

Open flux in Saturn's magnetosphere

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Abstract

We characterise the interaction between the solar wind and Saturn's magnetosphere by evaluating the amount of 'open' magnetic flux connected to the solar wind. This is deduced from a large set of Hubble Space Telescope images of the ultraviolet aurora, using the poleward boundary of the main aurora as a proxy for the open-closed field line boundary in the ionosphere. The amount of open flux is found to be 10–50 GWb, with a mean of 35 GWb. The typical change in open flux between consecutive observations separated by 10–60 h is –5 or +7 GWb. These changes are a result of imbalance between open flux creation at the dayside magnetopause and its closure in the magnetotail. The 5 GWb typical decrease in open flux is consistent with in situ measurements of the flux transported following a reconnection event. Estimates of average, net reconnection rates are found to be typically a few

tens of kV, with some extreme examples of unbalanced magnetopause or tail reconnection occurring at ~ 300 kV. The range of values determined suggest that Saturn's magnetosphere does not generally achieve a steady state between flux opening at the magnetopause and flux closure in the magnetotail. The percentage of magnetic flux which is open in Saturn's magnetosphere is similar to that measured at the Earth (2–11%), but the typical percentage that is closed between observations is significantly lower (13% compared to 40–70%). Therefore, open flux is usually closed in smaller (few GWb) events in Saturn's magnetosphere. The exception to this behaviour is large, rapid flux closure events which are associated with solar wind compressions. While the rates of flux opening and closure should be equal over long timescales, they are evidently different on shorter (up to tens of hours) timescales. The relative independence of the magnetopause and tail reconnection rates can be attributed to the long loading timescales required to transport open field lines into the tail.

17 *Keywords:* Saturn, magnetosphere, Aurorae, Ultraviolet observations

18 **1. Introduction**

19 The interaction of the solar wind and interplanetary magnetic field (IMF)
20 with a planetary magnetosphere is important for the transfer of plasma and
21 momentum between the different environments. In the Dungey (1963) de-
22 scription of an 'open' magnetosphere, this interaction is driven by magnetic
23 reconnection between the planetary and interplanetary fields when they have
24 an anti-parallel component at the dayside magnetopause. The open field lines
25 are then dragged anti-sunward by the solar wind flow to form long magne-

26 total lobes. A simple schematic of the open magnetosphere is shown in
27 Figure 1a. To complete the circulation of flux, reconnection occurs again in
28 the tail and results in closed planetary field lines planetward of the reconec-
29 tion site, which return to the dayside, and tailward, disconnected field lines.
30 The disconnected field lines can take the form of a closed loop, a plasmoid,
31 followed by the post-plasmoid plasma sheet (PPPS), which is produced by
32 rapid reconnection of open field lines planetward of the plasmoid (Richardson
33 et al., 1987). This scenario is illustrated in Figure 1d.

34 The ionospheric footprint of the open field lines forms the approximately
35 circular polar cap, the size of which is modulated by the balance between
36 opening of flux at the dayside magnetopause and closure in the magnetotail.
37 The side and polar views of the polar cap (bounded by the open-closed field
38 line boundary, OCB) are illustrated in Figure 1b and c. When unbalanced
39 magnetopause (flux-opening) reconnection occurs, the open-closed boundary
40 expands to lower latitudes to accommodate the new open flux. Conversely,
41 when open flux is removed via unbalanced tail reconnection, the open-closed
42 boundary contracts to higher latitudes. This is shown in Figure 1e and f.

43 Observations of Saturn’s aurorae show that they generally form a ‘main
44 oval’ ring of emission circling the poles although with considerable substruc-
45 ture imposed (Broadfoot et al., 1981; Clarke et al., 2005). These aurorae
46 are associated with an upward-directed (from the ionosphere) field-aligned
47 current which lies close to the boundary between open and closed magnetic
48 field lines, driven by the flow shear between sub-corotating open and outer
49 magnetosphere flux tubes, and the near-rigid corotating middle and inner
50 magnetospheric flux tubes (Cowley et al., 2004; Badman et al., 2006; Bunce

51 et al., 2008). The darker area poleward of the main auroral oval maps to
52 open field lines, and its size is determined by the balance between opening
53 of flux at the dayside magnetopause and closure in the magnetotail, as de-
54 scribed above. In this case, observations of Saturn’s aurora can be used to
55 estimate the amount of open flux in Saturn’s magnetosphere, and deduce the
56 balance between magnetopause and tail reconnection (Badman et al., 2005;
57 Belenkaya et al., 2007).

58 While the conditions which control the rate and location of reconnection
59 at Saturn’s magnetopause have been debated (Scurry and Russell, 1991; Gro-
60 cott et al., 2009; Lai et al., 2012; Masters et al., 2012), observations at the
61 magnetopause have provided evidence of an open magnetopause required to
62 sustain the open polar caps (Huddleston et al., 1997; McAndrews et al., 2008;
63 Lai et al., 2012; Badman et al., 2013). Likewise, reconnection events have
64 been identified in Saturn’s magnetotail (Bunce et al., 2005; Jackman et al.,
65 2007, 2008a; Hill et al., 2008). Jackman et al. (2011) performed a superposed
66 epoch analysis of 34 plasmoids identified so far, and found evidence for a sig-
67 nificant PPS at Saturn, representing the closure of a significant amount (3
68 GWb) of open flux in a typical reconnection event in Saturn’s tail.

69 In this study the open flux content of Saturn’s magnetosphere is estimated
70 using a large collection of images of the UV aurora, using the poleward edge
71 of the auroral emission as a proxy for the open-closed field line boundary.
72 Its variation and rate of change are also estimated and compared to values
73 obtained from in situ measurements by Cassini, and global MHD simulations,
74 in order to characterise the balance of magnetopause and tail reconnection
75 over different timescales at Saturn.

76 **2. Auroral Images**

77 This study employs 108 images of Saturn’s UV aurora obtained by the
78 Space Telescope Imaging Spectrograph (STIS) and Advanced Camera for
79 Surveys (ACS) instruments onboard the Hubble Space Telescope (HST) dur-
80 ing 2000–2013. The data were reduced and projected onto a latitude-local
81 time grid following the methods described by Grodent et al. (2003) and Gro-
82 dent et al. (2005) for STIS images during 2000–2005, and Clarke et al. (2009)
83 for ACS images from 2007 onward. The auroral morphology during each
84 campaign has been detailed by Gérard et al. (2004) (1997–2001 campaigns),
85 Clarke et al. (2005) and Grodent et al. (2005) (2004 campaign), Gérard et al.
86 (2006) (2005 campaign), Clarke et al. (2009) (2007–8 campaigns) and Nichols
87 et al. (in preparation) (2011–13 campaigns).

