ON A MODIFIED BILINEAR LAW TO MODEL BIT/ROCK INTERACTION IN PERCUSSIVE DRILLING

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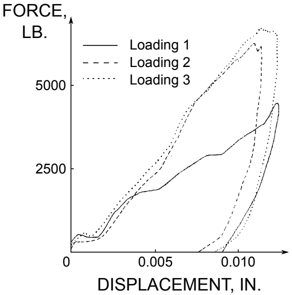
BIT/ROCK INTERACTION: SINGLE IMPACT

Experimental facts

- Multiple failure mechanisms
  - Indentation & crushing
  - Chipping
- 2 main phases
  - Loading (∼compression)
  - Unloading (∼expansion)
- Rate-independent

Hustrulid & Fairhurst (1972)



→ FORCE/PENETRATION ∼ BILINEAR LAW: (K_R, γ)

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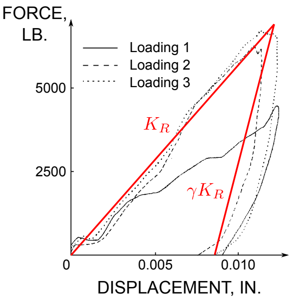
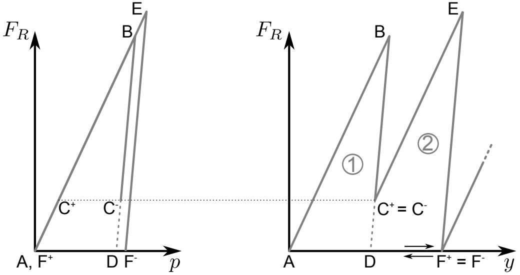


→ FORCE/PENETRATION ∼ BILINEAR LAW: (K_R, γ)

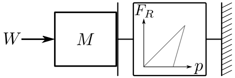
BIT/ROCK INTERACTION: REPEATED IMPACTS

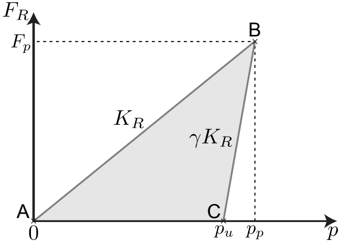
- Hyp: bilinear law holds
- Need: penetration definition

$$p^{(n)}(t) = y(t) - y_\ell^{(n)} + \frac{F_{R,\ell}^{(n)}}{K_R} \quad \rightarrow \quad F_R^{(n)}(t) = \begin{cases} K_R p^{(n)}(t) & \text{forward cont.} \\ \gamma K_R \left(p^{(n)}(t) - p_u^{(n)}\right) & \text{backward cont.} \\ 0 & \text{free flight} \end{cases}$$



ANALYSIS OF THE DRILLING CYCLE





$$\Delta K = M\frac{\dot{p}_2{}^2 - \dot{p}_1{}^2}{2} = W(p_2 - p_1) - W_{F_R}^{1\to 2}$$

$$p_p = \frac{W}{K_R}\left(1 + \sqrt{1 + \frac{K_R}{W}\frac{M\dot{p}_\ell^2}{W}}\right)$$

$$p_u = \frac{\gamma - 1}{\gamma}p_p$$

$$\dot{p}_u = -\frac{1}{\sqrt{\gamma}}\dot{p}_\ell$$

$$T = \sqrt{\frac{M}{K_R}}\left(1 + \frac{1}{\sqrt{\gamma}}\right)\left(\frac{\pi}{2} + \text{acos}\frac{\dot{p}_\ell}{\sqrt{\dot{p}_\ell^2 + \frac{W^2}{K_R M}}}\right)$$

Learnings

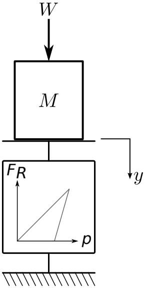
- ~~BC~~,~~FF~~ at the limit $\gamma \to \infty$
- Need to discern static from dynamic indentation

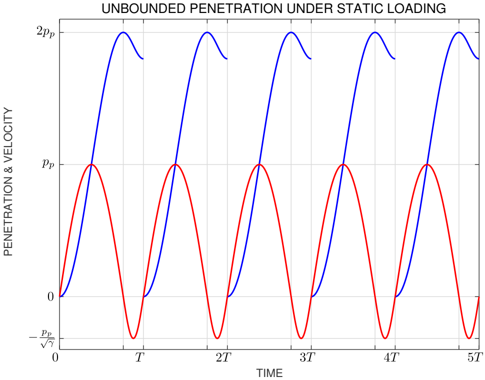
  $p_\ell = \dot{p}_\ell = 0 \to (p_u, \dot{p}_u) = (2W/K_R, 0)$

