

DEFORMATIONS IN DEEP CONTINUOUS REINFORCED CONCRETE TRANSFER GIRDERS

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SUMMARY

This paper presents a three-parameter kinematic model for the deformation patterns of deep continuous transfer girders. The three degrees of freedom of the model are the average strains along the top and bottom longitudinal reinforcements within each shear span, as well as the transverse displacement in the critical loading zone. The model is validated with the help of a large test of a two-span continuous beam performed at the University of Toronto. It is shown that the apparently complex deformation patterns of the specimen are captured well by the kinematic model.

Keywords: *transfer girders, deep beams, tests, shear deformations, kinematic model.*

1. INTRODUCTION

Continuous beams with small span-to-depth ratios (deep beams) are common in urban construction. For example transfer girders near the podium level of high rise buildings are often used to support heavy column loads across large open spaces. As these members are characterized by large shear distortions, their serviceability and displacement capacity can be assessed accurately only with the help of complex non-linear finite element models. This paper presents the basis of an alternative approach: a kinematic model with only three degrees of freedom (DOFs) capable of capturing the apparent complex deformation patterns of deep beams under double curvature. This model stems from an earlier two-parameter kinematic model for deep beams under single curvature [1][2].

2. DEFORMATIONS IN CONTINUOUS DEEP BEAMS

The deformation patterns of deep continuous transfer girders will be discussed with respect to the test specimen shown in Fig. 1. This symmetrical two span continuous beam is approximately a one-third scale model of a full storey height transfer girder. It contained 0.91% of longitudinal reinforcement top and bottom and 0.20% of transverse reinforcement.

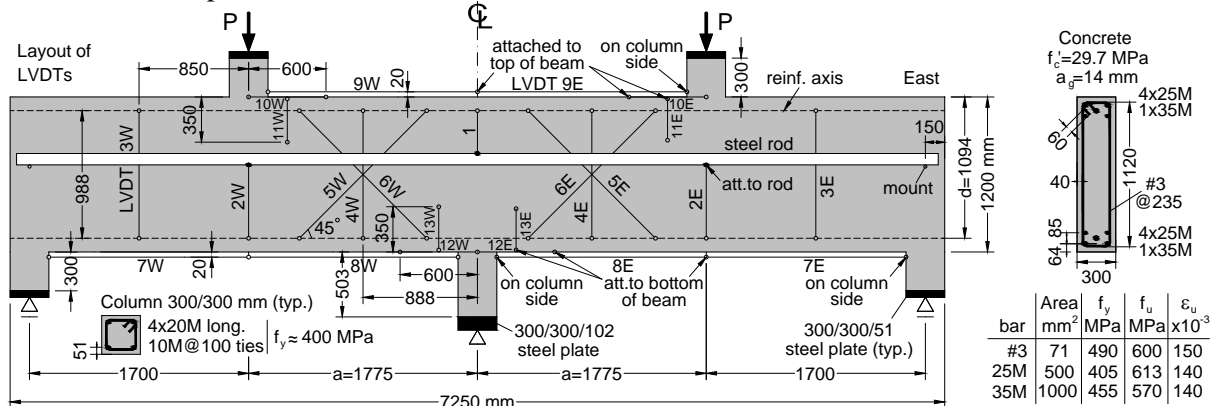


Fig. 1 Test specimen and layout of Linearly Variable Differential Transformers (LVDTs)

The beam was loaded to failure by two symmetrical loads P which were applied in nine load steps. At the end of each load step the load was lowered by about 10% in order to mark cracks and measure deformations on the concrete surface. The envelope of the measured load-deflection response of the specimen is shown in Fig. 2. The deflections under the two loads were measured with LVDTs 2E and 2W with respect to a steel rod going between two end mounts (Fig. 1). It can be seen that failure commenced in the east span after load stage 7 (LS7) when a small increase in load resulted in a large increase in deflection Δ_E and the shear force V_{int} reached its maximum value. The load P could still be increased because V_{ext} increased faster than V_{int} decreased. At LS9 a large diagonal crack had opened in the inner east shear span and concrete crushing was visible near the load, see Fig. 3. Strain measurements on the longitudinal reinforcement showed that the steel bars remained elastic.

