# RENORMALIZATION PROGRAMME FOR EFFECTIVE THEORIES *

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We summarize our latest developments in perturbative treating the effective theories of strong interactions. We discuss the principles of constructing the mathematically correct expressions for the S-matrix elements at a given loop order and briefly review the renormalization procedure. This talk shall provide the philosophical basement as well as serve as an introduction for the material presented at this conference by A. Vereshagin and K. Semenov-Tian-Shansky [1].

## 1 Introduction

In the papers [2] – [4] we started the systematic study of the special class of effective theories[1] of strong interactions, which we call *localizable*. Roughly speaking, these are theories with the $S$-matrix which can be obtained in a *perturbative way* in the frame of an *effective field theory* that contains auxiliary resonance fields along with the fields of true asymptotic states (those stable with respect to the strong interaction)[2]. Our goal is to construct an efficient method for calculating the amplitudes of physical processes. This means that we need to develop the systematic scheme of perturbative calculations in the framework of infinite component effective field theory.

There are few obstacles that usually prevents the effective theory concept to become a useful computational tool. The main one is the presence of an infinite number of coupling constants, which requires introducing an infinite number of renormalization prescriptions. It is clear that we hardly get some predictive power for our theory unless find some *regularity* in the system of those prescriptions. Further, if one admits an unlimited number of the resonance fields in a theory (as we do), then even the amplitude

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[1] Recall that we use this term in its original (though slightly modified) meaning suggested by Weinberg [5], see also [6]; the precise definitions can be found in [3, 4].

[2] The localizability property requires separate consideration which we do not present here due to lack of space. It will be done elsewhere, the preliminary discussion can be found in [3].

of a given loop order (say, tree level) acquires contributions from the infinite number of graphs. The problem of convergence of the latter *functional* series is the problem of correct definition of the loop expansion.

Our approach suggests a solution. First, we systematize the set of renormalization prescriptions. In [4] we demonstrated that only the combinations of coupling constants that we call as *resultant* parameters require fixing to obtain renormalized $S$-matrix at any given loop order. Next, we show that even these renormalization prescriptions cannot be taken arbitrary — it is the convergence requirements giving a hand. Here is the basic idea. Any requirement of convergence of functional series always can be thought as a *restriction for the parameters* appearing in those series. In other words, *the couplings, and so the renormalization prescriptions, are unavoidably restricted for each step of perturbation expansion to make sense.* It is this circumstance that eventually gives rise to the system of bootstrap equations for physical parameters (see [2, 3]) and allows to make numerical predictions.

Briefly speaking, we develop Dyson's perturbation scheme for the infinite component effective theories taking seriously the problems of mathematical correctness and self-consistency. In this talk we try to make a short overview of our strategy and explain the main postulates.

## 2 Resultant (minimal) parameters and renormalization prescriptions

The first (and, probably, the main) step toward a classification of the renormalization prescriptions is transition to the *minimal parametrization* discussed in [4]. This allows one to rewrite every given graph in terms of the minimal propagators and minimal effective vertices. The numerator of the minimal propagator is just a covariant spin sum considered as a function of four independent components of momentum. At tree level the minimal vertices are just the Hamiltonian on-shell vertices with the wave functions crossed out.

To be precise, one shall consider an $S$-matrix element of the formal infinite sum of all the Hamiltonian items constructed from a given set of, say, $k$ (normally ordered) field operators. The members of this sum differ from each other by the Hamiltonian coupling constants, by number of differentiation operators and/or, possibly, by their matrix structure. The matrix element under consideration should be calculated on the mass shell, presented in a Lorentz-covariant form and considered as a function of $4(k-1)$ independent components of the particle momenta. The wave functions should be crossed

out. The resulting structure we call as the $k$-leg *minimal effective vertex of the 0-the level*. Every minimal effective vertex presents a finite sum of scalar formfactors dotted by the corresponding tensor structures, each formfactor being a formal power series, or even just a polynomial in relevant scalar kinematical variables. The *$l$-th level* effective vertex only differs from that described above by the presence of bubbles[3] ($l$ loops in total) attached to the same point as the external legs. The numerical coefficients (eventually supplied with the index $l$) in the formal power series that describe the corresponding formfactors are called as the *$l$-th level minimal parameters*. The above-mentioned resultant parameters are certain sums of the minimal ones.

The resultant parameters are all independent, as far as one considers as independent all the Hamiltonian couplings. Besides, as shown in [4], every *amplitude graph*[4] depends only on minimal parameters. In turn, the full sum of $L$-th loop order graphs describing certain scattering process can be expressed solely in terms of the resultant parameters with level index $l \leq L$.

At least a few words should be said here about the renormalization prescriptions. In [4] it is shown that, if the renormalization point is taken on shell, the resultant parameters are the only quantities that require formulating renormalization prescriptions. Actually, our use of the minimal parametrization implies that we rely upon the scheme of *renormalized perturbation theory*. In this scheme one starts from the physical action written in terms of *physical* parameters and adjusts the counter terms in such a way that the numerical values of those parameters remain unchanged after renormalization. It is this fact that allows us to obtain the bootstrap restrictions for the physical (in principle *measurable*) parameters later on.

## 3 Polynomial boundedness and summability

One of the most important requirements which we make use of when constructing the meaningful items of Dyson's perturbation series is that of polynomial boundedness. Namely, the full sum of $S$-matrix graphs with a given set of external lines and fixed number $L$ of loops must be polynomially bounded in every pair energy at fixed values of the other kinematical variables[5]. There are two basic reasons for imposing this limitation. First, from the general postulates of quantum field theory (see, e.g., [7]) it follows that

[3] Some of them may have multi-loop inner structure.

[4] That computed on the mass shell of all external particles and dotted by the relevant wave functions.

