Quark transverse charge densities in the $\Delta(1232)$ resonance

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Using recent lattice QCD results on the $\Delta(1232)$ electromagnetic form factors we map out the quark transverse charge density in the Δ as viewed from a light front moving towards the baryon. The charge densities for a transversely polarized Δ are characterized by monopole, dipole, quadrupole, and octupole patterns. We discuss the "natural" values for the magnetic dipole, charge quadrupole and magnetic octupole moments which a point spin-3/2 particle, such as a gravitino of $\mathcal{N} = 2$ supergravity, displays. The moments of the transverse charge densities in a transversely polarized spin-3/2 particle are shown to be sensitive only to the anomalous parts, i.e. deviations from their "natural" values, of the magnetic dipole, charge quadrupole, and magnetic octupole moments, and vanish for a particle without internal structure. The lattice calculations show that the quark charge density in a Δ^+ of maximal transverse spin projection is elongated along the axis of the spin.

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I. INTRODUCTION

...a lot of information exists on the nucleon electromagnetic form factors, see Refs. [1–3] for some recent reviews.
...transition of the nucleon to its first excited state, $\Delta(1232)$, see Ref. [4] for a recent review.

II. THE $\gamma^*\Delta\Delta$ VERTEX AND FORM FACTORS

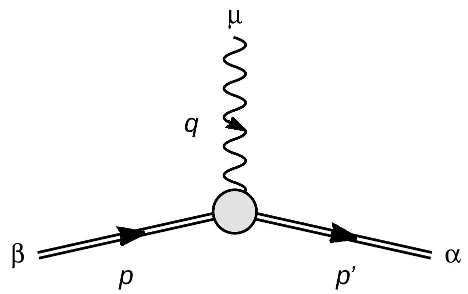

FIG. 1: The $\gamma^*\Delta\Delta$ vertex. The four-momenta of the initial (final) Δ and of the photon are given by p (p') and q respectively. The four-vector indices of the initial (final) spin-3/2 fields are given by β (α), and μ is the four-vector index of the photon field.

Consider the coupling of a photon to a Δ, Fig. 1. The matrix element of the electromagnetic current operator J^μ between spin-3/2 states can be decomposed into four multipole transitions: a Coulomb monopole (E0), a magnetic dipole (M1), a Coulomb quadrupole (E2) and a magnetic octupole (M3). We firstly write a Lorentz-covariant decomposition for the on-shell $\gamma^*\Delta\Delta$ vertex which exhibits manifest electromagnetic gauge-invariance [4, 5]:

$$
\begin{aligned}
\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda)\rangle \;=\; -\bar{u}_\alpha(p',\lambda') \Bigg\{ & \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \gamma^\mu \\
& + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} \Bigg\} u_\beta(p,\lambda),
\end{aligned} \tag{1}
$$

where $M_\Delta = 1.232$ GeV is the Δ mass, u_α is the Rarita-Schwinger spinor for a spin-3/2 state, and λ (λ') are the initial (final) Δ helicities. Furthermore, $F^*_{1,2,3,4}$ are the $\gamma^*\Delta\Delta$ form factors, and $F_1^*(0) = e_\Delta$ is the Δ electric charge in units of e (e.g., $e_{\Delta^+} = +1$). For further use we also define the quantity $\tau \equiv Q^2/(4M_\Delta^2)$.

A physical interpretation of the four electromagnetic $\Delta \to \Delta$ transitions can be obtained by performing a multipole decomposition [5, 6]. For this purpose it is convenient to consider the Breit frame, where $\vec{p} = -\vec{p}\,' = -\vec{q}/2$. Furthermore, we choose $\vec{q}$ along the z-axis and denote the initial (final) Δ spin projections along the z-axis by s (s') respectively. In this frame, the matrix elements of the charge operator define the Coulomb monopole (charge) and Coulomb quadrupole form factors as :

$$\langle \frac{\vec{q}}{2}, s' \,|\, J^0(0) \,|\, -\frac{\vec{q}}{2}, s\rangle \equiv (2M_\Delta)\, \delta_{s'\,s} \left\{ \left(\delta_{s\,\pm\frac{3}{2}} + \delta_{s\,\pm\frac{1}{2}}\right) G_{E0}(Q^2) - \frac{2}{3}\tau \left(\delta_{s\,\pm\frac{3}{2}} - \delta_{s\,\pm\frac{1}{2}}\right) G_{E2}(Q^2)\right\}. \tag{2}$$

Using Eq. (1), we can express the Coulomb monopole and quadrupole form factors in terms of $F^*_{1,2,3,4}$ as :

$$G_{E0} = (F_1^* - \tau F_2^*) + \frac{2}{3}\tau G_{E2}, \tag{3}$$

$$G_{E2} = (F_1^* - \tau F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* - \tau F_4^*)\,. \tag{4}$$

In an analogous way, the matrix elements of the current operator define the magnetic dipole and magnetic octupole form factors. E.g. for the transverse spherical component $J_{+1} \equiv -\frac{1}{\sqrt{2}}(J^1 + iJ^2)$ this reads :

$$\begin{aligned}\langle \frac{\vec{q}}{2}, s' \,|\, J_{+1}(0) \,|\, -\frac{\vec{q}}{2}, s\rangle \;\equiv\; & (-\sqrt{2})(2M_\Delta)\,\frac{\sqrt{\tau}}{\sqrt{3}} \left\{ \left(\delta_{s'\,+\frac{3}{2}}\,\delta_{s\,+\frac{1}{2}} + \delta_{s'\,-\frac{1}{2}}\,\delta_{s\,-\frac{3}{2}} + \frac{2}{\sqrt{3}}\delta_{s'\,+\frac{1}{2}}\,\delta_{s\,-\frac{1}{2}}\right) G_{M1}(Q^2)\right. \\ & \left. -\frac{4}{5}\tau \left(\delta_{s'\,+\frac{3}{2}}\,\delta_{s\,+\frac{1}{2}} + \delta_{s'\,-\frac{1}{2}}\,\delta_{s\,-\frac{3}{2}} - \sqrt{3}\delta_{s'\,+\frac{1}{2}}\,\delta_{s\,-\frac{1}{2}}\right) G_{M3}(Q^2)\right\}. \end{aligned} \tag{5}$$

Using Eq. (1), we can express the magnetic dipole and octupole form factors in terms of $F^*_{1,2,3,4}$ as :

$$G_{M1} = (F_1^* + F_2^*) + \frac{4}{5}\tau G_{M3}, \tag{6}$$

$$G_{M3} = (F_1^* + F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* + F_4^*)\,. \tag{7}$$

At $Q^2 = 0$, the multipole form factors define the charge (e_Δ), the magnetic dipole moment (μ_Δ), the electric quadrupole moment (Q_Δ), and the magnetic octupole moment (O_Δ) as :

$$e_\Delta = G_{E0}(0) = F_1^*(0), \tag{8a}$$

$$\mu_\Delta = \frac{e}{2M_\Delta} G_{M1}(0) = \frac{e}{2M_\Delta}\left[e_\Delta + F_2^*(0)\right], \tag{8b}$$

$$Q_\Delta = \frac{e}{M_\Delta^2} G_{E2}(0) = \frac{e}{M_\Delta^2}\left[e_\Delta - \frac{1}{2}F_3^*(0)\right], \tag{8c}$$

$$O_\Delta = \frac{e}{2M_\Delta^3} G_{M3}(0) = \frac{e}{2M_\Delta^3}\left[e_\Delta + F_2^*(0) - \frac{1}{2}\left(F_3^*(0) + F_4^*(0)\right)\right]. \tag{8d}$$

