CPHT-PC098.1110, LPT-ORSAY 10-82

Baryon to meson transition distribution amplitudes and their spectral representation

B. Pire*, K. Semenov-Tian-Shansky*,† and L. Szymanowski**

*CPhT, École Polytechnique, CNRS, 91128, Palaiseau, France
†LPT, Université d'Orsay, CNRS, 91404 Orsay, France
**Soltan Institute for Nuclear Studies, Warsaw, Poland.

Abstract. We consider the problem of construction of a spectral representation for nucleon to meson transition distribution amplitudes (TDAs), non-diagonal matrix elements of nonlocal three quark operators between a nucleon and a meson states. We introduce the notion of quadruple distributions and generalize Radyshkin's factorized Ansatz for this issue. Modelling of baryon to meson TDAs in the complete domain of their definition opens the way to quantitative estimates of cross-sections for various hard exclusive reactions.

Keywords: Pion to nucleon TDAs, backward electroproduction of mesons, quadruple distributions
PACS: 13.60.-r, 13.60.Le, 14.20.Dh

INTRODUCTION

Nucleon to meson transition distribution amplitudes (TDAs) [1, 2, 3, 4] (also known as super skewed parton DAs) occur in the description of backward meson[1] electroproduction $\gamma^*(q)+N(p_1)\to N(p_2)+\pi(p_\pi)$ (as well as $N(p_1)+\bar{N}(p_2)\to\gamma^*(q)+\pi(p_\pi)$ in the near forward or backward regions) in the framework of the collinear factorization approach. Nucleon to meson TDAs represent new nonperturbative objects which extend the concept of generalized parton distributions (GPDs). Up to this moment nucleon to meson TDAs lacked any suitable phenomenological parametrization properly taking into account the requirement of Lorentz covariance which is manifest as the polynomiality property of the Mellin moment of TDAs in the relevant light-cone momentum fractions, as well as the support properties. For the case of GPDs this kind of requirements can be fulfilled employing the spectral representation in terms of double distributions (DDs) [5]. The corresponding spectral properties were established using the α-representation techniques [6]. The factorized Ansatz for DDs [7] became the basis for a variety of successful GPD models.

Following the line of [8] we address the problem of constructing a spectral representation for the case of nucleon to meson TDAs. This requires introduction of the notion of quadruple distributions. We also generalize Radyushkin's factorized Ansatz for these quantities. We are thus ready for phenomenological modelling of meson to nucleon TDAs and quantitative estimates of cross-sections of new classes of hard exclusive

[1] For definiteness we consider the pion case although the analysis is completely general.

reactions.

DEFINITION AND PROPERTIES OF πN TDAS

πN TDAs are defined as the Fourier transform of the matrix element between a nucleon and a pion states of the three-local light-cone quark operator. In the light-cone gauge $A^+ = 0$ their definition can be symbolically written as [1, 2]:

$$\int \left[\prod_{i=1}^{3} \frac{dz_i^-}{2\pi} e^{ix_i(P\cdot z_i)} \right] \langle \pi(P+\Delta/2)| \varepsilon_{abc} \psi^a_{j_1}(z_1) \psi^b_{j_2}(z_2) \psi^c_{j_3}(z_3) |N(P-\Delta/2)\rangle \Big|_{z_i^+=z_i^\perp=0}$$
$$\sim \delta(2\xi - x_1 - x_2 - x_3) H_{j_1 j_2 j_3}(x_1, x_2, x_3, \xi, u)\,. \qquad (1)$$

Here j_i stand for spin-flavor indices and a, b, c are color indices. The tensor decomposition of the r.h.s. of (1) involves a set of independent spin-flavor structures multiplied by corresponding invariant functions (πN TDAs). Each invariant function H depends on three longitudinal momentum fractions x_i ($i = 1,2,3$) subject to the constraint $x_1 + x_2 + x_3 = 2\xi$, skewness parameter $\xi = -(\Delta \cdot n)/(2P\cdot n)$ $(n^2 = 0)$ and the momentum transfer squared $u = \Delta^2$.

The support property in x_i is given by $-1+\xi \le x_i \le 1+\xi$. One can distinguish Efremov-Radyushkin-Brodsky-Lepage (ERBL)-like domain in which all three momentum fractions x_i are positive and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)-like domains in which one or two momentum fractions x_i are negative. It is convenient to introduce the so-called quark-diquark coordinates. There are three possible choices depending on which quarks are supposed to form a "diquark system". Below we employ the particular choice of quark-diquark coordinates: $v \equiv v_3 = \frac{x_1 - x_2}{2}$; $w \equiv w_3 = \frac{x_3 - x_1 - x_2}{2}$. We also introduce the notation $\xi' \equiv \xi'_3 = \frac{\xi - w_3}{2}$. The complete domain of definition of πN TDA in quark-diquark coordinates can be parameterized as follows: $-1 \le w \le 1$; $-1 + |\xi - \xi'| \le v \le 1 - |\xi - \xi'|$.

