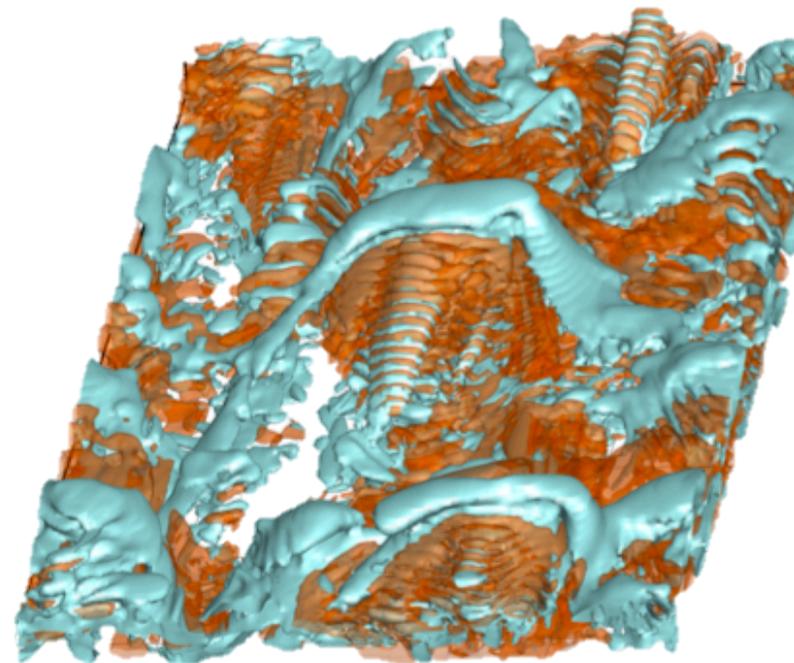


# Dynamics of Elasto-Inertial Turbulence in Flows with Polymer Additives

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Poitiers, 30 August 2013



# Acknowledgements

## Collaborators

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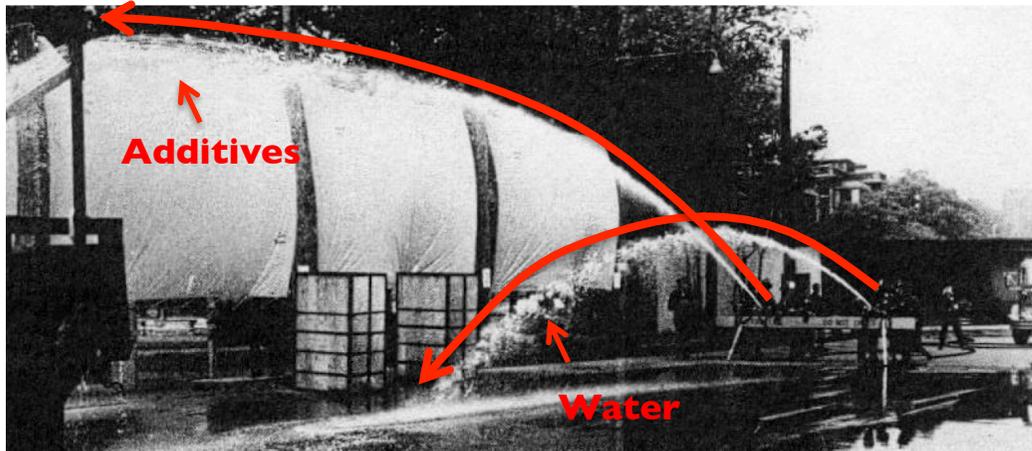
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- Marie Curie FP7 CIG
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- Australian Research Council
- Center for Turbulence Research Summer Program



# Polymers and turbulence

## Turbulent drag reduction

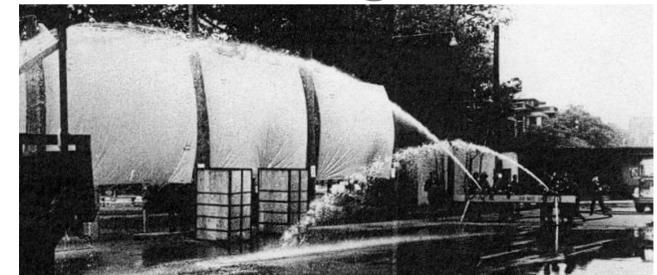


Fire hoses with and without polymer additives

- Up to 80% friction drag reduction, even at low concentration
- No significant effect on drag in laminar flows
- Bounded by Maximum Drag Reduction (MDR) asymptote

