

b - τ Yukawa non-unification in supersymmetric $SU(5)$ with an Abelian flavor symmetry

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We present a supersymmetric $SU(5) \times U(1)_H$ model, free from gauge anomalies, where the Abelian factor $U(1)_H$, introduced to account for the hierarchy of fermion masses and mixings, is broken by the same adjoint representations of $SU(5)$ that also break the GUT symmetry. The model predicts approximate, but never exact, b - τ Yukawa unification. The deviations are related to group theoretical coefficients and can naturally be at the level of 10%-20%.

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INTRODUCTION

The Standard Model (SM) of elementary particles represents one of the greatest achievements in theoretical physics of the past century. Particle interactions are derived from local symmetries and are explained at a fundamental level by means of the gauge principle. However, the SM does not provide any real clue for understanding other elementary particle properties, like the fermion masses and mixing angles, that are simply accommodated within the theory. Among the several approaches put forth to tackle this problem, the one proposed long ago by Froggatt and Nielsen (FN) [1] always attracted much interest in so far as it is able to account semi-quantitatively for the fermion mass pattern and to yield a couple of predictions. The basic ingredient is a horizontal Abelian symmetry $U(1)_H$ that forbids, at tree level, most of the fermion Yukawa couplings. The symmetry is spontaneously broken by the vacuum expectation value (vev) $\langle S \rangle$ of a SM singlet field. After $U(1)_H$ is broken a set of effective operators arises that couple the SM fermions to the electroweak Higgs boson, and that are induced by heavy vectorlike fields with mass M . The hierarchy of fermion masses results from the dimensional hierarchy among the various higher order operators, that are suppressed by powers of a small parameter $\epsilon = \langle S \rangle / M$. In turn, the suppression powers are determined by the horizontal charges assigned to the fermion fields. In the past, this mechanism was thoroughly studied in different contexts like the supersymmetric SM [2] or in frameworks where the horizontal symmetry is promoted to a gauge symmetry that can be anomalous [3, 4, 5, 6] or non-anomalous [7]. In this work we investigate the consequences of embedding the FN mechanism within a supersymmetric $SU(5)$ GUT. This appears as a natural step to take, given that the precise unification of the three gauge couplings within the minimal supersymmetric SM (MSSM) promoted the GUT idea to an almost compelling ingredient for a more fundamental theory.

However, a straightforward implementation of the FN mechanism within a Gran Unified model is not an easy task. Few models have been constructed in which, dif-

ferently from our case, an anomalous Abelian symmetry is used [8]. Indeed, in the context of GUTs, non-Abelian flavor symmetries have been often preferred for model building [9]. Within a GUT, the main difficulties with the FN mechanism arise because while in the SM there are five different multiplets per generation, and correspondingly five independent horizontal charges, this number is reduced to two in $SU(5)$, and to a single one in models in which a full fermion generation is assigned to the same gauge multiplet, like for example in $SO(10)$. Since the horizontal charges are free parameters of the model, it is clear that GUTs symmetries overconstrain the FN mechanism. Let us just mention the problem represented by mass ratios like $m_\mu/m_s \sim m_d/m_e \sim 3$ whose solution has always been a challenge for GUT model building [10]. Due to the fact that the leptons and the down-type quarks belong to the same gauge multiplets, it is clear that these mass relations cannot be explained simply by means of a suitable assignment of the horizontal charges.

The main aim of this work is to propose a mechanism that seems capable to account for the problematic mass relations, and therefore could reconcile the FN approach with the GUT idea. Here we will mainly focus on the m_b/m_τ mass ratio, and we will show that in our framework it could well deviate from unity, most naturally at the level of 10%-20%. Such a possibility appears important in view of the fact that the low energy values of the b and τ Yukawa couplings, when run up to the GUT scale by means of the MSSM renormalization group equations, do not unify with a precision comparable to gauge coupling unification. Yukawa unification at an acceptable level can be achieved only if strong restrictions are imposed on the MSSM parameter space, and a set of conditions for the supersymmetric particles masses are satisfied [11]. This fact also prompted for investigations of supersymmetric models with non-universal boundary conditions, that can better accommodate b - τ unification while satisfying other low energy constraints [12].