One may check that the (n_1, n_2, n_3)-th Mellin moment of πN TDAs (1) is related to the form factors of local three quark operators between nucleon and pion states and is a polynomial of ξ of order $n_1 + n_2 + n_3$ [2]. A phenomenological parametrization for nucleon to meson TDAs should properly incorporate the polynomiality property of the Mellin moments of TDAs in x_i as well as the support properties described above.

In our approach we use the relation between GPDs and DDs - which is known to be a particular case of the Radon transform [9] - as the building block for the spectral representation of the multipartonic generalization of GPDs with the restricted support properties. Indeed, representing GPDs as the Radon transform is the natural way to implement polynomiality property known as the Cavalieri conditions in the Radon transform theory and ensure the proper support in the longitudinal momentum fractions. In order to write down the spectral representation for πN TDAs we introduce for each of momentum fractions x_i the spectral parameters β_i, α_i. x_i are supposed to have the

---

[2] The problem of necessity of adding D-term like contributions to πN TDAs still lacks some analysis.

following decomposition in terms of these spectral parameters: $x_i = \xi + \beta_i + \alpha_i \xi$. In order to satisfy the constraint $x_1 + x_2 + x_3 = 2\xi$ we require that $\sum_i \beta_i = 0$ and $\sum_i \alpha_i = -1$.

This allows to write down the following spectral representation for πN TDAs:

$$H(x_1, x_2, x_3, \xi)|_{\sum x_i = 2\xi} = \Big[\prod_{i=1}^{3} \int_{\Omega_i} d\beta_i d\alpha_i\Big] \delta(x_1 - \xi - \beta_1 - \alpha_1 \xi)\, \delta(x_2 - \xi - \beta_2 - \alpha_2 \xi)$$
$$\times \delta(\beta_1 + \beta_2 + \beta_3)\delta(\alpha_1 + \alpha_2 + \alpha_3 + 1) F(\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3)\,. \quad (2)$$

By Ω_i we denote the copies of the usual DD square support in the parameter space: $\Omega_i = \{|\beta_i| \le 1; \quad |\alpha_i| \le 1 - |\beta_i|\}$. $F(\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3)$ depends on six spectral parameters which are however subject to constrains due to two last δ function in (2). Hence F is a quadruple distribution. After introducing the natural combinations of spectral parameters and switching to quark-diquark coordinates one may perform two integrations with the help of δ functions and obtain the following spectral representation for πN TDAs:

$$H(w, v, \xi)$$
$$= \int_{-1}^{1} d\sigma \int_{-1+|\sigma|/2}^{1-|\sigma|/2} d\rho \int_{-1+|\sigma|}^{1-|\rho-\sigma/2|-|\rho+\sigma/2|} d\omega \int_{-1/2+|\rho-\sigma/2|+\omega/2}^{1/2-|\rho+\sigma/2|-\omega/2} d\nu\, \delta(w - \sigma - \omega\xi)$$
$$\times \delta(v - \rho - \nu\xi) F(\sigma, \rho, \omega, \nu)\,, \quad (3)$$

where $F(\sigma, \rho, \omega, \nu) \equiv F(\rho - \sigma/2, -\rho - \sigma/2, \sigma, \nu - (1+\omega)/2, -\nu - (1+\omega)/2, \omega)$ is the quadruple distribution. One may check that the resulting πN TDA possesses the desired support properties in (w, v) variables. The polynomiality property of the Mellin moments in (w, v) is ensured by the very construction of the spectral representation (3).

RADYUSHKIN TYPE ANSATZ FOR πN TDAS

Now we discuss a possible approach for modelling quadruple distributions. In analogy with GPDs [7] we suggest to employ the following factorized Ansatz:

$$F(\sigma, \rho, \omega, \nu) = f(\sigma, \rho)\, h(\sigma, \rho, \omega, \nu)\,, \quad (4)$$

where $h(\sigma, \rho, \omega, \nu)$ is a profile function and $f(\sigma, \rho)$ determines the shape of πN TDA in the limit $\xi \to 0$. Exploiting further the analogy with the usual GPDs we assume that the shape of the profile h in (ω, ν) directions is determined by the asymptotic form of nucleon distribution amplitude $\Phi^{\rm as}(y_1, y_2, y_3) = 15/4 y_1 y_2 y_3$. This results in the following simple expression for the profile in terms of the initial constrained spectral parameters α_i and β_i:

$$h(\beta_1, \beta_2, \beta_3; \alpha_1, \alpha_2, \alpha_3)\Big|_{\substack{\sum_i \beta_i = 0 \\ \sum_i \alpha_i = -1}} = \frac{15}{4} \frac{\prod_{i=1}^{3} (1 + \alpha_i - |\beta_i|)}{(1 - \frac{1}{2}(|\beta_1| + |\beta_2| + |\beta_3|))^5}\Big|_{\substack{\sum_i \beta_i = 0 \\ \sum_i \alpha_i = -1}}\,. \quad (5)$$

