{-1}(\not{Q})\langle 0|\psi(0)|\vec{p}\alpha,\vec{q}a:\text{in}\rangle \equiv i[\Lambda_+(Q)K(W)+\Lambda_-(Q)K(-W)]G\gamma_5\tau_a u(\vec{p},\alpha)\,, \quad (1)$$

where $Q=p+q$, $\not{Q}=Q_\mu\gamma^\mu$, $W=(Q^2)^{1/2}$; ψ is the nucleon interpolating field, $\Lambda_\pm(Q)=(W\pm\not{Q})/2W$ the projector for a positive (negative) energy nucleon state,[5] and $S_F(\not{Q})=1/(\not{Q}-m)$ is the bare (Feynman) propagator for the nucleon with m the nucleon mass. $K(W)$ represents the one nucleon off-shell vertex function normalized as

$$K(m)=1\,,$$

so that G is identified as the renormalized πNN coupling constant.

In a similar manner the *proper* vertex function $\Gamma(W)$ may be introduced with the following replacements in Eq. (1):

$$K(\pm W)\to\Gamma(\pm W),\quad S_F(\not{Q})\to S_F'(\not{Q})\,,$$

where $S_F'(\not{Q})$ is the fully dressed nucleon propagator. Then we may write

$$\Gamma(W)=K(W)/J(W) \qquad (2)$$

with

$$J(W)=S_F'(W)/S_F(W)\,, \qquad (3)$$

from which it follows that [noting $J(m)=1$]

$$\Gamma(m)=1\,.$$

As far as their analytic structure is concerned, $K(W)$, $S_F'(W)$, and hence $\Gamma(W)$ all have branch cuts starting at $W=\pm(m+\mu)$ (μ is the pion mass). It is worthwhile to note that the vertex function Γ of NS ($\Gamma\equiv 1+\Gamma_1+\Gamma_2$) can be identified with our $\Gamma(W)$, while $1+\Gamma_1$, which is their first approximation to Γ, corresponds to our $K(W)$ when a correct renormalization procedure is carried out in the NS scheme.

The discontinuity relation for $K(W)$ reads[3]

$$\operatorname{Im}K(\pm W)=f_\pm(W)^*K(\pm W)+\sigma_\pm(W)\theta(W-m-2\mu) \qquad (W>m+\mu) \quad (4)$$

where $f_\pm(W)$ is the πN amplitude for P_{11} (positive sign) and S_{11} (negative sign) which takes the form

$$f_\pm(W)=\frac{\eta_\pm(W)\exp[2i\delta_\pm(W)]-1}{2i}\,,$$

with $\delta_\pm(W)$ and $\eta_\pm(W)$ the phase shift and inelasticity (+ for P_{11}, − for S_{11}), respectively. In addition $\sigma_\pm(W)$ in (4) is the contribution from higher mass (multiparticle) intermediate states.

In the absence of inelastic contributions, $K(W)$ could be written, through sideways dispersion relations, as a solution to the homogeneous Omnès-Muskhelishvili equation[3]:

$$K(W)=P(W)\exp\left\{\frac{W-m}{\pi}\int_{m+\mu}^{\infty}\left[\frac{\delta_+(W')}{(W'-m)(W'-W-i\epsilon)}-\frac{\delta_-(W')}{(W'+m)(W'+W+i\epsilon)}\right]dW'\right\}, \tag{5}$$

where $P(W)$ is an arbitrary polynomial satisfying $P(m)=1$. We choose $P(W)\equiv 1$ to avoid a possible danger of having a nondecreasing $|K(W)|$ for $W\to\infty$.

In practice, it is crucially important to take into account inelastic effects as both partial waves become highly inelastic above, say 1400 MeV. In our present work we shall neglect $\operatorname{Im}\sigma_\pm(W)$ in (4), but satisfy in part the above requirement by the following replacement in Eq. (5):

$$\delta_\pm(W)\to\hat{\delta}_\pm(W)=\tan^{-1}\left\{\frac{1-\eta_\pm(W)\cos[2\delta_\pm(W)]}{\eta_\pm(W)\sin[2\delta_\pm(W)]}\right\}; \tag{6}$$

in fact, this is the phase of $f_\pm(W)$.[6] We note that the above procedure is implicit in the NS approach.