88 For each campaign, when successive images were obtained on the same
89 HST orbit, i.e. within an observing interval of < 45 min, these have been
90 combined to increase the signal to noise. Although the instrument sensi-
91 tivities and data reduction methods varied between campaigns on different
92 years, in this study we are concerned only with relative intensity between the
93 bright auroral and dark polar cap regions for each image, rather than their
94 absolute values, so such differences do not affect our results.

95 **3. Determining the auroral boundary and open flux estimates**

96 Following previous studies (Badman et al., 2005) the poleward bound-
97 ary of the auroral emission is used as a proxy for the open-closed field line
98 boundary. The region poleward of this is generally much darker than the
99 main aurora, as expected in the open field line region. The poleward bound-

100 ary was identified at intervals of 10° longitude (ϕ) using a largely automated
 101 method. First, an automated procedure searched for the strongest positive
 102 gradient in intensity along each meridian from the pole. These points were
 103 checked by eye and any extreme outliers removed. At these locations and in
 104 regions of faint emission or where a strong gradient could not be identified,
 105 the boundary position was linearly interpolated between the values either
 106 side. Examples of the boundaries obtained from this method are shown by
 107 the red crosses on the images in Figures 2–3.

108 The boundary points obtained define the ‘polar cap’ area in Saturn’s
 109 ionosphere threaded by open field lines. To calculate the amount of open flux,
 110 Φ , a model of Saturn’s magnetic field (Burton et al., 2010) is integrated over
 111 the polar cap area, following the method detailed by Badman et al. (2005)
 112 and employing a flux function $F(r, \theta)$ (e.g. Cowley and Bunce (2003)):

$$\Phi = \Delta\phi \sum_{n=1}^{36} F(R(\theta_n), \theta_n), \quad (1)$$

113 where $\Delta\phi = 10^\circ$ is the width of each longitudinal sector, θ_n is the co-latitude
 114 of the boundary in longitude sector n , and $R(\theta_n)$ is the radius of the sur-
 115 face containing the auroral emissions at that co-latitude, which matches the
 116 altitude to which each HST image was projected. This surface is an oblate
 117 spheroid about the spin axis, with an equatorial radius R_e and polar radius
 118 R_p , i.e.

$$R(\theta) = \frac{R_e}{(1 + \epsilon \cos^2 \theta)^{1/2}} \quad (2)$$

119 where

$$\epsilon = \frac{R_e^2}{R_p^2} - 1 \quad (3)$$

120 The auroral images were all projected to the peak UV emission altitude of
121 1100 km above the 1 bar reference spheroid (G erard et al., 2009).

122 The flux contained within a circular polar cap region centred on Saturn’s
123 magnetic pole, calculated using this method, is shown as a function of circle
124 radius in degrees co-latitude in Figure 4. The solid line shows the relationship
125 for the southern hemisphere and the dashed line represents the northern
126 hemisphere. The difference between the two is caused by the quadrupole
127 component of Saturn’s magnetic field which results in a stronger surface field
128 strength in the north than the south at a given latitude (Burton et al., 2010).
129 Figure 4b shows a reduced range of radius and flux values relevant to those
130 discussed in this study.

131 The uncertainty in the open flux estimates can arise from uncertainties
132 in the projection method (including the fact that the finite altitudinal extent
133 of the auroral curtain is not accounted for), the boundary extrapolation in
134 regions of dim aurora, and the underlying approximation of the open-closed
135 boundary by the poleward boundary of the aurora. While the first of these
136 is readily quantified e.g. by Grodent et al. (2005) to be $\sim 1\text{--}2^\circ$ depending on
137 the position relative to the sub-observer point, the others are less precise. For
138 example, auroral emissions can be present on open field lines as a result of
139 field-aligned currents and particle precipitation associated with ongoing re-
140 connection at the dayside magnetopause (Bunce et al., 2005). These features
141 have been observed in Saturn’s aurora in both HST and Cassini observations
142 (G erard et al., 2004, 2005; Radioti et al., 2011; Badman et al., 2012). How-
143 ever, the area affected is generally a small fraction of the total open field
144 region and, in the absence of sequential images or corresponding in situ mea-

145 surements, it is difficult to confirm whether such features are indeed occurring
146 on open field lines. Furthermore, Cassini crossings of the high-latitude night-
147 side have shown that the region of upward field-aligned current associated
148 with the main auroral emission can be latitudinally displaced from the appar-
149 ent open-closed boundary determined from the particle flux measurements
150 (Talboys et al., 2011). This could lead to a systematic over-estimate of the
151 open flux using our method based on auroral observations, but it obviously
152 requires more detailed study to reconcile the observations made by different
153 instruments and at different local times (c.f. Bunce et al. (2008)). In the
154 absence of more comprehensive determination of the boundary location, we
155 therefore use the consistent approximation of the poleward boundary of the
156 UV emission to represent the open-closed boundary and include a reasonable
157 uncertainty in the boundary location of 2° latitude in our open flux estimates
158 to account for these combined uncertainties.

159 **4. Results**

160 *4.1. Open flux distribution*

161 The distribution of open flux values, Φ , estimated using the above method
162 is plotted in Figure 5. The lower panel shows a histogram of the values across
163 bins of width 10 GWb, while the upper panel shows each value and its error
164 bar. The distribution of values in the y-direction on the upper panel is simply
165 to space the values so each error bar can be seen. The distribution extends
166 between 10–50 GWb, with two outliers at 9.7 GWb and 50.6 GWb. The
167 minimum open flux value would be enclosed by a circular boundary, centered
168 on Saturn’s magnetic pole, with a radius of $\sim 7.5^\circ$ in the southern hemisphere

169 and $\sim 7^\circ$) in the northern hemisphere (see Figure 4b). The maximum flux
170 values correspond to circles of radii $\sim 17^\circ$ in the southern hemisphere and
171 $\sim 15.5^\circ$) in the northern hemisphere, hence there is considerable variability
172 in the size of Saturn’s polar cap.

173 The median value of the open flux distribution is ~ 35 GWb, marked
174 by the vertical dashed line on Figure 5. This amount of open flux would
175 be contained by a circular boundary centred on Saturn’s pole with radius
176 $\sim 14^\circ$ in the southern hemisphere and $\sim 13^\circ$ in the northern hemisphere.
177 The mean value is the same. The vertical dotted lines indicate the first
178 and third quartiles of the distribution, which are 29.8 GWb and 42.0 GWb,
179 respectively.

180 *4.2. Sequences of open flux estimates*

181 To investigate the time variability of the open flux content, the estimates
182 for each sequence of images from 2004–2013 are plotted versus time in Fig-
183 ure 6a–f. The grey and black dots mark the open flux estimate for each image
184 and the coloured shading gives the uncertainty range. The black dots in the
185 2007 and 2008 sequences highlight the estimates obtained from the images
186 shown in Figures 2 and 3. The time distributions are referenced to the time
187 of the minimum open flux value of each sequence, to facilitate comparison of
188 open flux loading and unloading trends.