BIT/ROCK INTERACTION: RATE-INDEPENDENCE

Domain of validity

- Upper bound: $\dot{p} < c_0$, verified in practice
- Lower bound: static vs. dynamic, requires adjustment of BRI (no $\rightleftharpoons$)





BIT/ROCK INTERACTION: RATE-INDEPENDENCE

Domain of validity

- Upper bound: $\dot{p} < c_0$, verified in practice
- Lower bound: static vs. dynamic, requires adjustment of BRI (no $\rightleftharpoons$)

$\rightarrow$ Introduce energy barrier & instantaneous dissipation

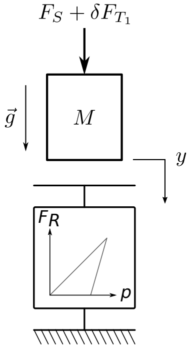
Definition

At the beginning of a drilling cycle, part of the bit kinetic energy is instantaneously dissipated by fast processes, e.g. wave radiation

$$\dot{y}_\ell^+ = \begin{cases} 0 & \text{if } \frac{1}{2} M \left(\dot{y}_\ell^-\right)^2 \le E_\ell^k \\ \sqrt{\left(\dot{y}_\ell^-\right)^2 - 2E_\ell^k / M} & \text{otherwise} \end{cases}$$

In accordance with Lundberg and Okrouhlik (2006): $\mathcal{O}(5)\%$ of percussive energy is dissipated by radiation

BIT DYNAMICS: IMPACT OSCILLATOR



Timescales

- T_1: Percussive activation period $\rightarrow \mathcal{O}(50)$ms
- T_2: Percussive impact duration $\rightarrow \mathcal{O}(1)$ms
- T_3: Drilling cycle duration $\rightarrow T_1 > T_3 > T_2$

$$T_3 \sim \sqrt{\frac{M}{K_R}}$$

Model specificities

- Periodic impulsive force activation

$$\delta F_{T_1}(t) = I \sum_{n \in \mathbb{Z}^+} \delta(t - nT_1 - t_s)$$

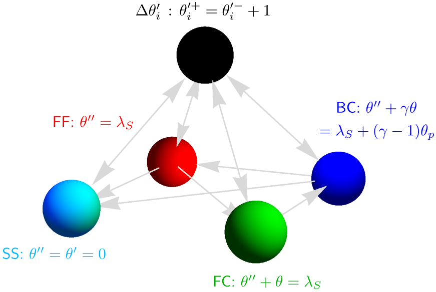
- Modified bilinear interaction law

TIMESCALE SEPARATION – $T_1, T_3 \gg T_2$

- Reference scales: $t_* = \sqrt{M/K_R}$, $\ell_* = I/\sqrt{MK_R}$
- Variables: $\tau = t/t_*$, $\theta = p/\ell_*$
- Parameters: $\gamma = \gamma K_R/K_R$, $\lambda_S = (Mg + F_S)t_*/I$

  $\psi = T_1/t_*$, $\kappa_0 = \sqrt{2E_\ell^k M/I}$

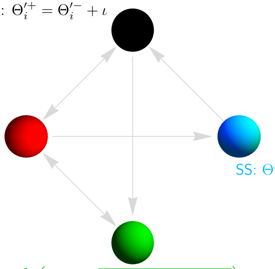
- Hybrid model: 5 modes, 10 transitions



TIMESCALE SEPARATION – $T_1 \gg T_2, T_3$

- Reference scales: $T_* = T_1$, $L_* = I/\sqrt{MK_R}$
- Variables: $T = t/T_*$, $\Theta = p/L_*$
- Parameters: $\gamma = \gamma K_R/K_R$, $\Lambda_S = (Mg + F_S)\,T_*^2/ML_*$

  $\iota = T_*\sqrt{K_R/M}$ $\kappa_0 = \sqrt{2E_\ell^k/M}\,T_*/L_*$
- Hybrid model: 4 modes, 5 transitions



TIME INTEGRATION: EVENT-DRIVEN SCHEME

Piecewise linear hybrid system

1. Integrate smooth vector field until next non-smooth event
2. Accurately locate non-smooth event
3. Identify next drilling phase and set appropriate initial conditions