In the following, we will also use the relations which express the form factors $F^*_{1,2,3,4}$ in terms of the multipole form factors :

$$\begin{aligned} F_1^* &= \frac{1}{1+\tau}\left\{G_{E0} - \frac{2}{3}\tau G_{E2} + \tau\Big[G_{M1} - \frac{4}{5}\tau G_{M3}\Big]\right\}, \\ F_2^* &= -\frac{1}{1+\tau}\left\{G_{E0} - \frac{2}{3}\tau G_{E2} - \Big[G_{M1} - \frac{4}{5}\tau G_{M3}\Big]\right\}, \\ F_3^* &= \frac{2}{(1+\tau)^2}\left\{G_{E0} - \left(1 + \frac{2}{3}\tau\right) G_{E2} + \tau\Big[G_{M1} - \left(1+\frac{4}{5}\tau\right) G_{M3}\Big]\right\}, \\ F_4^* &= -\frac{2}{(1+\tau)^2}\left\{G_{E0} - \left(1 + \frac{2}{3}\tau\right) G_{E2} - \Big[G_{M1} - \left(1+\frac{4}{5}\tau\right) G_{M3}\Big]\right\}. \end{aligned} \tag{9}$$

III. THE NATURAL VALUES

As first argued by Weinberg [9], based on the Gerasimov-Drell-Hearn (GDH) sum rule, there is a "natural" value for the magnetic moment of elementary (pointlike) particle with spin. This value corresponds to gyromagnetic ratio g equal to 2. It has later been observed that all consistent field theories of charged particles with spin respect this value, see e.g. [10, 11].

It is conceivable that the higher electromagnetic moments of pointlike particles are also fixed at some "natural" values. To extend Weinberg's argument to higher e.m. moments we would need to derive the sum rules for them, since we are not aware that such sum rules have been derived. Leaving this ambitious exercise for the future, we confine ourselves here to a more modest task of examining the values of e.m. moments of gravitinos in extended supergravity [12, 13]. The gravitino, if existed, would be a spin-3/2 particle described by a Rarita-Schwinger field which, in the framework of $\mathcal{N}=2$ supergravity, couples consistently to electromagnetism. We therefore expect that all the e.m. moments arising in this theory have "natural" values.

To find these values we depart from the following Lagrangian density: [25]

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_\mu \gamma^{\mu\nu\alpha}(i\partial_\alpha - eA_\alpha)\,\psi_\nu - m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu \\ &+ em^{-1}\,\bar{\psi}_\mu\,(\,i\kappa_1 F^{\mu\nu} - \kappa_2\gamma_5\tilde{F}^{\mu\nu}\,)\,\psi_\nu\,. \end{aligned} \tag{10}$$

It describes the spin-3/2 Rarita-Schwinger field (ψ_μ) with mass m coupled to the electromagnetic field (A_μ) via the minimal coupling (with positive charge e) and two non-minimal couplings κ_1 and κ_2. As is shown in [14], this is the most general such Lagrangian that gives the right number of spin degrees of freedom for a spin-3/2 particle. This theory, however, would still lead to rather subtle pathologies — non-causal wave propagation and such [15, 16], at least if no other fields are present. Adding gravity in a supersymmetric way, makes a fully consistent, from this viewpoint, theory, but also constrains the non-minimal couplings as follows:

$$\kappa_1 = \kappa_2 = 1. \tag{11}$$

We shall refer to these values as the 'SUGRA choice'.

The e.m. vertex stemming from Eq. (10) is

$$\Gamma^{\alpha\beta\mu}(p',p) = e\gamma^{\alpha\beta\mu} - \kappa_1(q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}) + i\kappa_2\gamma_5\varepsilon^{\alpha\beta\mu\varrho}q_\varrho\,, \tag{12}$$

where $q = p' - p$. It is easy to verify that this coupling conserves the e.m. current:

$$q_\mu\,\bar{u}_\alpha(p')\,\Gamma^{\alpha\beta\mu}(p',p)\,u_\beta(p) = 0\,, \tag{13}$$

as well as, for the SUGRA choice, the supersymmetric current:

$$(p'_\alpha - \tfrac{1}{2}m\gamma_\alpha)\,\Gamma^{\alpha\beta\mu}(p',p)\,u_\beta(p)\,\epsilon_\mu(q) = 0\,, \tag{14a}$$

$$\bar{u}_\alpha(p')\,\Gamma^{\alpha\beta\mu}(p',p)\,(p_\beta - \tfrac{1}{2}m\gamma_\beta)\,\epsilon_\mu(q) = 0\,, \tag{14b}$$

where $\epsilon_\mu(q)$ is the photon polarization vector, and $q\cdot\epsilon = 0 = q^2$ in Eqs. (14).

The matrix elements of this vertex can be compared with the general decomposition of the spin-3/2 e.m. current Eq. (1), and in doing so we obtain the following result:

$$F_1^* = 1 + 2(\kappa_1+\kappa_2)\,\tau \overset{\text{sugra}}{=} 1 + 4\tau\,, \tag{15a}$$

$$F_2^* = 2\kappa_1 \overset{\text{sugra}}{=} 2\,, \tag{15b}$$

$$F_3^* = 4(\kappa_1+\kappa_2) \overset{\text{sugra}}{=} 8\,, \tag{15c}$$

$$F_4^* = 0\,, \tag{15d}$$

with $\tau = Q^2/(2m)^2$. Thus, the values of gravitino's e.m. moments in $\mathcal{N}=2$ supergravity are:

$$G_{E0}(0) = 1,\;\; G_{M1}(0) = 3,\;\; G_{E2}(0) = -3,\;\; G_{M3}(0) = -1. \tag{16}$$

IV. THE $\gamma^*\Delta\Delta$ LIGHT-FRONT HELICITY AMPLITUDES

In the following we consider the electromagnetic (e.m.) $\Delta \to \Delta$ transition when viewed from a light front moving towards the Δ. Equivalently, this corresponds to a frame where the baryons have a large momentum-component along the z-axis chosen along the direction of $P = (p + p')/2$, where p (p') are the initial (final) baryon four-momenta. We indicate the baryon light-front + component by P^+ (defining $a^\pm \equiv a^0 \pm a^3$). We can furthermore choose a symmetric frame where the virtual photon four-momentum q has $q^+ = 0$, and has a transverse component (lying in the xy-plane) indicated by the transverse vector $\vec{q}_\perp$, satisfying $q^2 = -\vec{q}_\perp^{\,2} \equiv -Q^2$. In such a symmetric frame, the virtual photon only couples to forward moving partons and the + component of the electromagnetic current J^+ has the interpretation of the quark charge density operator. It is given by : $J^+(0) = +2/3\,\bar{u}(0)\gamma^+ u(0) - 1/3\,\bar{d}(0)\gamma^+(0)d(0)$, considering only u and d quarks. Each term in the expression is a positive operator since $\bar{q}\gamma^+ q \propto |\gamma^+ q|^2$.