[5] This is not precise enough: in the case of several variables one has first to fix the concrete choice of them. We shall not discuss it in detail here.

the full (non-perturbative) amplitude must be a polynomially bounded function of its variables. Second, from experiment it follows that this is quite a reasonable requirement.

Since we never fit data with non-perturbative expressions for the amplitude, it is natural to impose the polynomial boundedness requirement on a sum of terms up to any fixed loop order and, hence, on the sum of terms of each order. Similar argumentation also works with respect to the bounding polynomial degrees. To avoid unnecessary mutual contractions between different terms of the loop series, it makes sense to attract the following *asymptotic uniformity* requirement. *The degrees of the bounding polynomials which specify the asymptotics of the amplitude of a given loop order must be equal to those specifying the asymptotics of the full (non-perturbative) amplitude of the process under consideration.* Surely, this latter degree may depend on the type of the process as well as on the values of the variables kept fixed.

The condition of asymptotic uniformity (or, simply, uniformity) is concerned with the asymptotic behavior of the items corresponding to different loop orders. It tells us not too much about the rules needed to convert the disordered sum of graphs with the same number of loops (and, of course, describing the same process) into the well-defined (*summable*) functional series. To solve the latter problem we rely upon another general principle which we call as *summability requirement*[6]. It is formulated as follows. *In every sufficiently small domain of the complex space of kinematical variables there must exist an appropriate order of summation of the formal sum of contributions coming from the graphs with a given number of loops, such that the reorganized series converges. Altogether, these series must define a unique analytic function with only those singularities which are presented in contributions of individual graphs.*

At first glance, the summability (analyticity) requirement may seem somewhat artificial. This is not true. There are certain mathematical and field-theoretical reasons for taking it as the guiding principle that provides a possibility to manage infinite formal sums of graphs in a way allowing to avoid inconsistencies. It is, actually, *both* the summability and uniformity principles that allow us to use the Cauchy formula and obtain well defined expression for the amplitude of a given loop order [1, 2, 3, 4].

We would like to stress that the requirements of uniformity and summability are nothing but independent subsidiary conditions fixing the type of perturbation scheme which we only work with. Surely, there is no guarantee that on this way one can construct the most general expressions for the S-matrix elements in the case of effective theory.

[6] By analogy with the maximal analyticity principle widely used in the analytic theory of S-matrix (see, e.g. [8]) sometimes we call it as *analyticity principle*.

Nevertheless, there is a hope to construct at least a meaningful ones presented by the Dyson's type perturbation series only containing the well-defined items.

## 4 Amplitude calculation and bootstrap

The explicit calculations are better illustrated by the concrete examples [1], here we just briefly sketch the strategy.

One needs to consider any process and classify all the resultant parameters contributing to it at a given loop order[7]. Then, using the stated above principles of uniformity and summability, one applies the well known Cauchy formula to the amplitude of the given order at various regions in the space of kinematical variables.

Several things happen during this process. First, the summability principle helps to identify the parameters of the amplitude singularities as a combination of resultant parameters which are already fixed by renormalization conditions. With this in hands, and with the degree of bounding polynomial given by uniformity principle[8], the Cauchy formula allows one to write down a *well defined expression for the amplitude in a given domain (layer) of the space of kinematical variables.*

Next, one obtains *different expressions for the same amplitude in different layers*, different couplings and masses contributing to each of them. The layers intersect, so one should equate the expressions in the common domains of validity to ensure self-consistency (usually, it appears to be equivalent to requirement of crossing symmetry of the given loop order amplitude).

The latter step gives an infinite number of numerical relations for the resultant parameters and thus, as explained above, for the renormalization prescriptions or, the same, for the physical parameters of a theory. We call these relations as the *bootstrap equations* of a given loop order. It should be noted, that, although the bootstrap for each loop order amplitude gives restrictions for *physical* parameters (the latter are independent of the loop order), the structure of bootstrap equations themselves varies from order to order. The search for the *solution* of all these equations is still beyond our scope[9], what we can only do so far is to test them with experimental data. The results are presented in two other talks, see [1].

[7] So far we performed explicit calculations only for the case of tree level processes. Although the procedure generalizes for any loop level, the calculations becomes cumbersome in the latter case. We shall demonstrate a concrete example of 1 loop calculation in the forthcoming publications.

[8] At this step we attract the known information on the Regge intercepts.

[9] See, however, the discussion in [3].

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**References**

[1] A. Vereshagin, *"Explicit equations for renormalization prescriptions in the case of pion-nucleon scattering"*, this conference; K. Semenov-Tian-Shansky, *"Renormalization in effective theories: prescriptions for kaon-nucleon resonance parameters"*, this conference.

[2] A. Vereshagin and V. Vereshagin, Phys. Rev. D **59** (1999) 016002.

[3] A. Vereshagin, V. Vereshagin, and K. Semenov-Tian-Shansky, Zap. Nauchn. Sem. POMI **291**, Part 17, (2002) 78 (in Russian). hep-th/0303242 (The English version will appear in "J. Math. Sci." (NY) **125**, iss. 2 (2005) 114).

[4] A. Vereshagin and V. Vereshagin, Phys. Rev. D **69** (2004) 025002.

[5] S. Weinberg, Physica A **96** (1979) 327.

[6] S. Weinberg, *The Quantum Theory of Fields*, vols. 1-3 (Cambridge University Press, Cambridge, England, 2000).

[7] N. N. Bogoliubov, A. A. Logunov and I. T. Todorov. *Foundations of the axiomatic method in quantum field theory* (Nauka, Moskow, 1969; in Russian).

[8] J. F. Chew. *The analytic S-matrix* (W. A. Benjamin, Inc. New York, 1966).