189 The distributions for all sequences are plotted together by the coloured
190 lines in Figure 6g. We consider the decrease in open flux to the minimum of
191 each sequence, for those where the minimum value was in the first quartile of
192 the open flux distribution (< 30 GWb, from Figure 5). Two different trends
193 are observed. The first is a steady decrease over ~ 5 days, as seen in the 2007

194 (cyan), 2011 (yellow), and 2012 (orange) sequences. Example auroral images
195 used to estimate the open flux content over the interval in 2007 encompassing
196 the minimum value are shown with the open flux boundaries in Figure 2. The
197 open flux decreased from ~ 40 GWb to ~ 18 GWb (Figures 2a–g) in 2007,
198 ~ 33 GWb to ~ 16 GWb in 2011, and ~ 43 GWb to ~ 26 GWb in 2012.

199 The second trend is a sharper decrease occurring over less than 2 days,
200 as identified in the 2004 (black), 2008 (green), and 2013 (red) sequences,
201 which is illustrated in Figure 3 for the 2008 sequence. The open flux content
202 reduced from ~ 32 GWb to ~ 10 GWb in 2004, ~ 35 GWb to ~ 18 GWb
203 (Figures 3a–b) in 2008, and ~ 32 GWb to ~ 24 GWb in 2013. The first of
204 these was the largest decrease in open flux (~ 22 GWb) estimated from all
205 pairs of consecutive images used in this study. These decreases are correlated
206 with the occurrence of solar wind compressions at Saturn identified by Clarke
207 et al. (2005, 2009); Badman et al. (2005); Belenkaya et al. (2008).

208 The 2005 sequence (dark blue) was unusual in showing very little vari-
209 ation in open flux (37–44 GWb) over its week-long duration. Gérard et al.
210 (2006) noted that this campaign took place under particularly ‘quiet’ mag-
211 netospheric conditions.

212 The recovery from the minimum flux value also displays different be-
213 haviour between campaigns. The 2007 images indicate the most rapid sub-
214 sequent increase in open flux content in this study, from ~ 22 GWb to
215 ~ 39 GWb in ~ 1 day (shown in Figures 2g–h). The 2004 (black) and 2008
216 (green) campaigns accumulate a similar amount of open flux in total but over
217 3–4 d. The latter is shown in Figure 3b–e.

218 *4.3. Changes in open flux*

219 To investigate the typical change in open flux content, pairs of successive
220 images spaced by $10 < \Delta t < 60$ h were selected. This resulted in 61 pairs
221 of images. The lower time limit is imposed because evaluating changes in
222 open flux with their implicit uncertainties over short timescales of a few
223 hours or less can lead to excessively high estimated reconnection rates. The
224 validity of this limit is also affirmed by the study by Jackman et al. (2011),
225 who detected multiple magnetic field signatures of plasmoids in Saturn's
226 magnetotail during an interval of ~ 3 h. These could be counted together as a
227 single flux closure event. The upper limit of 60 h corresponds to the expected
228 occurrence interval between tail reconnection events involving unloading of
229 open flux, as found in the same study. Changes in flux over longer time
230 intervals are more likely to be attributed to multiple, separate reconnection
231 events, which would become indistinguishable if a longer time interval were
232 used. Furthermore, we are interested in determining the changes in open
233 flux observed, which would tend to average to zero over increasingly long
234 timescales.

235 The changes in open flux, $\Delta\Phi$, estimated between two consecutive images
236 are plotted against the time interval between the images, Δt , in Figure 7a.
237 The error bars account for the uncertainty in the open flux estimates. It is
238 clear that a wide range of both positive (net flux opening) and negative (net
239 flux closure) changes in open flux content were observed over all the time
240 intervals considered. This indicates that the open flux content of Saturn's
241 magnetosphere is far from steady.

242 The occurrence distribution of $\Delta\Phi$ is plotted in Figure 7b. The grey

243 shaded distribution represents all the values estimated while the solid line
 244 represents the distribution of only those values of $\Delta\Phi$ larger than their asso-
 245 ciated errors (43 values in total). The vertical dashed lines show the median
 246 positive and negative values for the reduced distribution. These distributions
 247 show that most of the net changes in open flux observed, $\Delta\Phi$, were less than
 248 ± 5 GWb over the time intervals studied, but that approximately half of these
 249 were small compared to their associated uncertainty. The median increases
 250 and decreases in open flux for the reduced distribution (where $\Delta\Phi$ is larger
 251 than its uncertainty) were $+7$ GWb and -5 GWb. However, the maximum
 252 changes observed were larger than 20 GWb.

253 These estimates of decreases in open flux are in good agreement with
 254 estimates of the amount of newly-closed flux transported in the PPS made
 255 by Jackman et al. (2011): up to ~ 6 GWb in a 3 h case study of multiple
 256 plasmoid encounters, and an average of up to ~ 3 GWb per event for all
 257 observations made.

258 4.4. Average, net reconnection rates

259 The time over which these changes in open flux was observed must also be
 260 considered. To do this, the average, net reconnection rate was calculated for
 261 each pair of images using $V_{avg,net} = \Delta\Phi/\Delta t$. Of course this cannot distinguish
 262 the separate rates of flux opening and closure, but while the rates must be
 263 equal over long timescales, they may be different over shorter intervals of
 264 time, such as those considered here.

265 The distribution of the derived $V_{avg,net}$ values are plotted in bins of 50 kV
 266 width in Figure 7c. As in panel (b), the grey shaded distribution represents
 267 all the values estimated while the solid line represents the reduced distribu-

268 tion. The majority of the values are clustered between ± 100 kV but half of
269 these are not significant compared to their errors. The median positive and
270 negative $V_{avg,net}$ values of the reduced distribution are +80 kV and -60 kV.
271 The overall mean is 3 kV, i.e. close to zero, confirming that the flux opening
272 and closing rates are equal over a long time interval.

273 The median positive flux loading rate is similar to the average and spot
274 values derived for flux opening at the magnetopause in previous studies (Jack-
275 man et al., 2004; Badman et al., 2005; McAndrews et al., 2008; Radioti et al.,
276 2011). These values correspond to intermediate driving by the solar wind,
277 based on empirical estimates of magnetopause reconnection rates by Jack-
278 man et al. (2004), while the maximum value, up to 305 kV, corresponds to
279 strong driving in a solar wind compression region.