We start by expressing the matrix elements of the $J^+(0)$ operator in the Δ as :

$$\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda' | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle \;=\; (2P^+) e^{i(\lambda-\lambda')\phi_q}\, A_{\lambda'\,\lambda}(Q^2), \tag{17}$$

where λ, λ' denotes the Δ light-front helicities, and where $\vec{q}_\perp = Q(\cos\phi_q \hat{e}_x + \sin\phi_q \hat{e}_y)$. The helicity form factors $A_{\lambda'\,\lambda}$ depend on Q^2 only and can equivalently be expressed in terms of $F^*_{1,2,3,4}$ as :

$$\begin{aligned}
A_{\frac{3}{2}\frac{3}{2}} &= A_{-\frac{3}{2}-\frac{3}{2}} = F_1^* - \frac{\tau}{2}\, F_3^*, \\
A_{\frac{3}{2}\frac{1}{2}} &= -A_{-\frac{3}{2}-\frac{1}{2}} = A_{-\frac{1}{2}-\frac{3}{2}} = -A_{\frac{1}{2}\frac{3}{2}} = \frac{\tau^{1/2}}{\sqrt{3}} \left[2F_1^* - F_2^* - \tau \left(F_3^* - \frac{1}{2}\, F_4^* \right) \right], \\
A_{\frac{3}{2}-\frac{1}{2}} &= A_{-\frac{3}{2}\frac{1}{2}} = A_{-\frac{1}{2}\frac{3}{2}} = A_{\frac{1}{2}-\frac{3}{2}} = \frac{\tau}{\sqrt{3}} \left[-2F_2^* + \frac{1}{2}\, F_3^* + \tau\, F_4^* \right], \\
A_{\frac{3}{2}-\frac{3}{2}} &= -A_{-\frac{3}{2}\frac{3}{2}} = -\frac{1}{2}\tau^{3/2}\, F_4^*, \\
A_{\frac{1}{2}\frac{1}{2}} &= A_{-\frac{1}{2}-\frac{1}{2}} = \left(1 - \frac{4}{3}\tau \right) F_1^* + \frac{\tau}{3} \left[4F_2^* - \left(\frac{1}{2} - 2\tau \right) F_3^* - 2\tau F_4^* \right], \\
A_{\frac{1}{2}-\frac{1}{2}} &= -A_{-\frac{1}{2}\frac{1}{2}} = \frac{\tau^{1/2}}{3} \left[4F_1^* - 2\,(1 - 2\tau)\, F_2^* - 2\tau F_3^* + \tau \left(\frac{1}{2} - 2\tau \right) F_4^* \right].
\end{aligned} \tag{18}$$

As the $\Delta \to \Delta$ electromagnetic transition is described by four independent form factors, one finds two angular conditions among the helicity form factors of Eq. (18) :

$$\begin{aligned}
0 &= (1 + 4\tau)\,\sqrt{3} A_{\frac{3}{2}\frac{3}{2}} - 8\tau^{1/2} A_{\frac{3}{2}\frac{1}{2}} + 2A_{\frac{3}{2}-\frac{1}{2}} - \sqrt{3} A_{\frac{1}{2}\frac{1}{2}}, \\
0 &= 4\tau^{3/2} A_{\frac{3}{2}\frac{3}{2}} + \sqrt{3}\,(1 - 2\tau)\, A_{\frac{3}{2}\frac{1}{2}} + \frac{1}{2} A_{\frac{3}{2}-\frac{3}{2}} - \frac{3}{2} A_{\frac{1}{2}-\frac{1}{2}}.
\end{aligned} \tag{19}$$

V. THE TRANSVERSE CHARGE DENSITIES FOR A SPIN-3/2 PARTICLE

We define a quark charge density for a spin-3/2 particle, such as the $\Delta(1232)$, in a state of definite light-cone helicity λ, by the Fourier transform :

$$\begin{aligned}
\rho_\lambda^\Delta(b) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+ | P^+, \frac{-\vec{q}_\perp}{2}, \lambda\rangle \\
&= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q)\, A_{\lambda\lambda}(Q^2).
\end{aligned} \tag{20}$$

The two independent quark charge densities for a spin-3/2 state of definite helicity are given by $\rho^\Delta_{\frac{3}{2}}(b)$ and $\rho^\Delta_{\frac{1}{2}}(b)$.

The above charge densities provide us with two combinations of the four independent Δ FFs. To get information from the other FFs, we consider the charge densities in a spin-3/2 state with transverse spin. We denote this transverse polarization direction by $\vec{S}_\perp = \cos\phi_S \hat{e}_x + \sin\phi_S \hat{e}_y$, and the Δ spin projection along the direction of $\vec{S}_\perp$ by $s_\perp$. We first express the transverse spin basis in terms of the helicity basis for spin-3/2. For the states of transverse spin $s_\perp = \frac{3}{2}$ and $s_\perp = \frac{1}{2}$ this yields :

$$\begin{aligned}
|s_\perp = +\frac{3}{2}\rangle &= \frac{1}{\sqrt{8}} \left\{ e^{-i\phi_S}\, |\lambda = +\frac{3}{2}\rangle + \sqrt{3}\, |\lambda = +\frac{1}{2}\rangle + \sqrt{3}\, e^{i\phi_S}\, |\lambda = -\frac{1}{2}\rangle + e^{2i\phi_S}\, |\lambda = -\frac{3}{2}\rangle \right\}, \\
|s_\perp = +\frac{1}{2}\rangle &= \frac{1}{\sqrt{8}} \left\{ \sqrt{3}\, e^{-i\phi_S}\, |\lambda = +\frac{3}{2}\rangle + |\lambda = +\frac{1}{2}\rangle - e^{i\phi_S}\, |\lambda = -\frac{1}{2}\rangle - \sqrt{3}\, e^{2i\phi_S}\, |\lambda = -\frac{3}{2}\rangle \right\},
\end{aligned} \tag{21}$$

where the states on the *rhs* are the spin-3/2 helicity eigenstates.

We can then define the charge densities in a spin-3/2 state with transverse spin $s_\perp$ as :

$$\rho^{\Delta}_{T\, s_\perp}(\vec{b}) \;\equiv\; \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \rangle. \tag{22}$$

By working out the Fourier transform in Eq. (22) for the two cases where $s_\perp = \frac{3}{2}$ and $s_\perp = \frac{1}{2}$, using the Δ helicity form factors of Eq. (18), one obtains :

$$\begin{aligned}
\rho^{\Delta}_{T\,\frac{3}{2}}(\vec{b}) = \int_0^{+\infty} \frac{dQ}{2\pi}\, Q \quad \Big[\quad & J_0(Qb)\, \frac{1}{4} \left(A_{\frac{3}{2}\frac{3}{2}} + 3 A_{\frac{1}{2}\frac{1}{2}} \right) \\
- \;& \sin(\phi_b - \phi_S)\, J_1(Qb)\, \frac{1}{4} \left(2\sqrt{3} A_{\frac{3}{2}\frac{1}{2}} + 3 A_{\frac{1}{2}-\frac{1}{2}} \right) \\
- \;& \cos[2(\phi_b - \phi_S)]\, J_2(Qb)\, \frac{\sqrt{3}}{2} A_{\frac{3}{2}-\frac{1}{2}} \\
+ \;& \sin[3(\phi_b - \phi_S)]\, J_3(Qb)\, \frac{1}{4} A_{\frac{3}{2}-\frac{3}{2}} \Big],
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
\rho^{\Delta}_{T\,\frac{1}{2}}(\vec{b}) = \int_0^{+\infty} \frac{dQ}{2\pi}\, Q \quad \Big[\quad & J_0(Qb)\, \frac{1}{4} \left(3A_{\frac{3}{2}\frac{3}{2}} + A_{\frac{1}{2}\frac{1}{2}} \right) \\
- \;& \sin(\phi_b - \phi_S)\, J_1(Qb)\, \frac{1}{4} \left(2\sqrt{3} A_{\frac{3}{2}\frac{1}{2}} - A_{\frac{1}{2}-\frac{1}{2}} \right) \\
+ \;& \cos[2(\phi_b - \phi_S)]\, J_2(Qb)\, \frac{\sqrt{3}}{2} A_{\frac{3}{2}-\frac{1}{2}} \\
- \;& \sin[3(\phi_b - \phi_S)]\, J_3(Qb)\, \frac{3}{4} A_{\frac{3}{2}-\frac{3}{2}} \Big],
\end{aligned} \tag{24}$$

where we defined the angle ϕ_b in the transverse plane as, $\vec{b} = b(\cos\phi_b \hat{e}_x + \sin\phi_b \hat{e}_y)$. One notices from Eqs. (23,24) that the transverse charge densities display monopole, dipole, quadrupole, and octupole field patterns, which respectively are determined by the helicity form factors with zero, one, two, or three units of helicity flip between the initial and final Δ states.