THE MODEL

In $SU(5)$ GUTs, the $SU(2)$ lepton doublets L and the down-quark singlets d^c are assigned to the fundamental

conjugate representation of the group $\bar{\mathbf{5}}$, while quark doublets Q , up-type quark singlets u^c and lepton singlets e^c fill up the antisymmetric $\mathbf{10}$. The Higgs field ϕ_d responsible for the down-quarks and lepton masses belong to another $\bar{\mathbf{5}}^{\phi_d}$, while ϕ_u responsible for the masses of the u -quarks is assigned to a fundamental $\mathbf{5}^{\phi_u}$. Schematically, the Yukawa Lagrangian reads

$$\mathcal{L}_Y = y_{ij}^d \bar{\mathbf{5}}_i \mathbf{10}_j \bar{\mathbf{5}}^{\phi_d} + y_{ij}^u \mathbf{10}_i \mathbf{10}_j \mathbf{5}^{\phi_u}, \quad (1)$$

where y^d represent the down-quarks and leptons Yukawa couplings, y^u the up-quark couplings, and $i, j = 1, 2, 3$ are generation indices.

Under the assumption that an additional $U(1)_H$ flavor symmetry is present and that the fermion masses are generated via the FN mechanism, the couplings $y_{ij}^{d,u}$ are no more simple dimensionless numbers, but embed suppression factors that account for the fact that the two terms in (1) now are effective operators of the low energy theory, that only arise after the breaking of $U(1)_H$. By introducing the notation f_i for the horizontal charge of $\bar{\mathbf{5}}_i$, t_i for the charge of $\mathbf{10}_i$ and f_u and f_d for the multiplets containing the up and down-type Higgs fields, the suppression factors can be written explicitly as:

$$y_{ij}^d = Y_{ij}^d \epsilon^{f_i+t_j+f_d}, \quad y_{ij}^u = Y_{ij}^u \epsilon^{t_i+t_j+f_u}. \quad (2)$$

As mentioned above, ϵ is a small number, with horizontal charge -1 , that arises from the ratio between the vev that breaks $U(1)_H$ and the large mass M of the heavy vectorlike FN fields needed to generate the effective operators. The numbers $Y_{ij}^{d,u}$ in (2) are assumed to be all of order unity, as is indeed more natural for dimensionless couplings. Phenomenologically, the mass hierarchy for the up-type quarks is much stronger than for the down quarks and leptons. This feature can be easily reproduced by means of the following charge assignment: $t_1 = t_2 + 1 = t_3 + 2$ and $f_i = f$ for each of the three $\bar{\mathbf{5}}$. This yields $m_u : m_c : m_t \approx \epsilon^4 : \epsilon^2 : 1$ and $m_{d,e} : m_{s,\mu} : m_{b,\tau} \approx \epsilon^2 : \epsilon : 1$. By choosing $\epsilon \approx 1/25$ the resulting mass ratios are qualitatively correct. Since the top Yukawa coupling is of order unity, it must be allowed by the horizontal symmetry. This implies $2t_3 + f_u = 0$ and suggests the simple choice $t_3 = f_u = 0$. The hierarchy $m_{b,\tau}/m_t \ll 1$ can also be easily accounted for by choosing $f + f_d = t_3 + f_u + 1 = 1$. Let us note that a redefinition $f \rightarrow f - x$ and $f_d \rightarrow f_d + x$ with x an arbitrary number, while it can affect the superpotential Higgs mass parameter $\mu \phi_d \phi_u$, it leaves invariant all the Yukawa couplings in (2). This is enough freedom to ensure that it is always possible to set to zero the $SU(5)^2 \times U(1)_H$ mixed anomaly $\propto 3f + \sum_i t_i + f_d + f_u - 2x$. The pure $U(1)_H^3$ and the mixed gravitational anomalies can always be canceled by adding SM singlets with suitable horizontal charges, and therefore $U(1)_H$ can be straightforwardly gauged, without implying additional constraints on the horizontal charges. This is quite different from the SM case

where consistency with phenomenology implies that either $U(1)_H$ is an anomalous symmetry [4, 6] or that the mass of the up quark must vanish [7].