For the moment we lack the knowledge of what may be the shape of πN TDAs in the unphysical limit $\xi \to 0$. In the numerical exercises of [8] we used a rather simple toy Ansatz $f(\beta_1,\beta_2,\beta_3)|_{\sum_i \beta_i=0} = 40/47\prod_{i=1}^{3}\theta(|\beta_i|\leq 1)(1-\beta_i^2)\big|_{\sum_i \beta_i=0}$ normalized to unity. This form ensures the good convergence of the relevant integrals expressing πN TDAs in the spectral representation (3). A more realistic model should take into account that in the limit $\xi \to 1$ πN TDAs are fixed due to the soft pion theorem [12] and are expressed through the nucleon DAs. In principle this allows to tune the shape of $f(\sigma,\rho)$. The recent calculations [10] of πN TDAs in the meson cloud model may also be useful for establishing the overall normalization. It is worth to mention that employing the factorized Ansatz (4) with the profile function (5) results in the reliable methods for the calculation of the convolution integrals in x_i occurring for $\gamma^* N \to \pi N$ amplitude.

CONCLUSIONS

We introduced the notion of quadruple distributions and construct the spectral representation of TDAs involving three parton correlators which arise in the description of baryon to meson transitions. We generalize Radyushkin's factorized Ansatz for the case of quadruple distributions and provide the explicit expression for the corresponding profile function. Our construction opens the way to quantitative modelling of baryon to meson TDAs in their complete domain of definition. The main part of our analysis can be directly applied to the nucleon to photon TDAs [11]. However the modelling of the corresponding quadruple distributions has to account for the anomalous nature of a photon [13]. This is a nontrivial task which deserves separate studies.

We are thankful to I. Anikin, J.-Ph. Lansberg, A. Radyushkin and S. Wallon for many discussions and helpful comments. K.S. also acknowledges the partial support by Consortium Physique des Deux Infinis (P2I). This work was supported by the Polish Grant N202 249235.

REFERENCES

1. L. L. Frankfurt, P. V. Pobylitsa, M. V. Polyakov and M. Strikman, *Phys. Rev. D* **60**, 014010 (1999) [arXiv:hep-ph/9901429].
2. L. Frankfurt, M. V. Polyakov, M. Strikman, D. Zhalov and M. Zhalov, "Novel hard semiexclusive processes and color singlet clusters in hadrons," in *Newport News 2002, Exclusive Processes at High Momentum Transfer*, edited by A. Radyushkin and P. Stoler; World Scientific, Singapore, 2002, pp.361-368 [arXiv:hep-ph/0211263].
3. B. Pire and L. Szymanowski, *Phys. Lett. B* **622**, 83 (2005) [arXiv:hep-ph/0504255].
4. J. P. Lansberg, B. Pire and L. Szymanowski, *Phys. Rev. D* **75**, 074004 (2007) [Erratum- *ibid. D* **77**, 019902 (2008)] [arXiv:hep-ph/0701125]; *Phys. Rev. D* **76**, 111502 (2007)[arXiv:0710.1267 [hep-ph]].
5. A. V. Radyushkin, *Phys. Rev. D* **56**, 5524 (1997) [arXiv:hep-ph/9704207].
6. A. V. Radyushkin, *Theor. Math. Phys.* **61**, 1144 (1985) [*Teor. Mat. Fiz.* **61**, 284 (1984)];
7. I. V. Musatov and A. V. Radyushkin, *Phys. Rev. D* **61**, 074027 (2000) [arXiv:hep-ph/9905376].
8. B. Pire, K. Semenov-Tian-Shansky and L. Szymanowski, *Phys. Rev. D* **82**, 094030 (2010) [arXiv:1008.0721 [hep-ph]].
9. O. V. Teryaev, *Phys. Lett. B* **510**, 125 (2001) [arXiv:hep-ph/0102303].
10. B. Pasquini, M. Pincetti and S. Boffi, *Phys. Rev. D* **80**, 014017 (2009) [arXiv:0905.4018 [hep-ph]].
11. B. Pire and L. Szymanowski, *Phys. Rev. D* **71**, 111501 (2005) [arXiv:hep-ph/0411387].
12. P. V. Pobylitsa, M. V. Polyakov and M. Strikman, *Phys. Rev. Lett.* **87**, 022001 (2001) [arXiv:hep-ph/0101279].
13. E. Witten, *Nucl. Phys. B* **120**, 189 (1977).