It is instructive to evaluate the electric dipole moment (EDM) corresponding with the transverse charge densities $\rho^{\Delta}_{T\,s_\perp}$, which is defined as :

$$\vec{d}^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\, \vec{b}\, \rho^{\Delta}_{T\,s_\perp}(\vec{b}). \tag{25}$$

Eqs. (23,24) yield :

$$\vec{d}^{\Delta}_{\frac{3}{2}} = 3\, \vec{d}^{\Delta}_{\frac{1}{2}} = -\left(\vec{S}_\perp \times \hat{e}_z \right) \{ G_{M1}(0) - 3 e_\Delta \} \left(\frac{e}{2M_\Delta} \right). \tag{26}$$

Expressing the spin-3/2 magnetic moment in terms of the g-factor, which yields $G_{M1}(0) = g\frac{3}{2}e_\Delta$, one sees that the induced EDM $\vec{d}^{\Delta}_{s_\perp}$ is proportional to $g - 2$. The same result was found before for the case of spin-1/2 particles in [7] and spin-1 particles in [8]. One thus observes as universal feature that for a particle without internal structure (corresponding with $g = 2$ [10, 11]), there is no induced EDM.

We next evaluate the electric quadrupole moment corresponding to the transverse charge densities $\rho^{\Delta}_{T\,s_\perp}$. Choosing $\vec{S}_\perp = \hat{e}_x$, the electric quadrupole moment can be defined as :

$$Q^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\, (b_x^2 - b_y^2)\, \rho^{\Delta}_{T\,s_\perp}(\vec{b}). \tag{27}$$

From Eqs. (23,24) one obtains :

$$Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} \;=\; \frac{1}{2} \{ 2\,[G_{M1}(0) - 3e_\Delta] + [G_{E2}(0) + 3e_\Delta] \} \left(\frac{e}{M_\Delta^2} \right). \tag{28}$$

We may note that for a spin-3/2 particle without internal structure, for which $G_{M1}(0) = 3e_\Delta$ and $G_{E2}(0) = -3e_\Delta$ according to Eq. (16), the quadrupole moment of the transverse charge densities vanishes. It is thus interesting to observe from Eq. (28) that $Q^\Delta_{s_\perp}$ is only sensitive to the anomalous parts of the spin-3/2 magnetic dipole and electric quadrupole moments, and vanishes for a particle without internal structure. The same observation was made for the case of a spin-1 particle in Ref. [8]. Furthermore, the factor 1/2 multiplying the curly brackets on the *rhs* of Eq. (28) can be understood by relating the quadrupole moment of a 3-dimensional charge distributions (which we denote by Q_{3d}) to the quadrupole moment of a 2-dimensional charge distribution (denoted by Q_{2d}) both defined w.r.t. the spin axis. By taking the spin axis along the x-axis, the quadrupole moment for a 3-dimensional charge distribution ρ_{3d} is defined as :

$$\begin{aligned} Q_{3d} &\equiv \int dxdydz\, (3x^2 - r^2)\, \rho_{3d}(x,y,z), \\ &= \int dxdydz\, \left[(x^2 - y^2) + (x^2 - z^2)\right]\, \rho_{3d}(x,y,z). \end{aligned} \tag{29}$$

For a 3-dimensional charge distribution which is invariant under rotations around the axis of the spin, the two terms proportional to $(x^2 - y^2)$ and $(x^2 - z^2)$ in Eq. (29) give equal contributions yielding :

$$Q_{3d} = 2 \int dxdydz\, (x^2 - y^2)\, \rho_{3d}(x,y,z). \tag{30}$$

Introducing the 2-dimensional charge density in the xy-plane as :

$$\rho_{2d}(x,y) = \int dz\, \rho_{3d}(x,y,z), \tag{31}$$

one immediately obtains the relation

$$Q_{3d} = 2\, Q_{2d}, \tag{32}$$

with the quadrupole moment of the 2-dimensional charge density defined as :

$$Q_{2d} \equiv \int dxdy\, (x^2 - y^2)\, \rho_{2d}(x,y). \tag{33}$$

Because Q_{3d} is proportional to $G_{E2}(0)$ in our case, we see that Eq. (32) yields a Q_{2d} which is half the value of $G_{E2}(0)$, consistent with Eq. (28).

We can also evaluate the electric octupole moment corresponding with the transverse charge densities $\rho^\Delta_{T\,s_\perp}$. Choosing $\vec{S}_\perp = \hat{e}_x$, the electric octupole moment can be defined as :

$$\begin{aligned} O^\Delta_{s_\perp} &\equiv e \int d^2\vec{b}\, b^3\, \sin(3\phi_b)\, \rho^\Delta_{T\,s_\perp}(\vec{b}), \\ &= e \int d^2\vec{b}\, b_y\, (3b_x^2 - b_y^2)\, \rho^\Delta_{T\,s_\perp}(\vec{b}). \end{aligned} \tag{34}$$

From Eqs. (23,24) one obtains :

$$O^\Delta_{\frac{3}{2}} = -\frac{1}{3} O^\Delta_{\frac{1}{2}} = \frac{3}{2} \Big\{ -G_{M1}(0) - G_{E2}(0) + G_{M3}(0) + e_\Delta \Big\} \left(\frac{e}{2M_\Delta^3} \right). \tag{35}$$

We may note that for a spin-3/2 particle without internal structure, for which $G_{M1}(0) = 3e_\Delta$, $G_{E2}(0) = -3e_\Delta$, and $G_{M3}(0) = -e_\Delta$ according to Eq. (16), the electric octupole moment of the transverse charge densities vanishes.

VI. RESULTS FOR THE $\Delta(1232)$ TRANSVERSE CHARGE DENSITIES

In this section we show results for the $\Delta^+(1232)$ charge densities. The experimental information on the $\gamma\Delta\Delta$ vertex is very scarce. Only some data exist on the Δ magnetic dipole moment. For instance, the current Particle Data Group value of the Δ^+ magnetic dipole moment [17]:

$$\mu_{\Delta^+} = 2.7^{+1.0}_{-1.3}(\text{stat.}) \pm 1.5(\text{syst.}) \pm 3(\text{theor.})\, \mu_N\,, \tag{36}$$

with $\mu_N = e/2M_N$ the nuclear magneton, was obtained from *radiative photoproduction* ($\gamma N \to \pi N \gamma'$) of neutral pions in the $\Delta(1232)$ region by the TAPS Collaboration at MAMI [18]. Using a phenomenological model of the $\gamma p \to \pi^0 p \gamma'$ reaction [19], the value of Eq. (36) was extracted. Eq. (36) implies for the Δ^+ :

$$G_{M1}(0) = 3.5\,^{+1.3}_{-1.7}(\text{stat.}) \pm 2.0(\text{syst.}) \pm 3.9(\text{theor.})\,, \tag{37}$$

The size of the error-bar is rather large due to both experimental and theoretical uncertainties. Recently a dedicated experimental effort is underway at MAMI using the Crystal Ball detector [20], improving on the statistics of the TAPS data by almost two orders of magnitude.