The function $J(W)$ may be given in terms of the Kamefuchi-Umezawa-Lehmann-Källén (KULK) spectral representation:

$$J(W)=1+(W-m)\times\int_{m+\mu}^{\infty}\left[\frac{\rho_+(M)}{W+i\epsilon-M}+\frac{\rho_-(M)}{W+i\epsilon+M}\right]dM. \tag{7}$$

In the above expression the spectral functions $\rho_\pm(M)$ are non-negative and take the following form:

$$\rho_\pm(W)=\pm\frac{(2\pi)^3}{2}\sum_n\delta^4(Q-P_n)\times\operatorname{tr}\{\Lambda_\pm(Q)\langle 0|\psi(0)|n\rangle\langle n|\bar{\psi}(0)|0\rangle\}. \tag{8}$$

Here the summation runs over all the physical states n with the same quantum numbers as those of the nucleon. Next we isolate in $\rho_\pm(M)$ the contribution from $n=\pi N$ states $\rho_\pm^{\pi N}(M)$. The remaining contribution $\delta\rho_\pm(M)$ is then neglected in our present work. Now we may write

$$\rho_\pm^{\pi N}(M)=\frac{3G^2}{32\pi^2}\frac{[(M\mp m)^2-\mu^2]^{3/2}[(M\pm m)^2-\mu^2]^{1/2}}{(M\mp m)^2M^3}\times|K(\pm M)|^2. \tag{9}$$

Because of the effective phase in Eq. (6) mentioned earlier some part of the multiparticle (inelastic) contribution is effectively included in the above spectral functions. Thus once we get $K(W)$ from Eqs. (5) and (6), $J(W)$ and then $\Gamma(W)$ follow from Eqs. (7) and (2).

Phase shifts (and inelasticity parameters) are taken from (i) an analytic expression[7] up to $W\sim 1300$ MeV, (ii) the most recent amplitude analysis by Pietarinen[8] for the intermediate region (1300 MeV $<W<$ 4000 MeV), and (iii) a Regge model[9] for higher energies where no data are available. Owing to the once-subtracted form of the integral in Eq. (5), no artificial damping of the phase shifts to zero is necessary for $W\to\infty$ [our $\hat{\delta}_\pm(W)$ approaches $\pi/2$ in that limit]. Such damped phases would make $K(W)$ constant (>0) asymptotically.[10] The improper vertex function $K(W)$ thus obtained is shown in Figs. 1 and 2 (in solid lines). Like the NS model, its structure is governed by the resonances P_{11} (1470 and 1780) for $W>m+\mu$ and S_{11} (1520 and 1700) for $W<-(m+\mu)$. It may be worth pointing out that there is a third bump in $\operatorname{Im}K(W)$ above $W=2000$ MeV. Its origin is not known to us; perhaps it may be associated with a possible new resonance coming out of the Pietarinen analysis.

With regard to the value of $K(W)$ below the physical threshold we find

$$K(-m)=1.03$$

consistent with the soft pion result: $K(-m)\simeq 1/g_A(0)$,[11] where $g_A(t)$ is the nucleon axial-vector form factor. We note in passing that it is important to keep $\hat{\delta}_+(W)$ in the evaluation of $K(W<0)$ as its omission leads to $K(-m)=2.83$. e.g. On the other hand, a neglect of $\hat{\delta}_-(W)$ for $K(W>0)$ makes only a small change, of the order of 10%. The

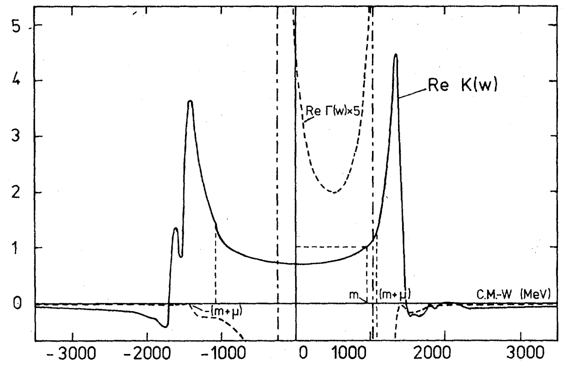

FIG. 1. The real part of $K(W)$ and $\Gamma(W)$: $K(W)$ in solid and $\Gamma(W)$ in dashed lines, respectively. Two vertical dot-dashed lines denote the pole positions of $\Gamma(W)$. Note that the actual value of Re $\Gamma(W)$ is five times smaller than that read off from the figure.

reason lies in the different denominator factors $(W' \pm m)$ assigned to respective phases in Eq. (5).