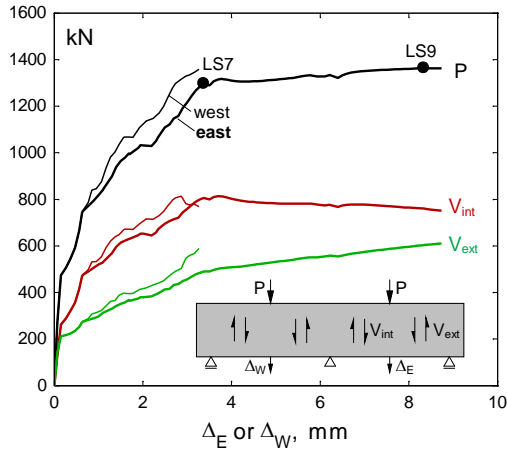


Fig. 2 Load-displacement response

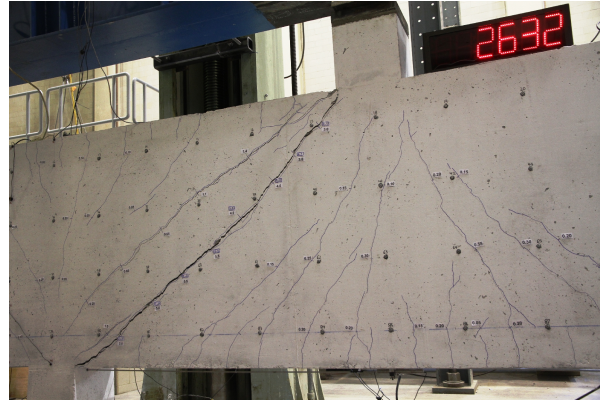


Fig. 3 East span of specimen at load stage 9

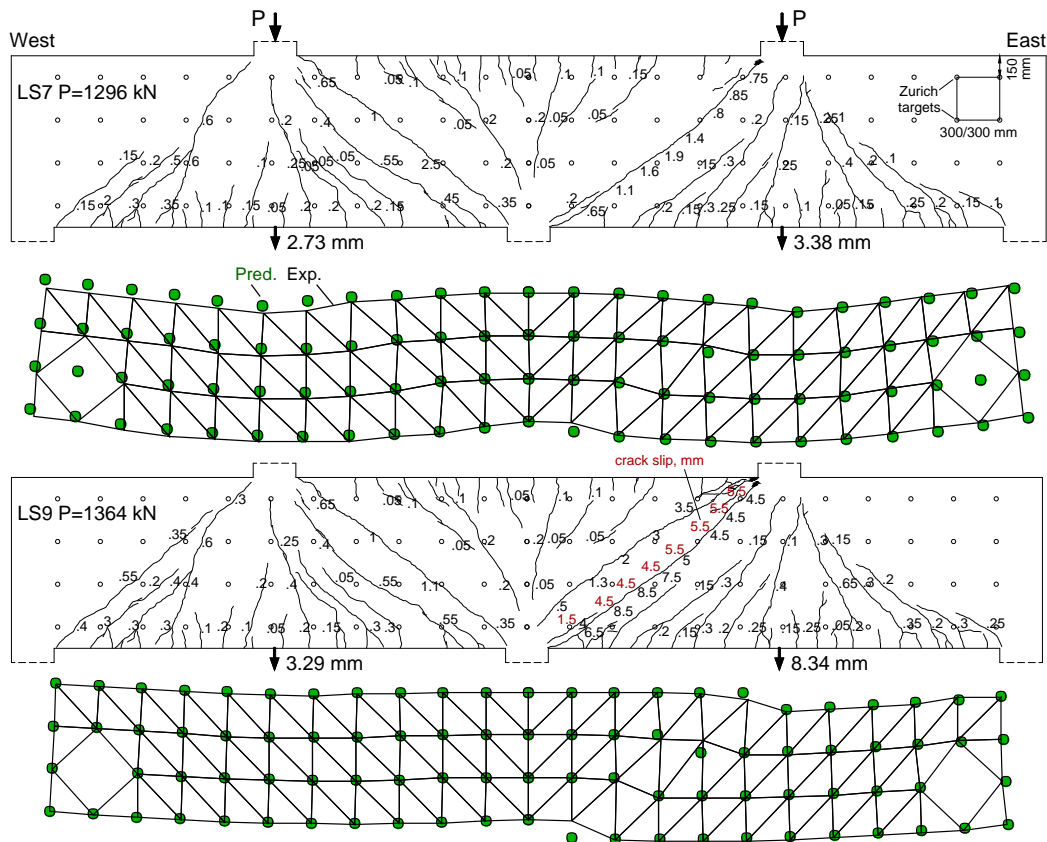


Fig. 4 Deformations in test specimen

Figure 4 shows the crack diagrams and the measured deformed shapes of the specimen at LS7 and LS9. The deformed shapes were calculated from the readings of demountable displacement transducers taken on a 300 mm by 300 mm grid of targets and are drawn so that for each diagram the maximum displacement plots as if it was 150 mm. It can be seen that the crack patterns consist of three “fans” of cracks radiating out of the two loading points and the middle support. At LS7 the width of the critical diagonal crack in the inner east shear span reached 1.9 mm while the rest of the crack widths in this shear span did not exceed 0.55 mm. Despite the wide shear cracks, the deformed shape of the specimen at LS7 is dominated by flexural deformations characterized by “positive” curvatures in the spans and “negative” curvatures above the middle support. The shear deformations are evident from the distorted shape of the triangles along the major diagonal cracks. At LS9 when the east inner shear span was post-peak, the shear distortions dominated the measured deformed shape. The crack measurements along the critical diagonal crack show that its width reached 8.5 mm while the maximum crack slip was 5.5 mm. Significant crushing of concrete was observed at the top and bottom ends of the critical crack in the critical loading zones (CLZ) [2]. A simple description of these apparently complex deformation patterns is discussed next.

3. THREE-PARAMETER KINEMATIC MODEL