For the Δ electric quadrupole moment, no direct measurements exist. However the electric quadrupole moment for the $N \to \Delta$ transition has been measured accurately from the $\gamma N \to \pi N$ reaction at the Δ resonance energy, and yields [21] :

$$Q_{p\to\Delta^+} \;=\; -\,(0.0846 \pm 0.0033)\;\; \mathrm{e}\cdot\mathrm{fm}^2. \tag{38}$$

The large N_c limit of QCD then allows to obtain relations between the Δ and $N \to \Delta$ quadrupole moments [22]:

$$\frac{Q_{\Delta^+}}{Q_{p\to\Delta^+}} \;=\; \frac{2\sqrt{2}}{5} + \mathcal{O}\left(\frac{1}{N_c^2}\right). \tag{39}$$

Using the phenomenological value of Eq. (38) for $Q_{p\to\Delta^+}$, the large N_c relation of Eq. (39) yields for the Δ^+ quadrupole moment:

$$Q_{\Delta^+} \;=\; -\,(0.048 \pm 0.002)\;\; \mathrm{e}\cdot\mathrm{fm}^2, \tag{40}$$

accurate up to corrections of order $1/N_c^2$. Eq. (40) corresponds with $G_{E2}(0) = -1.87 \pm 0.08$.

As the $\Delta(1232)$ electromagnetic form factors are not known experimentally, apart from the scarce phenomenological information described above, we will rely on recent lattice QCD calculations [23, 24] for these form factors. We will compare three recent lattice calculations. In Ref. [23] quenched Wilson lattice QCD calculations have been performed for a lattice volume of (2.95 fm)^3 for three different pion masses down to a lowest value of $m_\pi = 0.411(4)$ GeV. In Ref. [24], two full lattice QCD calculations have been presented. The first is a dynamical $N_f = 2$ Wilson calculation for a lattice volume of (1.85 fm)^3 for three different pion masses down to a lowest value of $m_\pi = 0.384(8)$ GeV. The second dynamical calculation is a hybrid lattice calculation using domain-wall valence quarks on a staggered sea, which was performed for a lattice volume of (3.5 fm)^3, and a pion mass $m_\pi = 0.353(2)$ GeV. In the lattice calculations, the $\Delta^+(1232)$ form factors were calculated over range $0 < Q^2 < 1.7$ GeV^2. For G_{E0}, we consider a dipole parameterization of the lattice results :

$$G_{E0}(Q^2) \;=\; \frac{1}{\left(1 + Q^2/\Lambda_{E0}^2\right)^2}, \tag{41}$$

with Λ_{E0}^2 treated as a free parameter. For G_{M1} and G_{E2}, we consider exponential parameterizations of the lattice results with two free parameters each :

$$\begin{aligned} G_{M1}(Q^2) \;&=\; G_{M1}(0)\, e^{-Q^2/\Lambda_{M1}^2}, \\ G_{E2}(Q^2) \;&=\; G_{E2}(0)\, e^{-Q^2/\Lambda_{E2}^2}. \end{aligned} \tag{42}$$

For G_{M3}, the lattice calculation of Ref. [23] has a slight tendancy for a negative value of G_{M3} albeit with large error bars. Within its obtained precision, the present lattice results are compatible with $G_{M3}(Q^2) \simeq 0$. The above choice of parameterization for the Δ multipole FFs ensures that the helicity conserving FFs $A_{\frac{3}{2}\frac{3}{2}}$ and $A_{\frac{1}{2}\frac{1}{2}}$ behave as $1/Q^4$ for large Q^2. The values of the parameters entering the dipole fit of Eq. (41) for G_{E0}, and the exponential fits of Eq. (42) for G_{M1} and G_{E2}, fitted to the three different lattice calculations described above are listed in Table I.

In Fig. 2, we show the FFs G_{E0}, G_{M1}, and G_{E2} for the 3 fits to the lattice calculations described above. One sees that for G_{E0} all three calculations give similar results. For G_{M1} and in particular G_{E2}, the fits show a larger spread. For $G_{E2}(0)$ the hybrid lattice calculation yields a quadrupole moment consistent with the large-N_c value, Eq. (40), whereas the quenched and dynamical $N_f = 2$ Wilson calculations yield only about half this value. It should be noticed however that the present dynamical calculations still have substantial larger error bars than the quenched calculations. We will use the central values of these fits to evaluate the quark transverse densities in the $\Delta^+(1232)$. A systematic error analysis of the lattice fits will be performed in a follow-up study.

	m_π (in GeV)	Λ^2_{E0} (in GeV^2)	$G_{M1}(0)$	Λ^2_{M1} (in GeV^2)	$G_{E2}(0)$	Λ^2_{E2} (in GeV^2)
Quenched Wilson	0.411 (4)	1.101 (10)	2.635 (61)	0.978 (26)	-0.810 (291)	0.696 (200)
Dynamical $N_f = 2$ Wilson	0.384 (8)	1.161 (57)	2.344 (169)	1.022 (73)	-0.784 (47)	1.938 (143)
Hybrid	0.353 (2)	1.126 (31)	3.101 (74)	0.895 (26)	-1.851 (0.427)	0.542 (122)

TABLE I: Parameters for the $\Delta^+(1232)$ form factors for three different lattice QCD calculations [23, 24]. For G_{E0}, the dipole parameterization of Eq. (41) is used. For G_{M1} and G_{E2}, the exponential parameterization of Eq. (42) is used.

In Fig. 3, we compare the Δ^+ transverse densities in helicity states of $\lambda = 3/2$ and $\lambda = 1/2$ for the quenched lattice calculations. A comparison reveals that both are very similar, with the density in a $\lambda = 1/2$ state slightly more concentrated.

In Fig. 4, the transverse densities are compared for a Δ^+ which has a transverse spin. It is seen that the quark charge density in a Δ^+ in a state of transverse spin projection $s_\perp = +3/2$ is elongated along the axis of the spin (prolate deformation) whereas in a state of transverse spin projection $s_\perp = +1/2$ it is elongated along the axis perpendicular to the spin.

The corresponding dipole, quadrupole and octupole field patterns in the transverse quark charge density for a transversely polarized Δ are shown in Fig. 5.