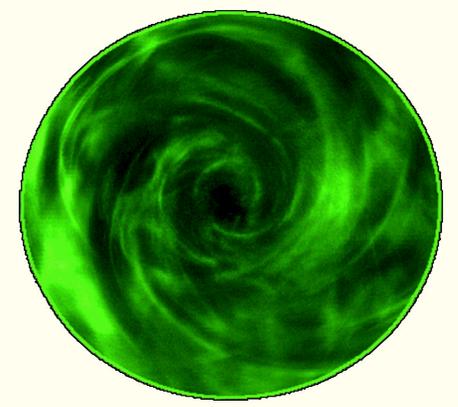
• Pipeline

## Turbulent drag reduction



# Polymers and turbulence

## Elastic turbulence

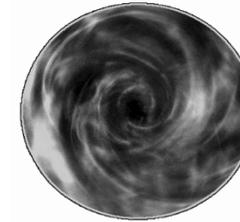


Chaotic motion of a polymer solution in micro-channel  
(Groisman & Steinberg, 2000)

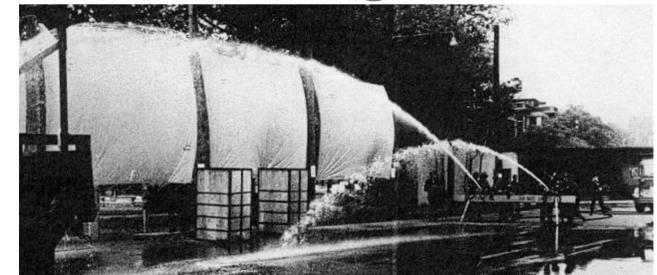
- Existence of elastic turbulence in flows with curved streamlines
- Observed at low Reynolds number
- Strong increase in mixing properties

- Blood flow
- Micro-channel flow

## Elastic turbulence

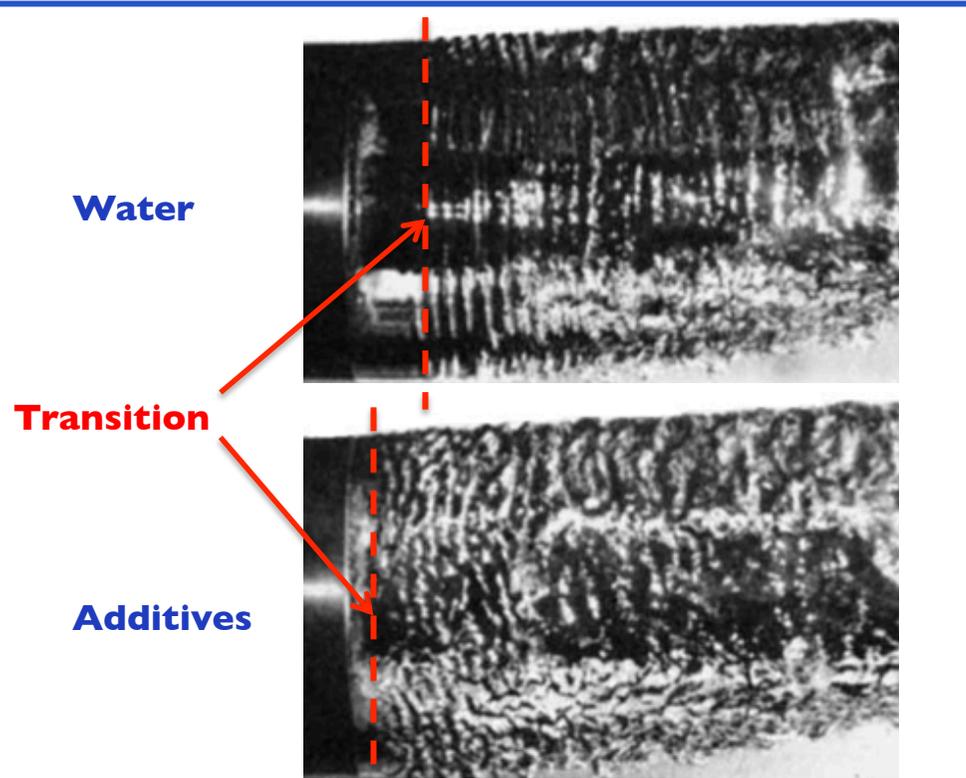


## Turbulent drag reduction



# Polymers and turbulence

## Early turbulence

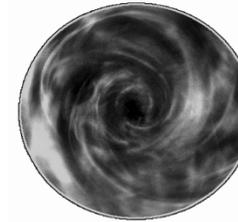


Transition to turbulence around an ogival head with ventilated cavity (Hoyt, 1977)

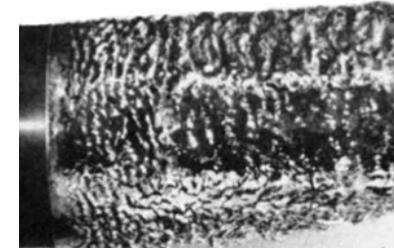
- Transition to turbulence promoted by polymers