280 4.5. Conditioning

281 We next consider whether there is any dependence of the net reconnection
282 rate on the initial or final amount of open flux present for those cases where
283 the changes in flux are larger than the associated uncertainties. Figure 8a
284 shows the distribution of average, net reconnection rates, $V_{avg,net}$ versus the
285 initial amount of open flux, Φ_1 , estimated from the first of the two consecutive
286 images. Similarly, Figure 8b shows the distribution of $V_{avg,net}$ versus the final
287 amount of open flux, Φ_2 , estimated from the second of the two consecutive
288 images. The distributions in the lower panels, c and d, of Figure 8 show the
289 relative occurrence of the positive (upper, dark grey shading) and negative
290 (lower, light grey) values of $V_{avg,net}$ in each 10 GWb open flux bin.

291 The relative heights of the bars in Figure 8c show that larger values of
292 open flux tend to be followed by negative net reconnection rates, i.e. large

293 open flux content tends to decrease. If the initial amount of open flux was
294 above 30 GWb, negative net reconnection rates were more often deduced to
295 follow, while if the starting amount of open flux was lower than 30 GWb,
296 positive reconnection rates were more often deduced (smaller open fluxes
297 tended to increase).

298 When comparing the net reconnection rates to their ‘final’ open flux val-
299 ues, Φ_2 , shown in Figure 8b and d, a trend in the opposite sense is observed.
300 Low open fluxes of < 30 GWb were three times more likely to be observed
301 after intervals of net flux closure. Net positive reconnection rates were more
302 frequently deduced preceding larger (> 40 GWb) open flux values.

303 While these trends seem intuitive, the fact that they are evident in a
304 large selection of images reveals that the reconnection rates are usually sig-
305 nificantly unbalanced over the various timescales considered in this study
306 (10–60 h). That is, Saturn’s magnetosphere does not generally display a
307 balanced ‘steady-state’ of solar wind interaction.

308 A final way to quantify this trend is to estimate the average and maximum
309 amount of open flux closed as a fraction of the initial open flux, i.e. $\Delta\Phi/\Phi_1$.
310 The maximum is found to be 69%, and the median (mean) across all pairs
311 of images is 13(18)%. The significance of these values will be discussed more
312 below.

313 5. Discussion

314 In the previous sections the averages and extrema of the open flux content
315 of Saturn’s magnetosphere, and their net rates of change, have been deduced.
316 Next, these values will be interpreted in comparison with estimates for the

317 Earth and Mercury, and their implications for the magnetospheric interac-
318 tion with the solar wind will be discussed. In this study, as described in the
319 Introduction, magnetic flux opening is considered to occur at the dayside
320 magnetopause, wherever the magnetic fields have an anti-parallel compo-
321 nent, while flux closure is generally considered to occur in the magnetotail.
322 The closure of magnetic flux at the dayside magnetopause via dual lobe re-
323 connection under southward IMF is not expected to be significant at Saturn
324 due to the predominantly azimuthal orientation of the IMF (Cowley et al.,
325 2008; Jackman et al., 2008b).

326 There is no routine upstream monitoring of the IMF at Saturn such that
327 we cannot comprehensively assess the IMF dependence of the open flux es-
328 timates obtained. IMF measurements were made by the Cassini spacecraft
329 during a few auroral imaging sequences, most notably the January 2004
330 campaign (Clarke et al., 2005). During these intervals the estimates of open
331 flux deduced from auroral images have been related to the IMF magnitude,
332 direction, and the solar wind dynamic pressure (Badman et al., 2005; Be-
333 lenkaya et al., 2007, 2008, 2010). These studies found that the open flux
334 increased with increasing northward IMF (more positive B_Z) and decreased
335 with increasing southward IMF (more negative B_Z). The reduction in open
336 flux under southward IMF was less when a strong B_Y component was also
337 present. The amount of open flux decreased following increases in solar wind
338 dynamic pressure. We expect the same general dependences to occur for all
339 the imaging sequences. However, the description in the following sections of
340 the increases and decreases in open flux is based only on the observed changes
341 in polar cap size, and is not conditional on assuming a certain prevalent IMF

342 orientation.

343 *5.1. Open flux content*

344 The amount of open flux in Saturn’s magnetosphere has been estimated to
345 be between 10 and 50 GWb, corresponding to 2–11% of the total magnetic
346 flux in one hemisphere. This is essentially the same as the proportion of
347 magnetic flux that has been identified by Milan et al. (2004) as open in the
348 Earth’s magnetosphere: 2.5–12%. At Mercury the estimated range is rather
349 higher, with $\sim 30\%$ of the planetary flux contained in an open magnetotail
350 during moderate loading events, and the suggestion that the magnetosphere
351 could approach 100% open under extreme loading conditions (Slavin et al.,
352 2010). Comparison of these values suggests that Saturn and the Earth have
353 a similar average interaction with the solar wind and IMF, leading to similar
354 open flux content, while Mercury’s magnetosphere is generally more open.

355 Jia et al. (2012) performed a global MHD simulation of Saturn’s mag-
356 netosphere under time-varying solar wind conditions. They found that the
357 amount of open flux varied between ~ 20 and ~ 35 GWb under northward or
358 azimuthal IMF conditions (implying anti-parallel or component reconnection
359 at the dayside magnetopause). These values are below the average estimated
360 from the auroral images in this study. The range of the values is also rather
361 smaller than estimated from the images (10–50 GWb). The reason for these
362 differences is not obvious and, as the reconnection rates in MHD simulations
363 depend strongly on numerical diffusion in the code, we do not attempt to
364 draw detailed conclusions on this.

365 The net amount of open flux closed over intervals between successive
366 images was found to be ~ 5 GWb, which agrees well with estimates made

367 from in situ magnetometer data (Jackman et al., 2011). Similar estimates of
368 the flux closed in tail reconnection events have also been obtained by a global
369 MHD simulation of Saturn’s magnetosphere by Jia et al. (2012), who found a
370 range of 1–10 GWb, with a mean of 3.5 GWb. Expressing the amount of flux
371 closed as a percentage of open flux originally present yields a median (mean)
372 value of $\sim 13(18)\%$ per interval, with a maximum of $\sim 69\%$. In only 2 of
373 25 cases was the net flux closed greater than 40% of the open flux originally
374 present.

375 This is in contrast to observations of the Earth’s magnetotail, where typ-
376 ically 40–70% of the open flux in the magnetotail is closed in a substorm
377 (flux closure event), and these large reconnection bursts provide the major
378 or only source of flux closure (Milan et al., 2003, 2007). It seems, therefore,
379 that while the average amount of planetary flux connected to the solar wind
380 is the same for the Earth and Saturn, the processes leading to open flux load-
381 ing and unloading may be quite different. Small amounts of open flux could
382 frequently be closed in post-plasmoid lobe reconnection events, such as those
383 described by Jackman et al. (2011), while the large-scale compressions of
384 the magnetosphere associated with solar wind shocks result in less-frequent,
385 large flux closure events, more like terrestrial substorms, and may be induced
386 by increased magnetic pressure in the compressed magnetotail (e.g. Badman
387 et al., 2005; Jia et al., 2012). It is important to remember that only the net
388 changes in open flux are deduced in this study and the amounts of open flux
389 loading and unloading in each interval cannot be separated without an up-
390 stream solar wind monitor. If, however, the open flux is usually removed via
391 small closure events, the open flux loading events should similarly be small

392 or occurring over long timescales.