Example – $T_1, T_3 \gg T_2$ – IC: (FC, θ_0, θ_0')

1. Exploit closed-form solution of governing ODEs

$$\text{FC}: \theta'' + \theta = \lambda_S \quad \rightarrow \quad \theta(\tau) = (\theta_0 - \lambda_S)\cos\tau + \theta_0' \sin\tau + \lambda_S$$

2. Locate closest non-smooth event among all possible events

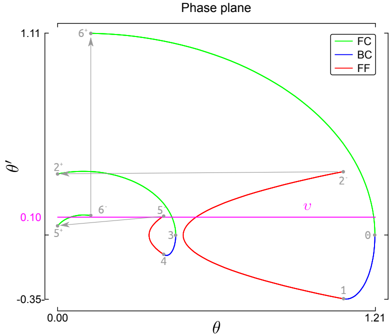
$$\tau_{EVT} = \min(\tau_p, \tau_i) \text{ with } \tan\tau_p = \frac{\theta_0'}{\theta_0 - \lambda_S} \text{ and } \tau_i = n\psi + \tau_s$$

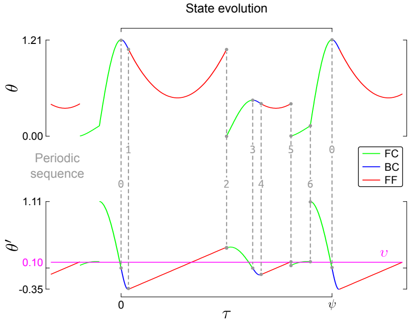
3. Set adequate initial conditions

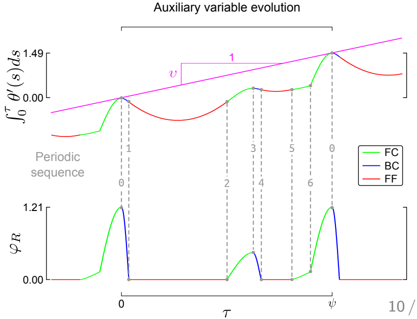
$$\tau_{EVT} = \tau_i : \text{DP} = \text{FC}, \quad \theta^+ = \theta^-, \quad \theta'^+ = \theta'^- + 1$$
$$\tau_{EVT} = \tau_p : \text{DP} = \text{BC}, \quad \theta^+ = \theta^-, \quad \theta'^+ = \theta'^-$$

PERIODIC SOLUTIONS – $T_1, T_3 \gg T_2$

- $(\gamma, \lambda_S, \psi, \kappa_0)$
  $= (10, 0.1, 15, 0.30)$
- Shooting method: θ_p, τ_s
- Descriptors:
  - $m/n = 2/1$
  - $((\mathrm{FC} \to \mathrm{BC} \to \mathrm{FF})_2 \to \mathrm{FC} \to \Delta\theta'_i)_{\circlearrowleft}$
  - Phase portrait

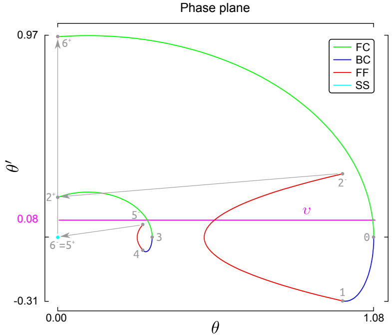


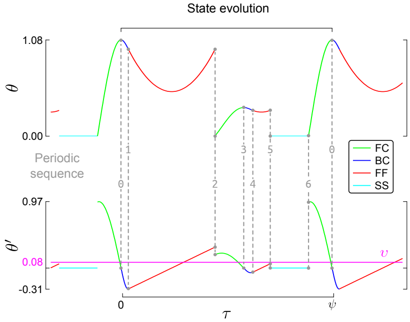


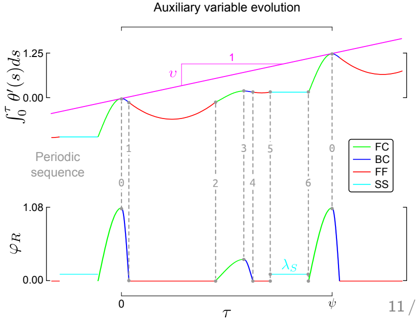


PERIODIC SOLUTIONS – $T_1, T_3 \gg T_2$

- $(\gamma, \lambda_S, \psi, \kappa_0)$
  $= (10, 0.1, 15, 0.75)$
- Shooting method: θ_p, τ_s
- Descriptors:
  - $m/n = 2/1$
  - $((\text{FC} \to \text{BC} \to \text{FF})_2 \to \text{SS} \to \Delta\theta'_i)_{\circlearrowleft}$
  - Phase portrait