VII. CONCLUSIONS

Acknowledgments

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[1] C. E. Hyde-Wright and K. de Jager, Ann. Rev. Nucl. Part. Sci. **54**, 217 (2004).
[2] J. Arrington, C. D. Roberts and J. M. Zanotti, J. Phys. G **34**, S23 (2007).
[3] C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **59**, 694 (2007).
[4] V. Pascalutsa, M. Vanderhaeghen and S. N. Yang, Phys. Rept. **437**, 125 (2007).
[5] S. Nozawa and D. B. Leinweber, Phys. Rev. D **42**, 3567 (1990).
[6] H. J. Weber and H. Arenhovel, Phys. Rept. **36**, 277 (1978).
[7] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. Lett. **100**, 032004 (2008).
[8] C. E. Carlson and M. Vanderhaeghen, arXiv:0807.4537 [hep-ph].
[9] S. Weinberg in "Lectures on Elementary Particles and Quantum Field Theory", Volume 1, Brandeis University Summer Institute 1970 (S. Deser, M. Grisaru and H. Pendleton, editors, M.I.T. Press, Cambridge, 1970).
[10] S. Ferrara, M. Porrati and V. L. Telegdi, Phys. Rev. D **46**, 3529 (1992).
[11] B. R. Holstein, Am. J. Phys. **74**, 1104 (2006).
[12] S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. Lett. **37**, 1669 (1976).
[13] P. Van Nieuwenhuizen, Phys. Rept. **68**, 189 (1981).
[14] S. Deser, V. Pascalutsa and A. Waldron, Phys. Rev. D **62**, 105031 (2000).
[15] K. Johnson and E. C. Sudarshan, Annals Phys. **13**, 126 (1961)
[16] G. Velo and D. Zwanziger, Phys. Rev. **186**, 1337 (1969).
[17] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
[18] M. Kotulla *et al.*, Phys. Rev. Lett. **89**, 272001 (2002).
[19] D. Drechsel and M. Vanderhaeghen, Phys. Rev. C **64**, 065202 (2001).
[20] M. Kotulla, AIP Conf. Proc. **904** (2007) 203.
[21] L. Tiator, D. Drechsel, S. S. Kamalov and S. N. Yang, Eur. Phys. J. A **17**, 357 (2003).
[22] A. J. Buchmann, J. A. Hester and R. F. Lebed, Phys. Rev. D **66**, 056002 (2002).
[23] C. Alexandrou, T. Korzec, T. Leontiou, J. W. Negele and A. Tsapalis, PoS **LAT2007**, 149 (2007).
[24] C. Alexandrou, talk at the ECT* workshop on "Hadron Electromagnetic Form Factors", May 12-23, 2008, Trento, Italy.
[25] In our conventions: $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, $\gamma^{\mu\nu\alpha} = \frac{1}{2}\{\gamma^{\mu\nu}, \gamma^\alpha\}$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\varrho\lambda}\partial_\varrho A_\lambda$.

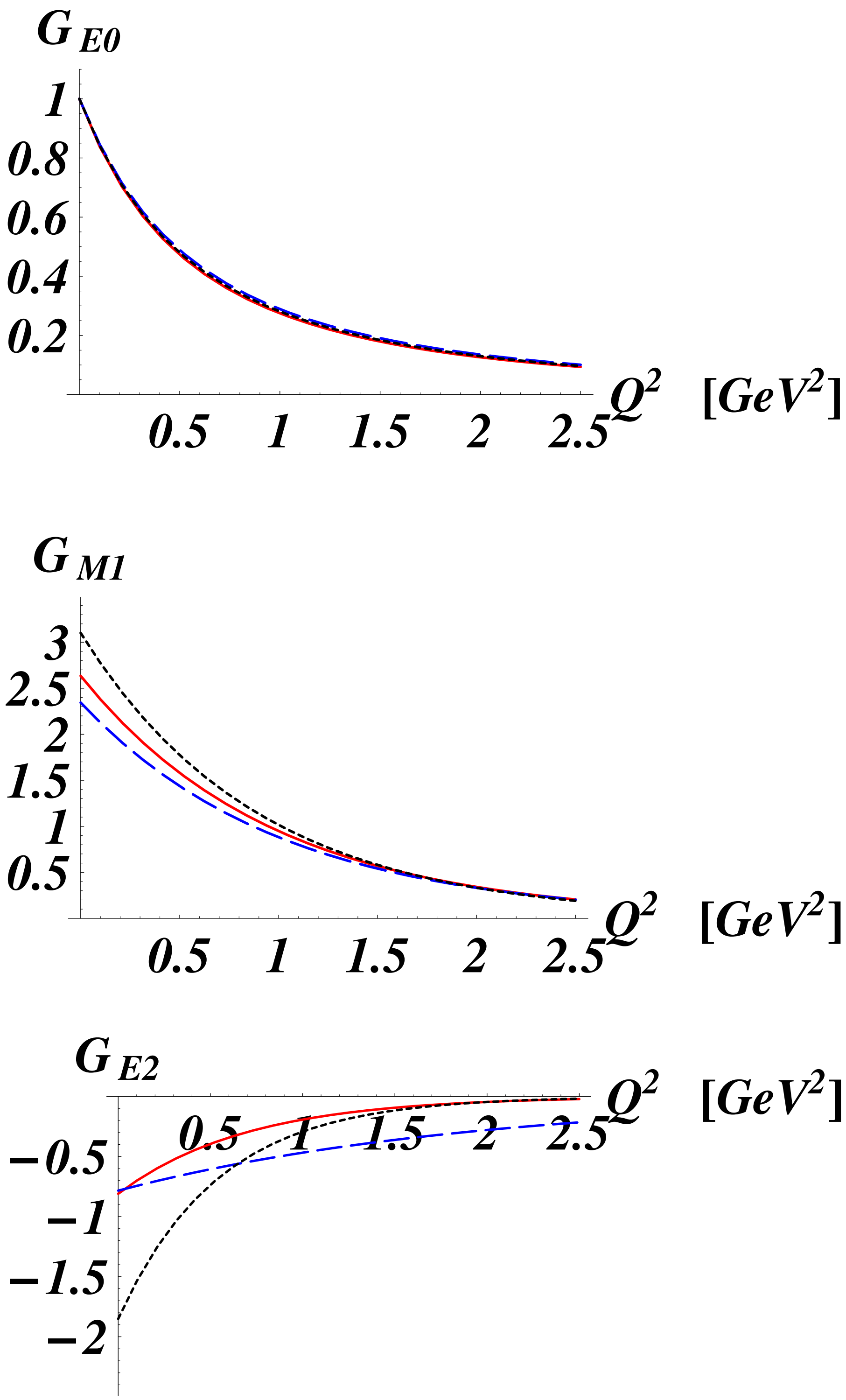

FIG. 2: Comparison of three different QCD lattice calculations for the $\Delta^+(1232)$ e.m. FFs G_{E0}, G_{M1}, and G_{E2}, according to the fit of Table I. Solid (red) curves : quenched Wilson result [23]; long dashed (blue) curves : dynamical $N_f = 2$ Wilson result [24]; dotted (black) curves : hybrid lattice calculation [24].