393 *5.2. Reconnection rates*

394 In the absence of simultaneous in situ measurements of the separate tail
395 and magnetopause reconnection rates, we have been able to deduce only
396 the net change in open flux from the auroral images. It is likely that the
397 reconnection rates in the tail or at the magnetopause will sometimes be
398 significantly higher than the values obtained in this study but proceeding in
399 both locations at the same time, as identified in the Earth’s magnetosphere
400 e.g. by Milan et al. (2007). Furthermore these are average values determined
401 over 10–60 h intervals, while the reconnection rates may be significantly
402 higher but lasting for correspondingly shorter intervals. These differences
403 have been estimated and discussed by Badman et al. (2005) for the 2004
404 dataset when Cassini was measuring the IMF upstream of Saturn.

405 The present analysis suggests that open flux is usually added to Saturn’s
406 polar cap at an average rate of a few tens of kV. Stronger loading events,
407 with average flux transfer greater than 200 kV are deduced in only one case.
408 Flux closure events usually proceed at a similar average rate of a few tens of
409 kV, with a single, maximum net flux transfer rate of 275 kV.

410 Despite the uncertainties described above, the values determined in this
411 study are in agreement with previous estimates of magnetopause reconec-
412 tion voltages. For example, Jackman et al. (2004) used an empirical algo-
413 rithm scaled from studies at the Earth to estimate the rate of flux opening at
414 Saturn’s magnetopause. They found average reconnection rates of between
415 ~ 10 kV and ~ 400 kV in rarefied and compressed solar wind conditions,
416 respectively. McAndrews et al. (2008) estimated the reconnection voltage

417 from magnetic field and plasma data acquired during a crossing of the mag-
418 netopause by Cassini, and found an intermediate value of 48 kV.

419 Furthermore, because of the long timescales for transport of newly-opened
420 flux tubes from the dayside magnetopause to the magnetotail lobes (few days,
421 (Jackman et al., 2004)), the tail dynamics and possible terrestrial substorm-
422 like activity (i.e. flux closure events) are not expected to respond immediately
423 to dayside driving, therefore it is reasonable to expect that magnetopause
424 and tail reconnection can proceed independently of each other. We therefore
425 conclude that our net voltage estimates are representative of the average
426 magnetopause and tail reconnection rates which occurred. Overall, the fact
427 that a wide range of both positive and negative net reconnection rates have
428 been derived, including some particularly large values, suggests that Saturn’s
429 magnetosphere does not achieve a steady interaction with the solar wind over
430 the timescales considered.

431 **6. Conclusions**

432 The open flux content of Saturn’s magnetosphere has been estimated
433 based on a large set of auroral images, and found to lie within 10–50 GWb,
434 with a mean of 35 GWb. These values, and their variability are considerably
435 higher than those determined from global simulations of Saturn’s magneto-
436 sphere e.g. Jia et al. (2012).

437 Estimates of average, net reconnection rates have also been made by com-
438 paring open flux estimates separated by intervals of 10–60 h, and are found
439 to be typically a few tens of kV, with some extreme examples of unbalanced
440 magnetopause or tail reconnection occurring at up to 270 kV. The average

441 increase in open flux between images was 7 GWb and the average decrease
442 was 5 GWb. The largest open fluxes (> 40 GWb) tended to decrease by
443 2–7 GWb. The smallest open fluxes (< 30 GWb) usually followed decreases
444 of 6–20 GWb. The range of values determined suggest that Saturn’s mag-
445 netosphere does not generally achieve a balance between flux opening at the
446 magnetopause and flux closure in the magnetotail.

447 A further clue to this behaviour is that while the amount of open flux at
448 Saturn is similar to that measured at the Earth (2–11%), the typical fraction
449 that is closed over the intervals studied is significantly lower (13% compared
450 to 40–70%). Therefore, open flux is usually closed in smaller (few GWb)
451 events in Saturn’s magnetosphere. The exception to this behaviour is the
452 large, rapid flux closure events which are associated with solar wind com-
453 pressions, as identified in the 2004 data set by Badman et al. (2005). While
454 the rates of flux opening and closure should be equal over long timescales,
455 they are evidently different on shorter (up to tens of hours) timescales. The
456 independence of the magnetopause and tail reconnection rates, compared to
457 those observed at the Earth can be attributed to the long loading timescales
458 required to transport open field into the tail.

459 These results provide useful constraints for models of magnetospheric
460 dynamics and the extent of the interaction with the solar wind, and for
461 diagnosing the time history of magnetospheric dynamics from remote auroral
462 observations.

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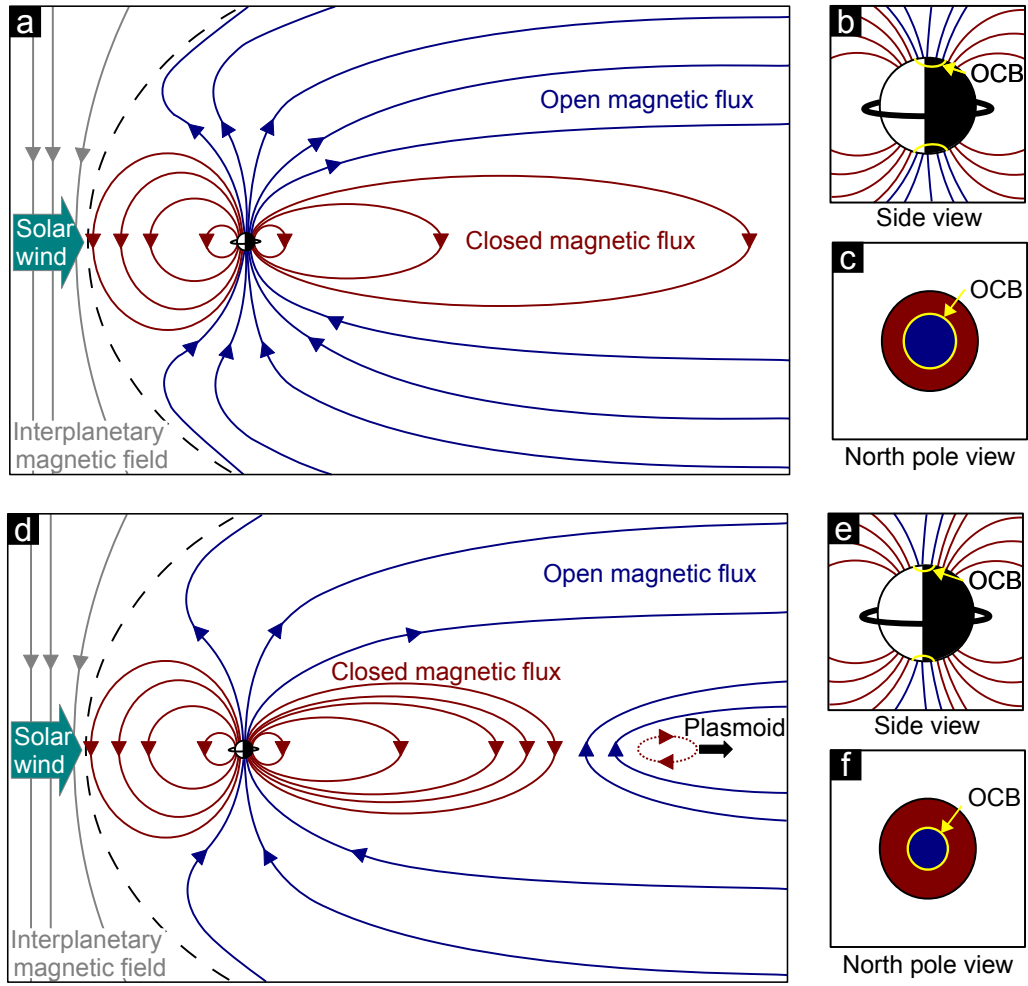