The integral (7) converged well with $\rho_\pm^{\pi N}(M)$ of Eq. (9) calculated from $K(W)$ above, giving the following interesting result: $J(W)$ has two zeros in the unphysical region, one ($\equiv W_-$) at ~ -125 MeV and the other ($\equiv W_+$) at ~ 1030 MeV. However, this is not an incongruity. In fact it can easily be shown through the spectral representation (7) that $S_F'(W)$ [and hence $J(W)$] can acquire at most two zeros, one in $(-m-\mu, m)$ and the other in $(m, m+\mu)$. As was noticed,[12] there should be no such zeros if the πN *strong* interaction is sufficiently *weak* to keep $\rho_\pm(M)$ small enough so that the bare nucleon propagator is a good approximation to the full one. A close examination indicates that the occurrence of zeros in our $J(W)$ is due mainly to the dominant

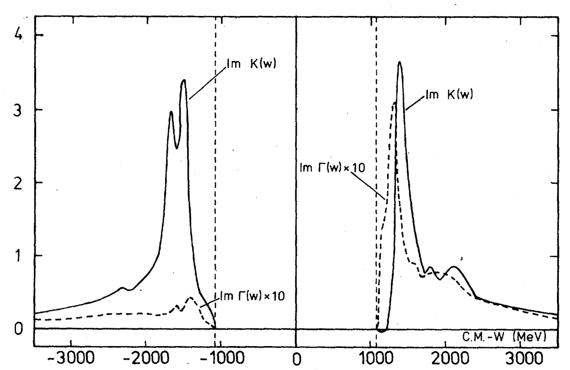

FIG. 2. The imaginary part of $K(W)$ and $\Gamma(W)$: $K(W)$ in solid and $\Gamma(W)$ in dashed lines. Im $\Gamma(W)$ is plotted as ten times its actual value.

contribution from the resonances P_{11} (1470) and S_{11} (1520 and 1700) to the spectral functions at not too high energies. A correct inclusion of the multiparticle contributions $\delta\rho_\pm(M)$ would only increase the magnitude of $\rho_\pm(M)$ at higher energies; hence the possible existence of zeros in $J(W)$ would be further strengthened.[13]

However, one might ask how reliable the effective phase in (6) is for the accurate evaluation of $K(W)$ and hence of $\rho_\pm^{\pi N}(W)$ throughout those important inelastic resonance regions. A definite answer may only be given after solving for $K(W)$ with an inclusion of $\sigma_\pm(W)$ in (4). We are planning such a calculation, but for the time being we shall be content with a couple of sensitivity tests through varying $\rho_\pm^{\pi N}(M)$. First we use a different set of phase shifts and inelasticities to calculate them. For this purpose we have used the CERN theoretical phases[14] for $1300 < W < 1900$ (MeV) matched smoothly to both the lower and higher energy parts mentioned before. Its main differences from the Pietarinen phases are as follows: (i) On the average it is less inelastic above the main resonances and (ii) the S_{11} (1700) resonance is less eminent. The gross feature of $J(W)$ thus obtained is not much different from our former result except for a change in the locations of the zeros: $W_+ \sim 1025$ MeV, $W_- \sim -140$ MeV. In the second test, using the Pietarinen phases we take $C\rho_\pm^{\pi N}(M)$ in place of $\rho_\pm^{\pi N}(M)$ in Eq. (7) and vary the constant C. Already for $C = 0.5$, W_+ has been lost and in its place appears a zero of Re $J(W)$ above but not far from threshold. For $C = 0.1$, even Re $J(W)$ becomes nonzero for all $W > m + \mu$ but W_- still stays at ~ -700 MeV.[15] It is quite unlikely that we have overestimated our spectral functions by an order of magnitude. Thus at least one zero, W_-, will remain. On the other hand some attempt in putting the multiparticle effects [through $\sigma_\pm(W)$ and $\delta\rho_\pm(M)$] into $\rho_\pm(M)$ is certainly needed, together with an accurate phase shift determination in order to make a definite statement about W_+. Incidentally, there is another argument[16] for the existence of zero(s) in $J(W)$ from a somewhat different line of reasoning.

In Figs. 1 and 2 $\Gamma(W)$ is plotted in dashed lines. Owing primarily to the zeros in $J(W)$, it looks considerably different from $K(W)$, contrary to the NS result, indicating the importance of their Γ_2 correction for the nucleon self-energy. In fact, in the NS model, nucleon propagators are always put on the mass shell. Together with the multiplicative renormalization procedure adopted there, this approximation might eventually end up with small corrections to Γ from Γ_2, hence with $K(W) \simeq \Gamma(W)$.

Clearly, $\Gamma(W)$ acquires poles at the zeros of

$J(W)$, viz., $W_{\pm}$.[17] In contrast to $K(W)$ the proper vertex $\Gamma(W)$ stays quite small and negative throughout all the physical region. It seems that the poles in the unphysical region simply have absorbed the strength of $\Gamma(W)$.