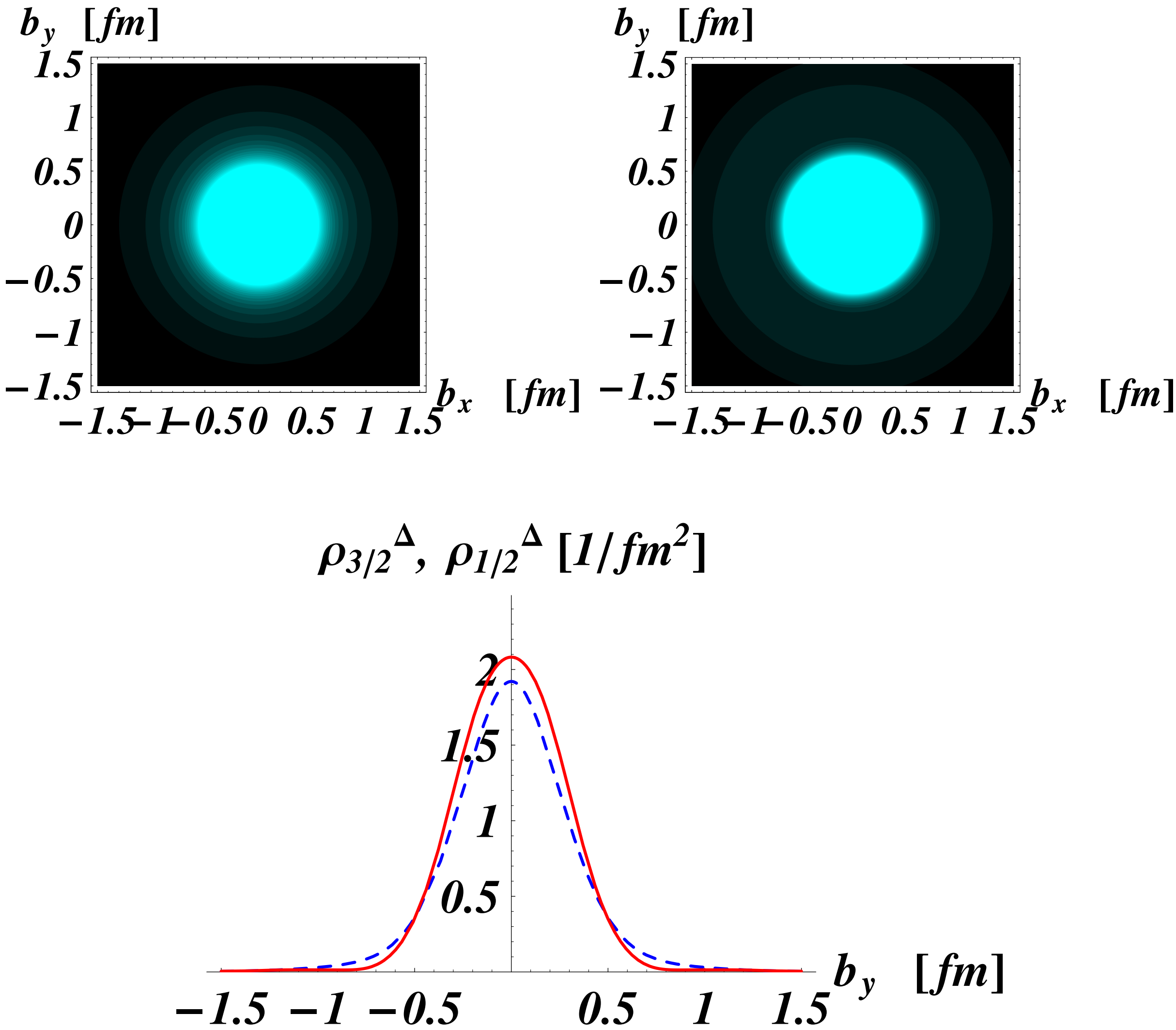

FIG. 3: Quark transverse charge densities in a $\Delta^+(1232)$ of definite light-cone helicity. Upper left panel : $\rho^{\Delta}_{\frac{3}{2}}$. Upper right panel : $\rho^{\Delta}_{\frac{1}{2}}$. The light (dark) regions correspond with largest (smallest) values of the density. The lower panel compares the density along the y-axis for $\rho^{\Delta}_{\frac{3}{2}}$ (dashed curve) and $\rho^{\Delta}_{\frac{1}{2}}$ (solid curve). For the Δ e.m. FFs, we use the lattice quenched QCD calculations (fit of Table I) of Alexandrou et al. [23].

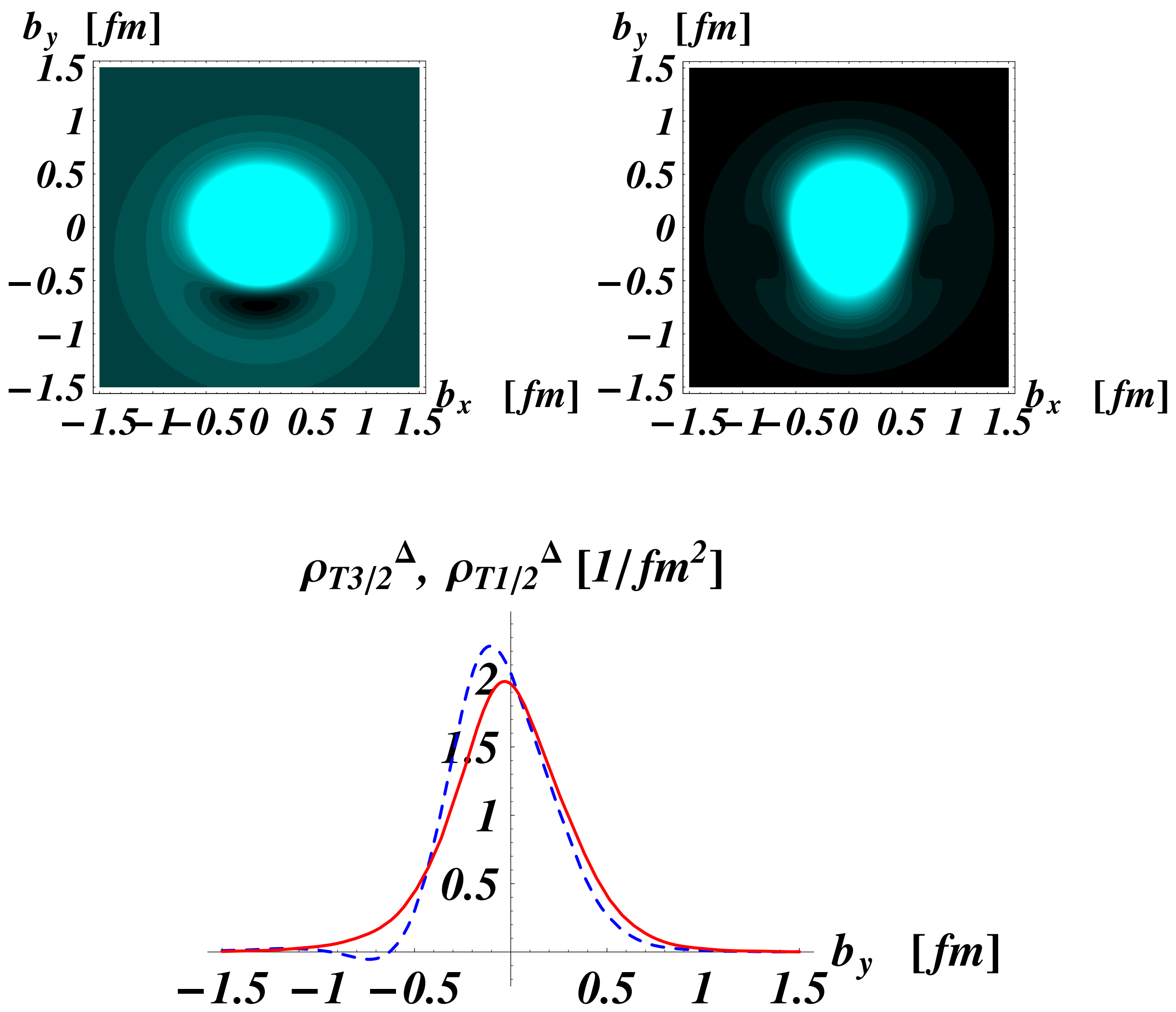

FIG. 4: Quark transverse charge densities in a $\Delta^+(1232)$ which is polarized along the positive x-axis. Upper left panel : $\rho^{\Delta}_{T\,\frac{3}{2}}$. Upper right panel : $\rho^{\Delta}_{T\,\frac{1}{2}}$. The light (dark) regions correspond with largest (smallest) values of the density. The lower panel compares the density along the y-axis for $\rho^{\Delta}_{T\,\frac{3}{2}}$ (dashed curve) and $\rho^{\Delta}_{T\,\frac{1}{2}}$ (solid curve). For the Δ e.m. FFs, we use the lattice quenched QCD calculations (fit of Table I) of Alexandrou et al. [23].