It is noticeable how the gross features of the fermion mass hierarchy can be reproduced by such a simple scheme. Even more interestingly, when we assume that neutrinos masses are generated from dimension five operators yielding mass term $m_{ij}^\nu \bar{\mathbf{5}}_i \bar{\mathbf{5}}_j$, the scheme predicts no hierarchy between the neutrino masses and unsuppressed mixings. The last prediction is in qualitative agreement with the most recent results from neutrino oscillation experiments, while the first one is certainly a viable possibility. In fact, this simple scheme has been previously discussed in relation with anarchical neutrino mass matrices [13], and it has been shown that it is able to reproduce the known phenomenology in the neutrino sector (see however the criticisms in [14]).

Indeed it is somewhat disappointing that at a closer quantitative inspection the scheme produces too many wrong numerical results. Besides the two down-quark to leptons mass ratios mentioned above, the Cabibbo angle θ_C also turns out to be way too small, simply because the parameter ϵ that determines the suppression of the various mixings is much smaller than θ_C . In our opinion these discrepancies are too serious to be accounted for by simply appealing to random fluctuations in the values of the couplings Y_{ij} .

Nevertheless, as we will now discuss, the scheme can be retained if one of the initial ingredients of the FN mechanism is modified. While it is generally assumed that the horizontal symmetry is broken by a SM singlet, here we will use the $\mathbf{24}$ adjoint representation of $SU(5)$. We assign to the adjoint Σ that breaks $SU(5)$ down to the SM a horizontal charge -1 so that its vev $\langle \Sigma \rangle = V_a$ with $V_a = V \text{diag}(2, 2, 2, -3, -3)$ breaks also $U(1)_H$. Here the normalization factor $(2\sqrt{15})^{-1}$ of the $SU(5)$ generator T_{24} has been absorbed in V . It is clear that the suppression factor $\epsilon = V/M$ will now appear together with additional coefficients related to the different entries in V_a . We introduce also a second adjoint $\bar{\Sigma}$ with charge $+1$ in order to have a vectorlike representation $(\Sigma, \bar{\Sigma})$ under the horizontal symmetry, and allow for GUT scale masses. With this modification, the heavy FN fields responsible for inducing the mass operators of the light fermions are no more restricted to lie only in the $\mathbf{5}$ or $\mathbf{10}$ representations of the group, as is the case when the $U(1)_H$ breaking is triggered by a singlet. Higher dimensional representations like $\mathbf{15}$, $\mathbf{45}$, $\mathbf{70} \dots$ are now allowed, with the only restriction that the relevant vertices involved in the construction of the effective operators must be invariant under $SU(5) \times U(1)_H$.

Let us now see what are the implications for the b and τ mass operators that, being suppressed by only one power of ϵ , require just one insertion of $\langle \Sigma \rangle$. For the construction of these operators we assume one pair of vectorlike FN fields in the ten dimensional antisymmetric repre-

representations of $SU(5)$ ($\mathbf{10}$, $\overline{\mathbf{10}}$) and a second pair in the symmetric ($\mathbf{15}$, $\overline{\mathbf{15}}$). Recalling the light fermions charge assignments $f + f_d = 1$ and $t_3 = 0$, we need to assign a charge -1 to the $\mathbf{10}$ and $\mathbf{15}$, and $+1$ to the conjugate representations. The light eigenvalues m_b and m_τ of the resulting 3×3 mass matrices can be computed with a very good approximation by summing up the contributions of the two mass operators depicted in fig. 1:

$$\mathcal{L}_\pm = Y_\pm \bar{\mathbf{5}}_a \langle \bar{\mathbf{5}}_b^{\phi_d} \rangle \left(\frac{\delta_c^a \delta_d^b \pm \delta_d^a \delta_c^b}{2M} \right) \langle \Sigma_f^d \rangle \mathbf{10}^{fc}, \quad (3)$$