The deformation patterns of deep beams subjected to single curvature can be described by a two-parameter kinematic model suggested by Mihaylov et al. [1][2]. This model consists of the first two deformation patterns depicted in Fig. 5. The first pattern is associated with the elongation of the bottom longitudinal reinforcement (average strain $\epsilon_{b,avg}$) while the second pattern is associated with the transverse displacement Δ_c at the CLZ. The critical diagonal crack is assumed to extend from the inner edge of the support to the point along the loading plate where the shear force is zero or minimum. The cracked concrete below the crack is represented by a set of rigid radial struts pinned at the loading point and connected to the bottom reinforcement. The concrete above the critical crack is modelled as a rigid block. The combination of deformation patterns 1 and 2 is used to reproduce the complete deformed shape of the external shear spans of continuous deep transfer girders, see Fig. 6. The third deformation pattern in Fig. 5 is introduced in order to extend the two-parameter kinematic model to deep beams under double curvature. This pattern is associated with the elongation of the top longitudinal reinforcement (average strain $\epsilon_{t,avg}$) and is an inverted version of pattern 1. When the three patterns are superimposed, they model the complete deformed shape of the internal shear spans of continuous transfer girders as shown in Fig. 6.

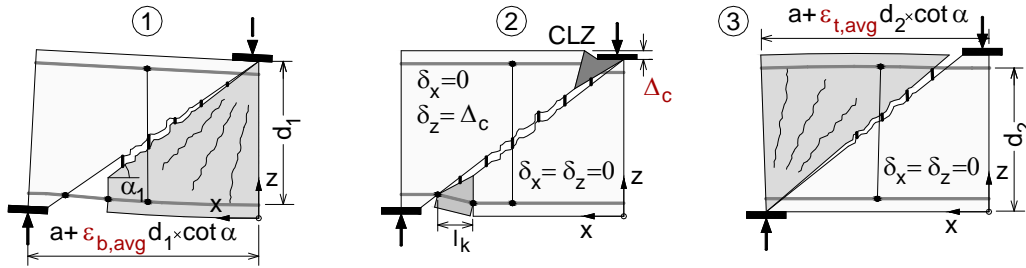


Fig. 5 Three-degree of freedom kinematic model for shear spans of deep beams

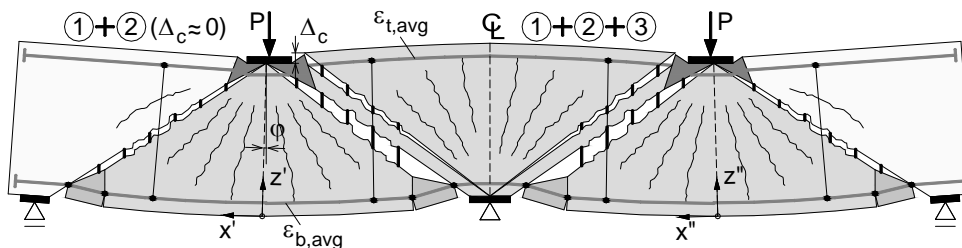


Fig. 6 Three-degree of freedom kinematic model applied to a symmetrical two-span deep transfer girder