Figure 1: Schematic of Saturn's open magnetosphere (a) before, and (d) after a tail reconnection event which closes some of the open flux in the tail lobes. (b) and (e) The corresponding locations of the open-closed field line boundary (OCB) in the ionosphere. (c) and (f) The polar view of the OCB.

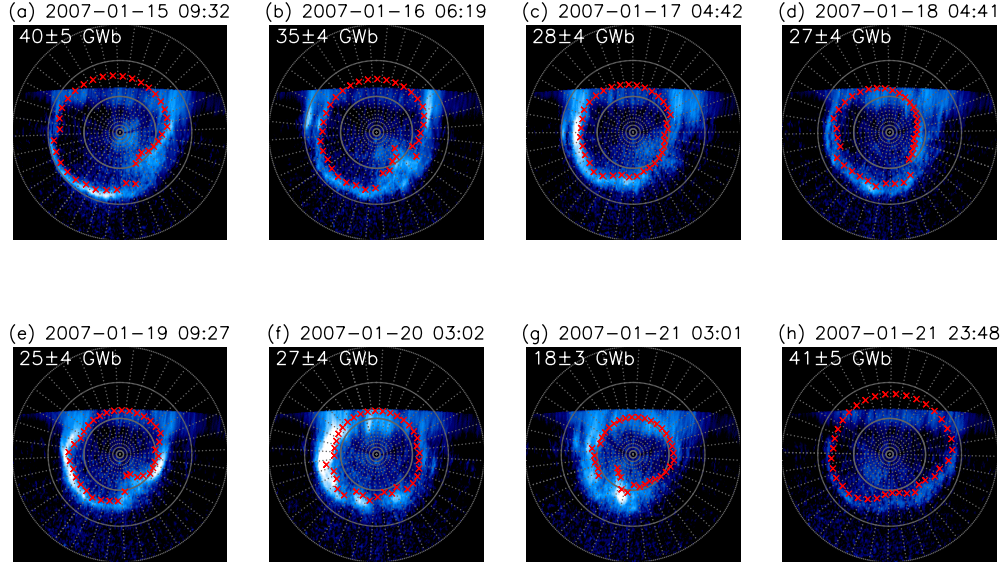


Figure 2: Sequence of images of the southern UV aurorae from the 2007 campaign. The images are polar projected with local noon to the bottom and dawn to the left. A portion of the nightside of each image is cut off where the viewing angle was 90° and higher because of uncertainties in the projection beyond this limit. The grey grid marks 10° lines of longitude and latitude. The start time of each image is labelled at the top. The red crosses mark the estimated boundary of the open flux region. The open flux estimate is labelled in the top left corner of each panel.

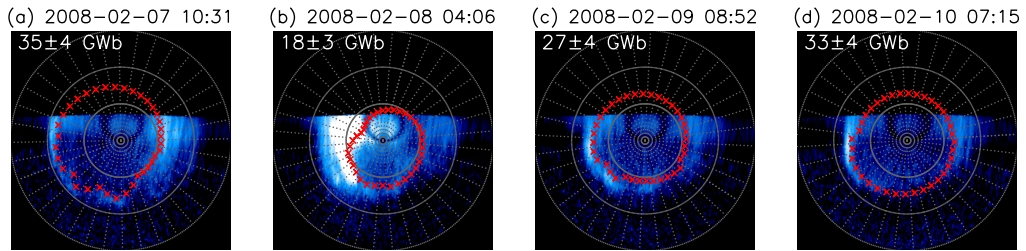


Figure 3: Sequence of images of the southern UV aurorae from the 2008 campaign in the same format as Figure 2.

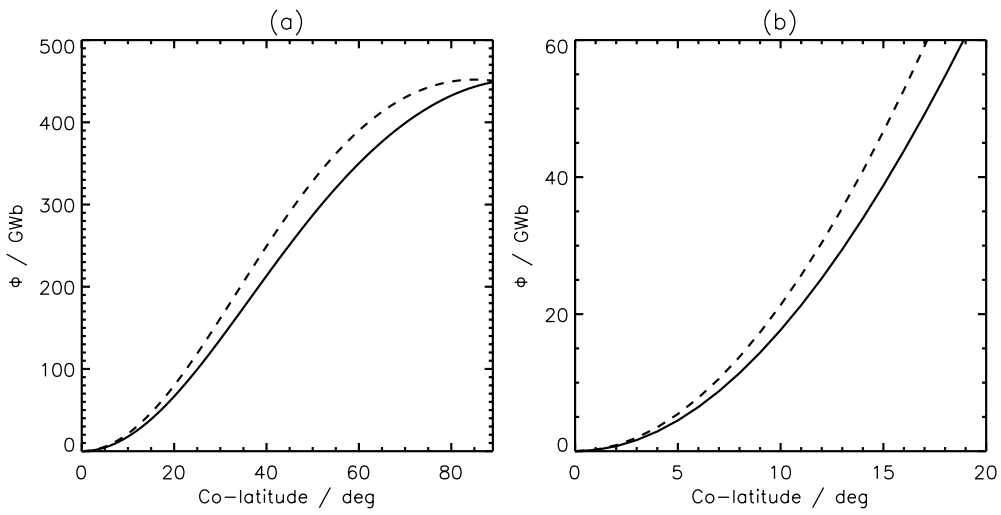


Figure 4: Magnetic flux, Φ , enclosed by a circular boundary centred on Saturn’s magnetic pole as a function of co-latitude radius. The solid line represents the southern hemisphere and the dashed line represents the northern hemisphere. (a) The full co-latitude range. (b) A reduced range pertinent to the values discussed in this study.