where a, b, \dots are $SU(5)$ indices. The term in parentheses arises from integrating out the heavy FN fields, with the plus (minus) sign corresponding to the $\mathbf{15}$ ($\mathbf{10}$), while Y_\pm are two numbers of order unity. By projecting out the vevs $\langle \bar{\mathbf{5}}_b^{\phi_d} \rangle = v \delta_b^5$ and $\langle \Sigma_f^d \rangle = V_a \delta_f^d$ we obtain:

$$\mathcal{L}_\pm = Y_\pm \frac{(-V_5 \pm V_a)}{2M} \bar{\mathbf{5}}_a 10^{a5} v, \quad (4)$$

with $V_5 = V_4 = -3V$ and $V_1 = V_2 = V_3 = 2V$. Recalling the $SU(5)$ field assignments $\mathbf{10}^{a5} = (d, d, d, e^c, 0)^T$ and $\bar{\mathbf{5}}_a = (d^c, d^c, d^c, e, -\nu)$, and summing up the two contributions, for the b quark ($a = 1, 2, 3$) and τ lepton ($a = 4$) masses we obtain

$$m_b = \frac{1}{2} (5Y_+ + Y_-) v \epsilon \quad (5)$$

$$m_\tau = \frac{1}{2} (6Y_-) v \epsilon = m_b + \delta m \quad (6)$$

$$\delta m = \frac{5}{2} (Y_- - Y_+) v \epsilon. \quad (7)$$

We see that mass unification ($\delta m = 0$) is achieved only if the two couplings Y_\pm are equal. However, there is no reason for this to happen. δm could be a positive or negative quantity, but in general will be non vanishing. The only requirement for Y_\pm is the same one that motivates the whole approach. Namely, they must be of order unity, implying that the hierarchy of the fermion masses and mixing angles is solely determined by the horizontal symmetry, while fluctuations of the dimensionless couplings around unity are only responsible for their exact numerical values. Since we do not have yet a theory for the order one couplings, it is not possible to state what is the size of these fluctuations, but it seems not an unreasonable guess to assume an order of at least a few percent, implying $\delta m/m \sim 10\% - 20\%$. From a bottom-up point of view, it is clear that the low energy values of the b and τ Yukawa couplings when run up to the scale where the GUT symmetry and the horizontal symmetry are broken will only approximately unify, even if the unification of more fundamental couplings like Y_+ or Y_- is exact. We stress that b and τ Yukawa non-unification is a general outcome of breaking $U(1)_H$ through the adjoint of $SU(5)$. The particular FN representations introduced, only determine the size of the deviations from unification. For

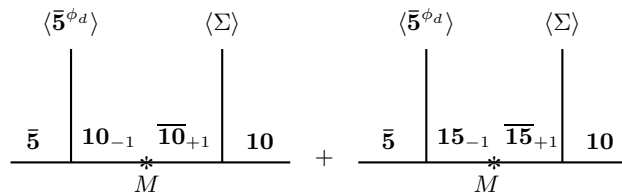


FIG. 1: Diagrammatic representation of the contributions to the masses of the τ lepton and of the b quark. The subscripts -1 and $+1$ represent the horizontal charges of the FN fields.

example, had we used just one pair of $(\mathbf{5}, \bar{\mathbf{5}})$ (and just one coupling Y_5) instead than $\mathbf{10}$ and $\mathbf{15}$, we would have obtained $m_b/m_\tau = 2/3$. However, this deviates too much from unity to be phenomenologically acceptable.

In this work we focused on the issue of b - τ Yukawa unification. The prediction of an approximate but never exact unification is an intriguing outcome of our framework. It would be interesting to develop further the model and try to explain mass ratios like m_e/m_d and m_μ/m_s that, due to their large deviation from unity, represent a real challenge to the idea of Gran Unification. We plan to explore these issues in a future publication [15].

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