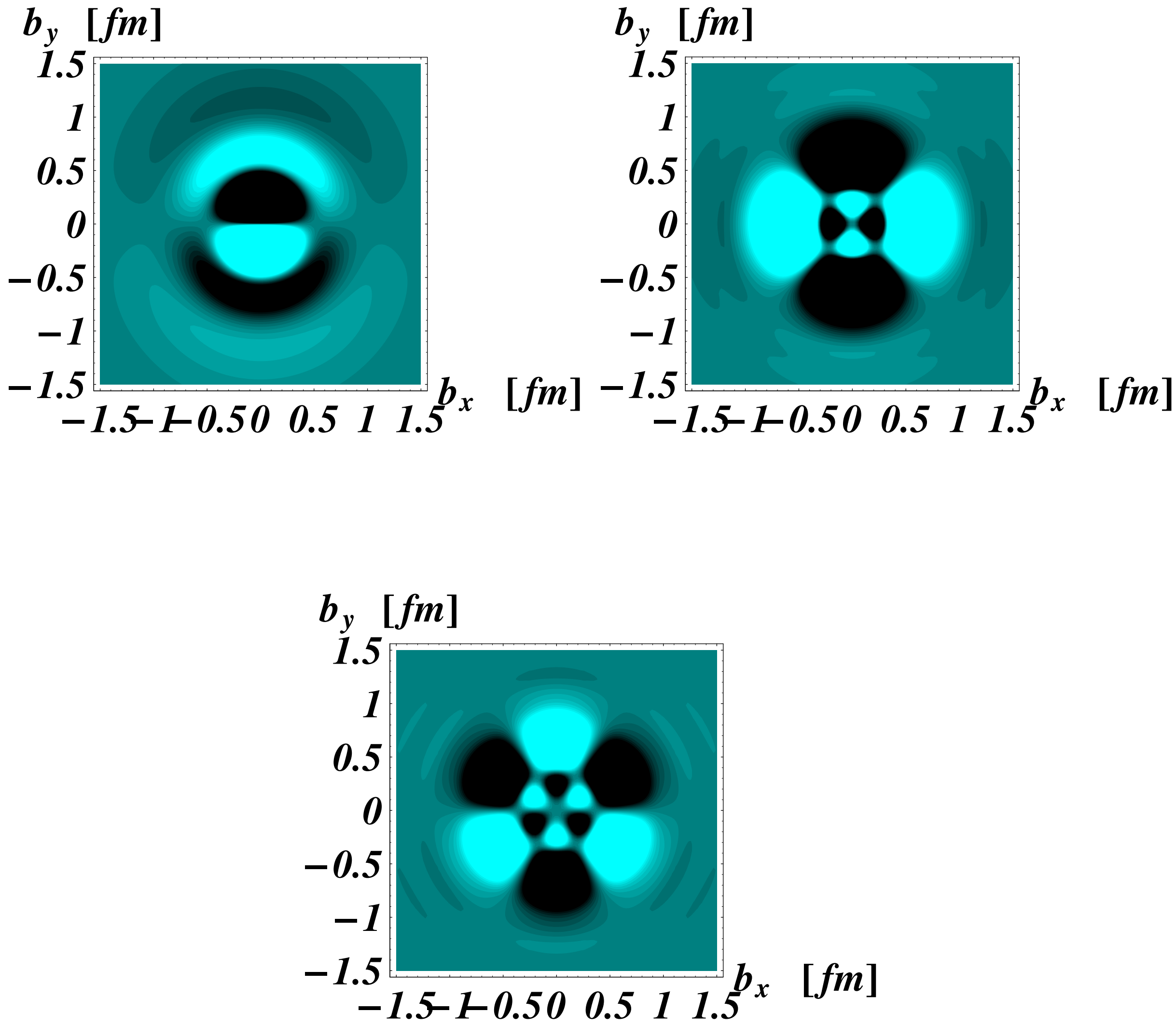

FIG. 5: Different field patterns in the quark transverse charge density $\rho^{\Delta}_{T\,\frac{3}{2}}$ in a $\Delta^{+}(1232)$ which is polarized along the positive x-axis. Upper left panel : dipole field pattern; upper right panel : quadrupole field pattern; lower panel : octupole field pattern. For the Δ e.m. FFs, we use the lattice quenched QCD calculations (fit of Table I) of Alexandrou et al. [23].

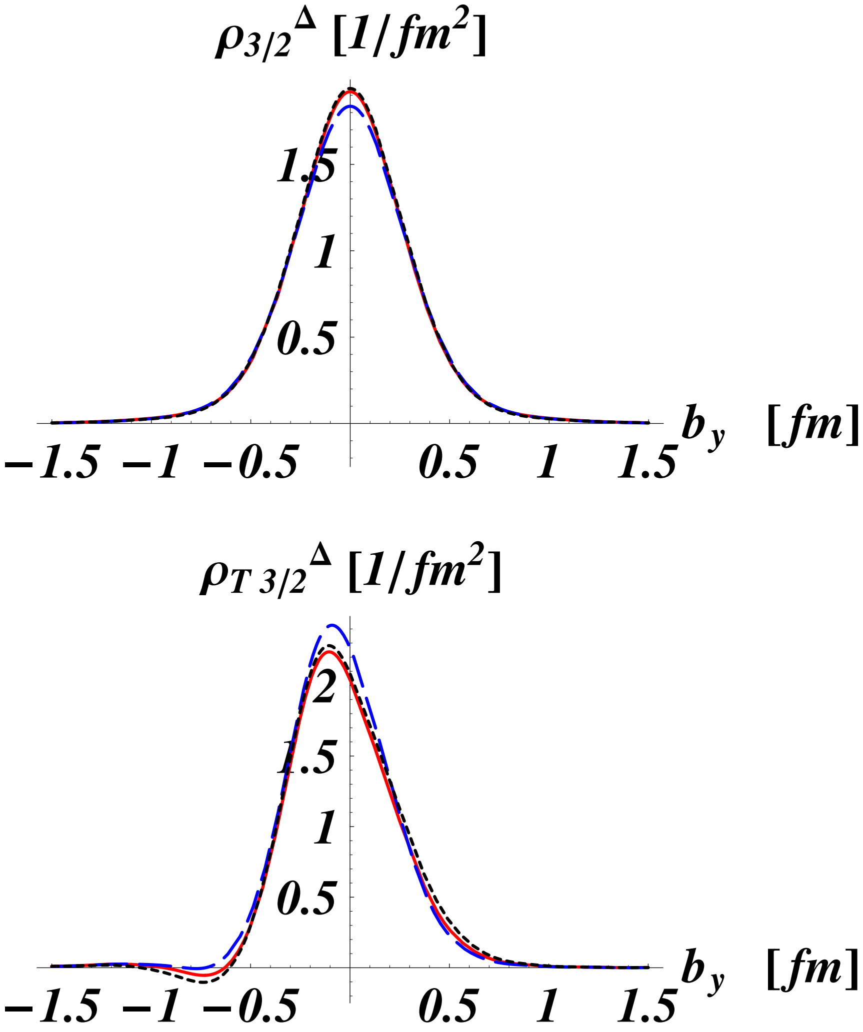

FIG. 6: Comparison of the densities along the y-axis for $\rho^{\Delta}_{\frac{3}{2}}$ (upper panel) and $\rho^{\Delta}_{T\,\frac{3}{2}}$ (lower panel), for three different QCD lattice calculations of the $\Delta^+(1232)$ e.m. FFs according to the fit of Table I. Solid (red) curves : quenched Wilson result [23]; long dashed (blue) curves : dynamical $N_f = 2$ Wilson result [24]; dotted (black) curves : hybrid lattice calculation [24].