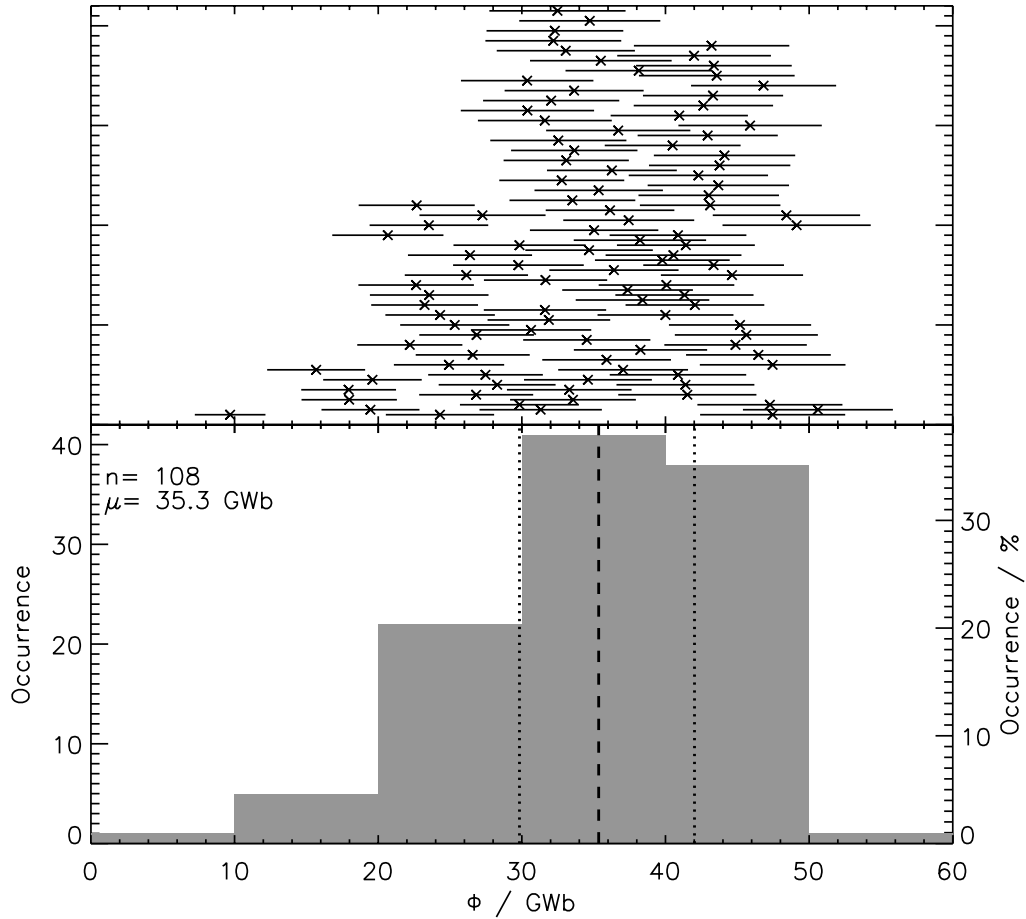


Figure 5: Distribution of estimated open flux, Φ , in 10 GWb bins. The lower panel shows a histogram of the values across bins of width 10 GWb, while the upper panel shows each value and its error bar. The distribution of values in the y-direction on the upper panel is simply to space the values so each error bar can be seen. The black dashed line on the lower panel marks the median of the distribution, and the two dotted lines mark the first and third quartiles.

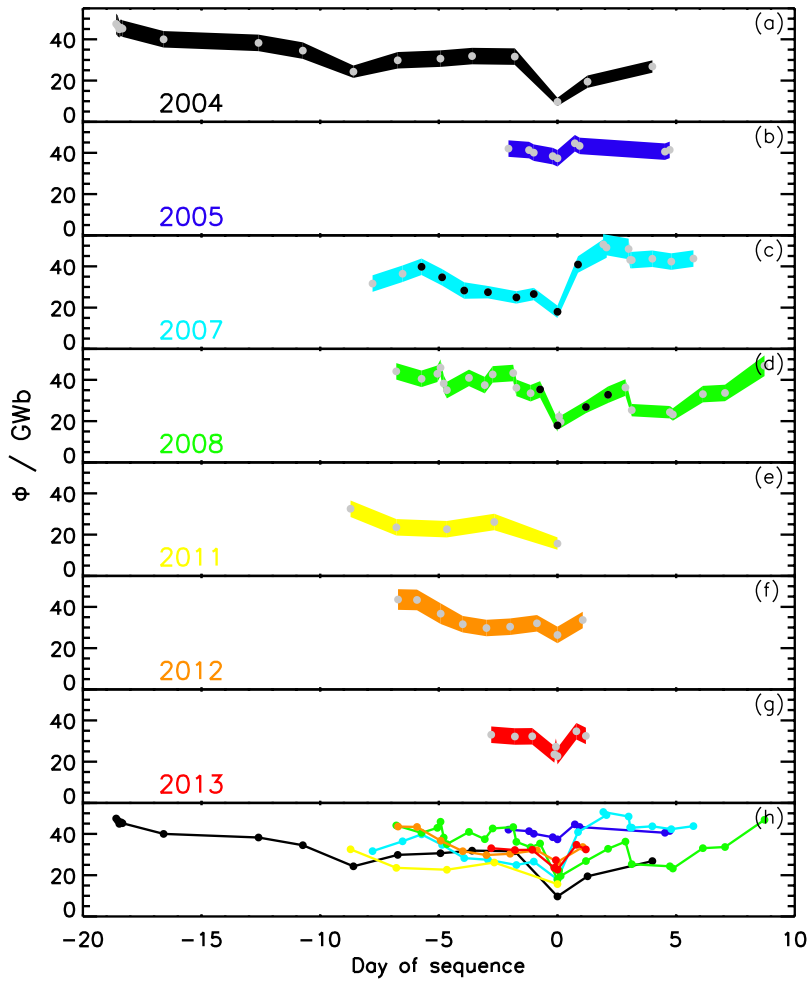


Figure 6: (a)–(f) Time series of open flux estimates for sequences of images in years 2004–2013. The coloured shading gives the uncertainty range on each estimate. Black dots indicate estimates obtained from images shown in Figures 2–3. Each time series is referenced to the time of the minimum open flux estimate in that sequence. (g) Open flux estimates for all campaigns from panels above.