The deformation patterns of the three-parameter kinematic model can be expressed by the following algebraic relationships as a function of the degrees of freedom of the model $\epsilon_{b,avg}$, $\epsilon_{t,avg}$, and Δ_c :

- Points below the critical diagonal crack

$$u_x(x, z) = v_{b,avg} x \quad (1)$$

$$u_z(x, z) = v_{b,avg} x^2 / (h - z) \quad (2)$$

- Points above the critical diagonal crack

$$u_x(x, z) = v_{b,avg} (h - z) \cot \gamma + v_{t,avg} (x - a + z \cot \gamma) \quad (3)$$

$$u_z(x, z) = v_{b,avg} x \cot \gamma + \Delta_c + v_{t,avg} (x - a + z \cot \gamma) (a - x) / z \quad (4)$$

To validate the model, equations (1)-(4) are applied to the test specimen based on the scheme depicted in Fig. 6. It is assumed that the transverse displacement in the CLZs of the external shear spans is negligible ($\Delta_c = 0$) as the shear failure occurred in the east inner shear span. The rest of the DOFs of the model, namely the average strains in the bottom and top reinforcement, and the transverse displacements of the inner CLZs, were obtained from the readings of LVDTs 7E/W, 8E/W, 9E/W, 11E/W, and 13E/W (Fig. 1). The values at LS7 and LS9 are summarized in Table 1.

Table 1 Measured degrees of freedom of the three-parameter kinematic model

Load Stage	P (kN)	West Span					East span				
		External		Internal Shear Span			Internal Shear Span			External	
		$v_{b,avg} \times 10^{-3}$	U_c (mm)	$v_{b,avg} \times 10^{-3}$	$v_{t,avg} \times 10^{-3}$	U_c (mm)	U_c (mm)	$v_{t,avg} \times 10^{-3}$	$v_{b,avg} \times 10^{-3}$	U_c (mm)	$v_{b,avg} \times 10^{-3}$
7	1296	1.207	0	1.060	0.574	0.435	0.943	0.726	1.105	0	1.155
9	1364	1.601	0	1.450	0.266	0.510	9.28	0.061	1.691	0	1.500

The resulting predicted locations of the points from the grid are shown with green circles in Fig. 4. It can be seen that the circles match well the vertices of the triangles depicting the measured locations of these targets. From Table 1 it can be seen that the average strains in the longitudinal reinforcement were almost symmetrical with respect to the middle section. This is due to the symmetrical bending moment diagram and the elastic longitudinal reinforcement. Furthermore, the average strains in the bottom reinforcement were almost constant along the two spans as characteristic of arch action in deep beams. The transverse displacement in the inner CLZs, on the other hand, differed significantly between the east and west shear spans since the shear failure occurred in the east span. Therefore, the three-parameter kinematic model shown in Fig. 5 can be used to accurately predict the entire deformed shapes of deep continuous girders such as the test specimen with the help of only four independent kinematic parameters: the average strain in the bottom reinforcement, the average strain in the top reinforcement between the loading points, and the transverse displacements of the two inner CLZs. These degrees of freedom can be predicted if the equations of the kinematic model are combined with equilibrium equations and appropriate constitutive relationships for the mechanisms of shear resistance of the member [2][3].

4. CONCLUSIONS

This paper showed that the apparently complex deformation patterns of deep continuous transfer girders can be accurately described by a kinematic model that uses only three degrees of freedom per shear span. These degrees of freedom are the average strains in the top and bottom longitudinal reinforcement, as well as the transverse displacement in the critical loading zone. The degrees of freedom of the model were measured in a test of a large shear-critical two-span continuous deep beam. It was shown that with the measured DOFs the three-parameter kinematic model provides an excellent approximation of the measured deformation patterns of the specimen. Approaches for predicting the values of the DOFs and using the model to estimate both shear strength and load distribution in such continuous girders near shear failure are discussed elsewhere [2][3].

References

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