Now one might ask if one could see the effect of these poles of $\Gamma(W)$ in some physical processes. For πN scattering in the P_{11} and S_{11} partial waves the amplitudes take the form[2,18]

$$f^{\text{tot}}(W) = -R(W)\Gamma(W)S_F'(W)\Gamma(W) + f^{\text{irr}}(W) \qquad (10)$$

$$\left.\begin{array}{l} W > m+\mu \text{ for } P_{11} \\ W < -(m+\mu) \text{ for } S_{11} \end{array}\right\} \text{physical scattering},$$

where the first term is the fully dressed nucleon pole contribution with $R(W)$ a kinematical factor and the second term the *one nucleon irreducible* amplitude. Obviously the first term gets poles at $W_{\pm}$. However, the residues of $\Gamma(W)S_F'(W)\Gamma(W)$ there can be shown to be *negative*, that is, they are *ghostlike*. Fortunately, it was shown[12,19] that those poles in the dressed nucleon pole term are canceled exactly by the poles in $f^{\text{irr}}(W)$, so that there is no inconsistent behavior in the πN scattering amplitude [it is not difficult to show[2,12,20] that $f^{\text{irr}}(W)$ is unitary by itself; $\arg\Gamma(W) = \arg f^{\text{irr}}(W)$ (modulo π), etc. for $|W| < m+\mu$, suggesting the existence of such poles in $f^{\text{irr}}(W)$]. This, however, makes it impossible to see the poles in pion-nucleon scattering. So if these poles are really there in $\Gamma(W)$, we are forced to look for a possibility of observing them in other processes involving pions e.g., in pion-nucleus systems.

Finally, it may be worth mentioning that the well-known unrealistic behavior of the S-wave πN amplitude, which is calculated from the "bare" nucleon pole term in the $\gamma_5(PS)$ theory, especially near threshold is found to be greatly improved by the "dressed" one. In fact, the improvement turns out to come from the existence of the $\Gamma(W)$ pole at W_-. A more detailed and extended version of the present paper will be reported elsewhere.

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[4] Our normalization convention is $\bar{u}(\vec{p},\alpha)u(\vec{p},\beta) = 2m\delta_{\alpha\beta}$, $\langle\vec{p}\alpha\,|\,\vec{q}\beta\rangle = 2p_0\delta_{\alpha\beta}\delta^3(p-q)$, $pq = p_\mu q^\mu = p_0q_0 - \vec{p}\vec{q}$.

[5] Note that $f(\not{Q})$ as a function of $\not{Q}$ may generally be written as $f(\not{Q}) = \Lambda_+(Q)f(W) + \Lambda_-(Q)f(-W)$.

[6] We prefer this choice over that of Goldberger and Treiman for the same reason as found in T. N. Pham and T. N. Truong, Phys. Rev. D 14 185 (1976).

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[10] We have checked that in fact the Goldberger-Treiman phases $\delta_{\mp}^{\text{GT}}(W)$ constructed from our $\delta_{\pm}(W)$ and $\eta_{\pm}(W)$ become very small at high energies and the resulting $K(W)$ indeed approaches some positive constant.

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[13] Our spectral functions give the renormalization constant Z_2 as

$$Z_2^{-1} = \lim_{W\to\infty} J(W) = 1 + \int_{m+\mu}^{\infty} [\rho_+(M) + \rho_-(M)]\,dM.$$

Its value turns out to be $Z_2 = 0.007$. For a composite nucleon such as a triple-quark bound state, Z_2 should be zero, so the above integral diverges.

[14] D. H. Herndon, A. Barbaro-Galtieri and A. H. Rosenfeld, LRL Report No. UCRL-20030 πN, 1970 (unpublished).

[15] As speculated, (see Ref. 12) the missing zero presumably turns around the threhold branch point and moves into the unphysical sheet along the real axis as we decrease the value of C.

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[17] One might reconsider the solution (5) with a polynomial $P(W)$ having zeros in such a way that they coincide with the zeros of $J(W)$, hence, $\lambda(W)$ is free of poles. However, this modification of $K(W)$ also requires a modification of $J(W)$ through (7) and (9). So the coincidence of the zeros in $J(W)$ and $K(W)$ is highly unlikely. We therefore discard this possibility.

[18] For a nonrelativistic derivation of this form, see Appendix A of T. Mizutani and D. S. Koltun, Ann. Phys. (N. Y.) 109, 1 (1977).

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