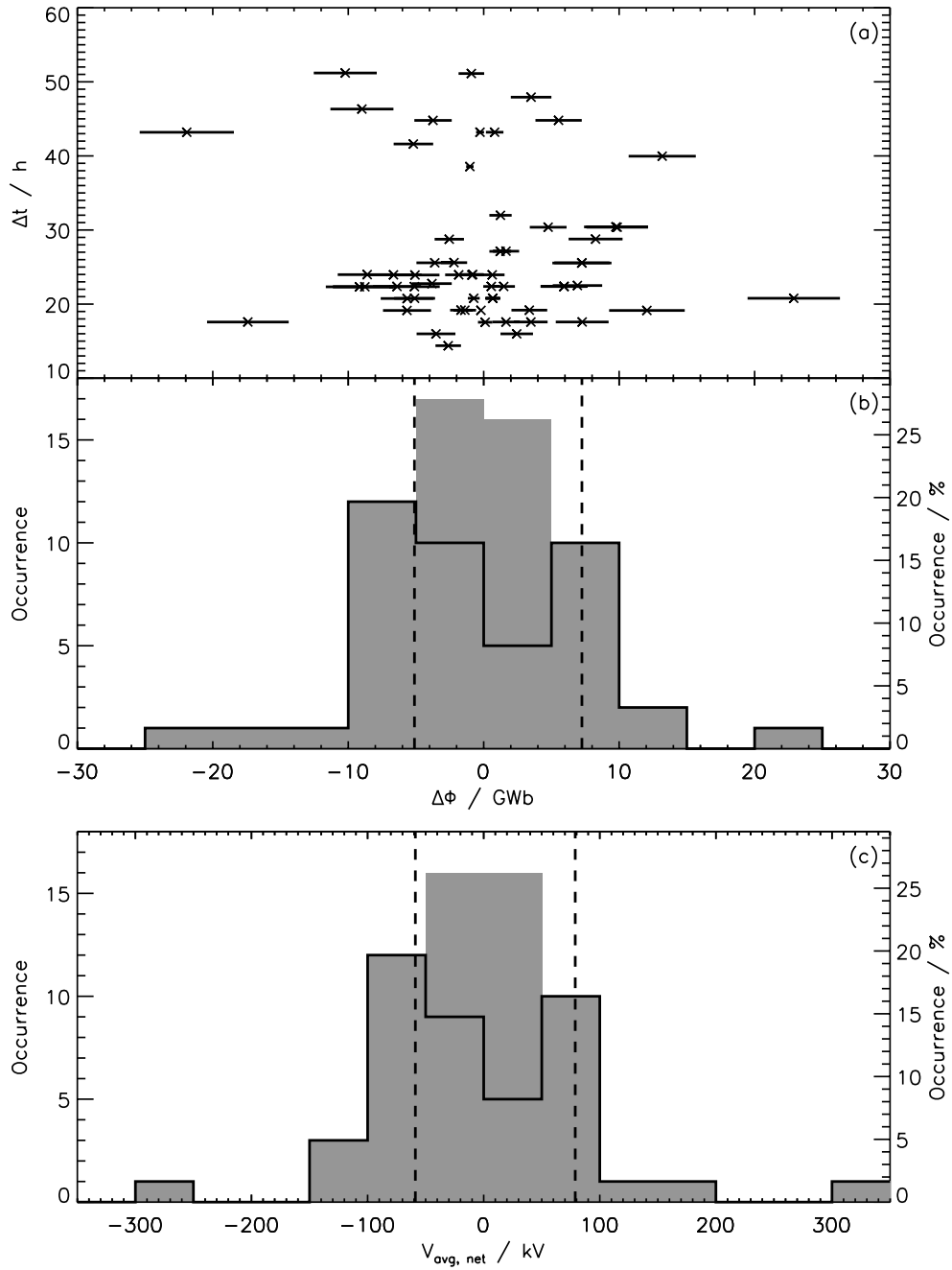


Figure 7: Changes in open flux, $\Delta\Phi$, and the derived average, net reconnection rates, $V_{avg,net}$. (a) $\Delta\Phi$ versus the time between images, Δt . (b) Distribution of $\Delta\Phi$ values. The grey shaded distribution represents all the values estimated, while the solid line represents the distribution of only those values of $\Delta\Phi$ larger than their associated errors. The vertical dashed lines show the median positive and negative values for the reduced distribution. (c) Distribution of $V_{avg,net}$ values in a similar format as (b).

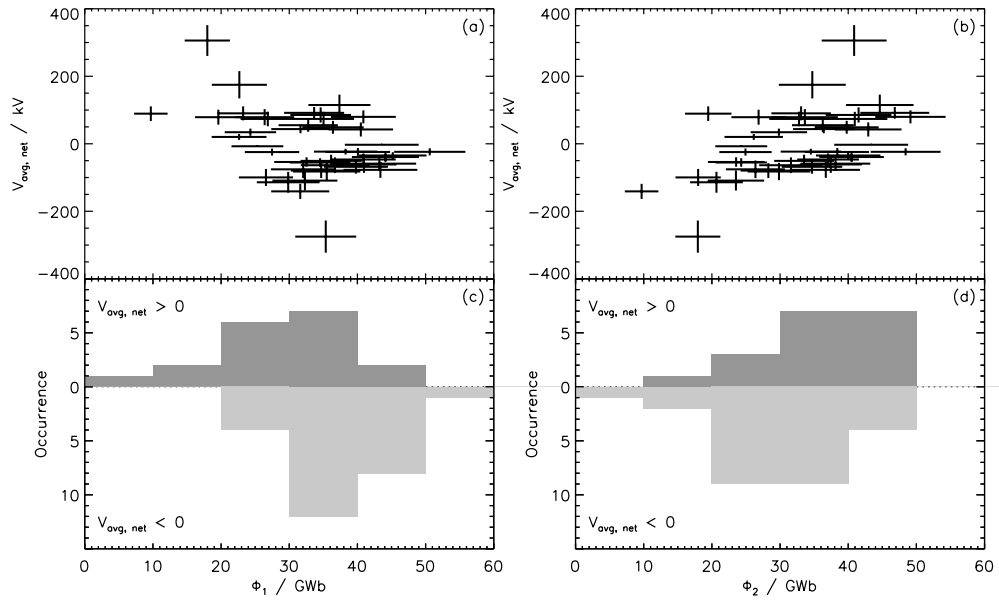


Figure 8: Average reconnection rates associated with each pair of successive open flux measurements Φ_1 and Φ_2 . (a) Initial open flux, Φ_1 , and $V_{avg,net}$ values and associated uncertainties. (b) Final open flux, Φ_2 , and $V_{avg,net}$ and associated uncertainties. (c) The number of positive (upper, dark grey) and negative (lower, light grey) $V_{avg,net}$ values in each 10 GWb bin of Φ_1 . (d) The same as (c) but for Φ_2 .