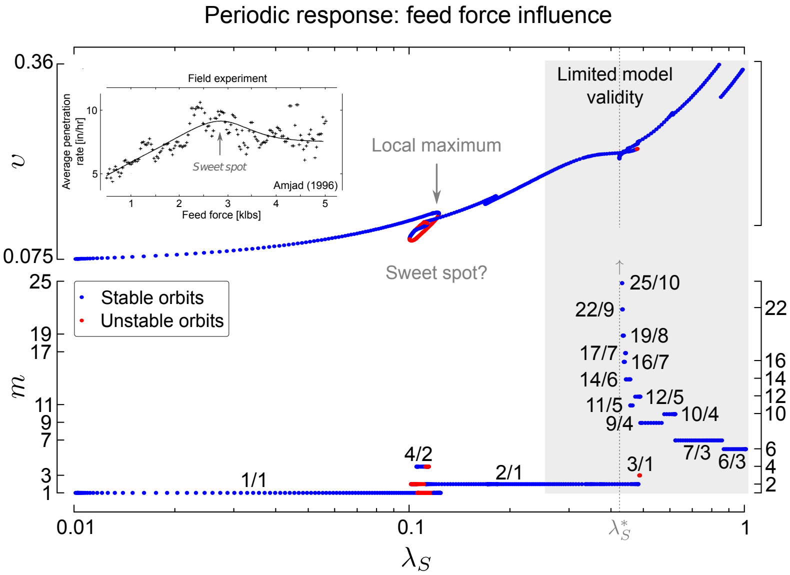




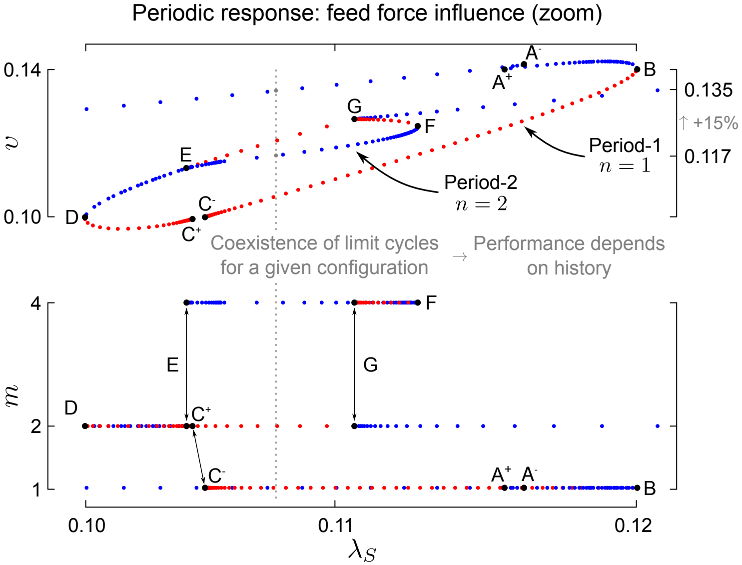


BIFURCATION DIAGRAM – $T_1, T_3 \gg T_2$

$(\gamma, \psi, \kappa_0) = (10, 10, 0.3)$



BIFURCATION DIAGRAM – $T_1, T_3 \gg T_2$
$(\gamma, \psi, \kappa_0) = (10, 10, 0.3)$ – $\lambda_S \in [0.10, 0.12]$



SUMMARY

Modified bilinear bit/rock interaction model

- Follows from experimental evidence
- Energy barrier is essential to discriminate static & dynamic contact

Impact oscillator + modified bilinear BRI model

- two possible models, depending on BRI timescale
- coexistence of periodic solutions
- clues of the experimentally-observed sweet spot

→ this suggests the sweet spot to result from the process dynamics, not from a change of bit/rock interaction mechanisms

For further details:

Depouhon, Denoël, Detournay – A drifting impact oscillator with periodic impulsive loading: Application to percussive drilling. *Physica D* 258 (2013) 1–10.

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