MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

BOOTSTRAP METHOD FOR THE PHYSICAL VALUES OF πN RESONANCE PARAMETERS

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Abstract

We argue how it is possible to apply the general scheme of the effective scattering theory (EST) to the description of the hadronic processes. The results of the numerical tests of sum rules for πN spectrum parameters that follow from the bootstrap system allow us to claim the consistency of the predictions obtained in the framework of our approach with the known experimental data.

1 Introduction

The essence of our work is an attempt to develop a self consistent Dyson perturbation technique for the infinite component effective scattering theory of strong interaction. It is quite reasonable to start from the definition of such a theory. We use a slightly modified version of the definition first given in [1]. The field theory is called effective if the quantum interaction Hamiltonian (in the interaction picture) contains all the monomials consistent with a given algebraic (linear) symmetry. The effective theories are as renormalizable as the ordinary renormalizable ones. The only difference is that one needs to formulate an infinite number of renormalization prescriptions (RPs) fixing the finite part of counterterms. Effective theories are intrinsically quantum constructions since we relay upon Weinberg scheme of constructing QFT

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(see e.g. [2]). This approach is adjusted for S-matrix calculations. We call a theory constructed with the use of this scheme as effective scattering theory (EST).

In our approach we deal only with a very narrow class of localizable effective scattering theories and introduce the notion of extended perturbative scheme (see the discussion in [3, 5]). The hypothetical localizable effective theory of strong interaction requires an infinite extension of perturbative scheme by introduction of an infinite tower of baryon and meson resonances of arbitrary high spin and mass. When dealing with such a theory one has first to point out a way to assign meaning to the perturbation series. The second problem is to somehow reduce the number of independent parameters for which it is necessary to formulate RPs fixing the physical contents of the theory. In Refs. [3–5] we propose a way to construct the meaningful perturbative scheme for such a theory.

2 Construction of the Cauchy Forms

The amplitude $M_{a\alpha}^{b\beta}$ of πN binary scattering can be presented in the following form (isotopic invariance is taken to be the exact symmetry of strong interaction):

$$M_{a\alpha}^{b\beta} = \delta_{ba}\delta_{\beta\alpha}M^{+} + i\varepsilon_{bac}(\sigma_c)_{\beta\alpha}M^{-}, \tag{1}$$

where

$$M^{\pm} = \overline{u}(p', \lambda') \left\{ A^{\pm} + \left(\frac{\widetilde{k} + \widetilde{k'}}{2} \right) B^{\pm} \right\} u(p, \lambda) \ . \tag{2}$$

The invariant amplitudes $A^{\pm}$, $B^{\pm}$ are certain functions of Mandelstam variables s, t, u.

The tree-level binary πN scattering amplitude calculated in the framework of our effective theory approach is the sum of all possible s-, t- and u- channel resonance exchanges plus the sum of contributions of all possible $\pi\pi N\overline{N}$ vertices. To assign meaning to this sum (which is certainly a formal one) one has to switch to minimal parametrization (see [4, 5]) and to use the method of Cauchy forms ([3] and Refs. therein). The transition to the minimal parametrization helps to get of those combinations of Hamiltonian couplings which appear only in off-shell matrix elements and hence does not require the formulation of RPs since we are only interested in the calculation of the S-matrix.

To construct the Cauchy forms one needs to fix the values of the residues at the relevant poles and to choose properly the bounding polynomial degree. Residues at poles of tree-level amplitudes are just the on-shell spin sums dotted by the minimal triple coupling constants. It is at this step that we take the main advantage of minimal parametrization since there is only a finite number of minimal triple vertices for each resonance with given quantum numbers. The bounding polynomial degrees are chosen in accordance with the known values of corresponding Regge intercepts.

This results in uniformly converging series of singular terms defining tree-level amplitude as the polynomially bounded meromorphic function in three mutually intersecting layers $B_x:\ (x \in \mathbb{R},\ x \sim 0;\ \ \nu_x \in \mathbb{C})$, where $x = s,\ t,\ u$ and $\nu_x,\ x\ (x = s, t, u)$: $\nu_s = u - t,\ \ \nu_t = s - u,\ \ \nu_u = t - s$, fixing the invariant amplitudes in the layers up to few unknown functions.

One of the principal results of [5] states that if one relays upon the renormalized perturbation theory scheme with on-shell renormalization point it is sufficient to formulate RPs only for minimal triple couplings and (real) resonance masses. The next step is to show that although the number of RPs fixing the physical contents of EST is still infinite these RPs are not independent.

3 Bootstrap System

Bootstrap system arise as the natural requirement that the Cauchy forms (different in different layers) should coincide in the domains of intersection of layers. This system constrains the allowed values of fundamental observables of the theory (triple minimal couplings and mass parameters). Besides it completely determines the allowed form of the four-leg pointlike vertex contributions and in this way helps to fix completely the binary scattering amplitude.

For example the set of bootstrap constrains for A^- in $B_t \cap B_u$ domain reads:

$$\Psi_s(A^-) \equiv [\text{Cauchy form in } B_u] - [\text{Cauchy form in } B_t] = 0 \ \text{ for } t,\ u \sim 0. \quad (3)$$

Expanding the bootstrap equation in powers of kinematical variables $t,\ u$ in the vicinity of $(t = 0,\ u = 0)$ one obtains an infinite set of sum rules for minimal (resultant) triple couplings and resonance masse parameters. These constrains $(m,\ n = \ 0,\ 1, ...)$ read as:

$$\sum_{\substack{\text{N},\,\Delta \\ \text{baryons}}} g^2_{R_B \pi N} V_{m,n}(M_{R_B}, J, \mathcal{N}, I) - \sum_{\substack{\text{Mesons with} \\ \text{I=1, odd J,P=}-1}} g_{R_M \pi\pi} \cdot g_{R_M N \overline{N}} W_{m,n}(M_{R_M}, J) = 0. \quad (4)$$

Here $g_{R_B \pi N}$ $(g_{R_M \pi\pi},\ g_{R_M N \overline{N}})$ stand for minimal triple couplings of baryon (meson resonances) with pions and nucleons. $V_{m,n}$ and $W_{m,n}$ are certain known functions depending on resonance quantum numbers (mass parameter, spin, normality and isospin). Bootstrap constrains are renorm-invariant in the sense that they are the equations for physical renormalization prescriptions (RPs): triple couplings and mass parameters.

Since bootstrap constrains connect physical quantities the sum rules (4) can be checked with help of experimental data. The numerical check (see [6]) demonstrates a good fit thus supporting the system of postulates we use in our EST approach.

4 Conclusions

We develop the logically complete scheme of EST suitable for the description of hadronic scattering processes. Numerical test of sum rules for πN (and also $\pi\pi$ and KN) resonance parameters show that the system of postulates forming the basis our approach is consistent with the presently known phenomenology. We also argue that the sum rules derived from the bootstrap system can be used as a powerful tool to study hadron spectrum.

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