• Biofluids

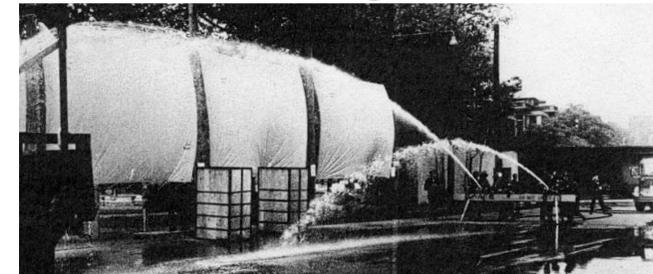
## Elastic turbulence



## Early turbulence



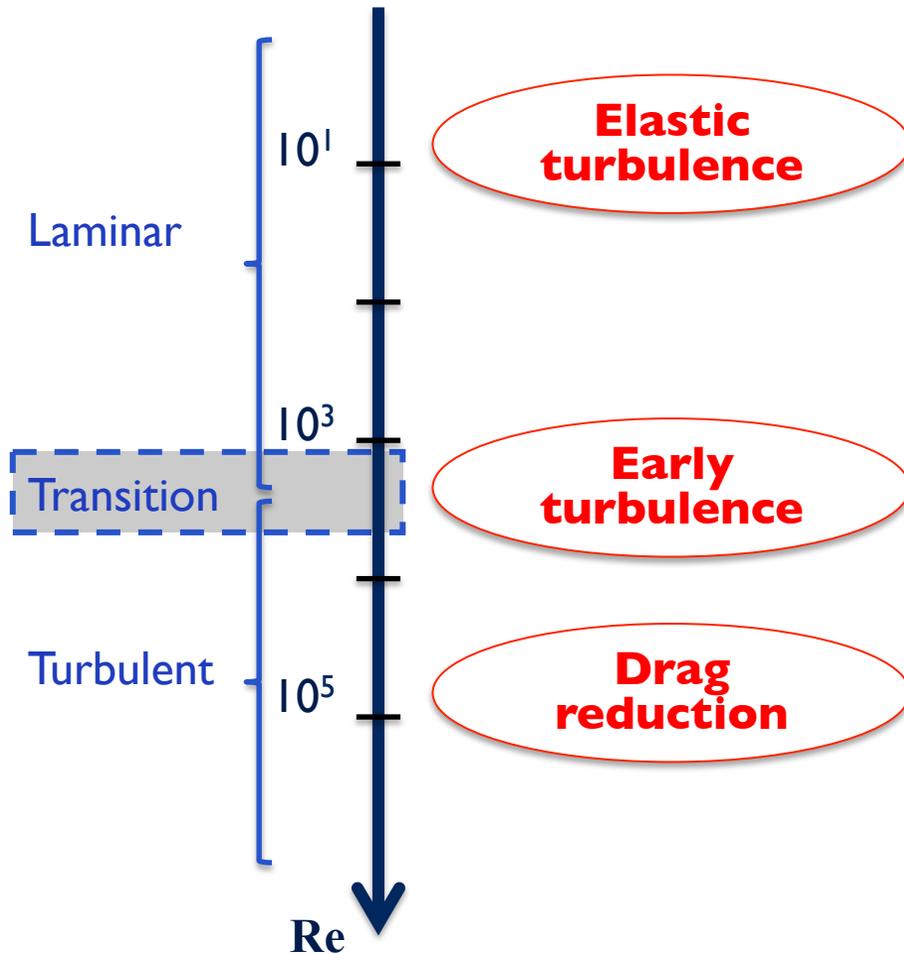
## Turbulent drag reduction



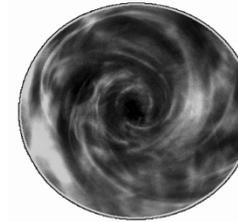
# Polymers and turbulence

## Newtonian

## Viscoelastic



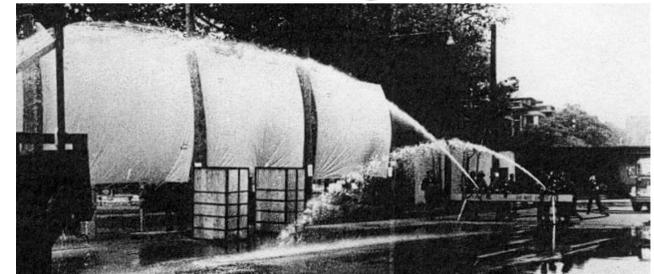
## Elastic turbulence



## Early turbulence



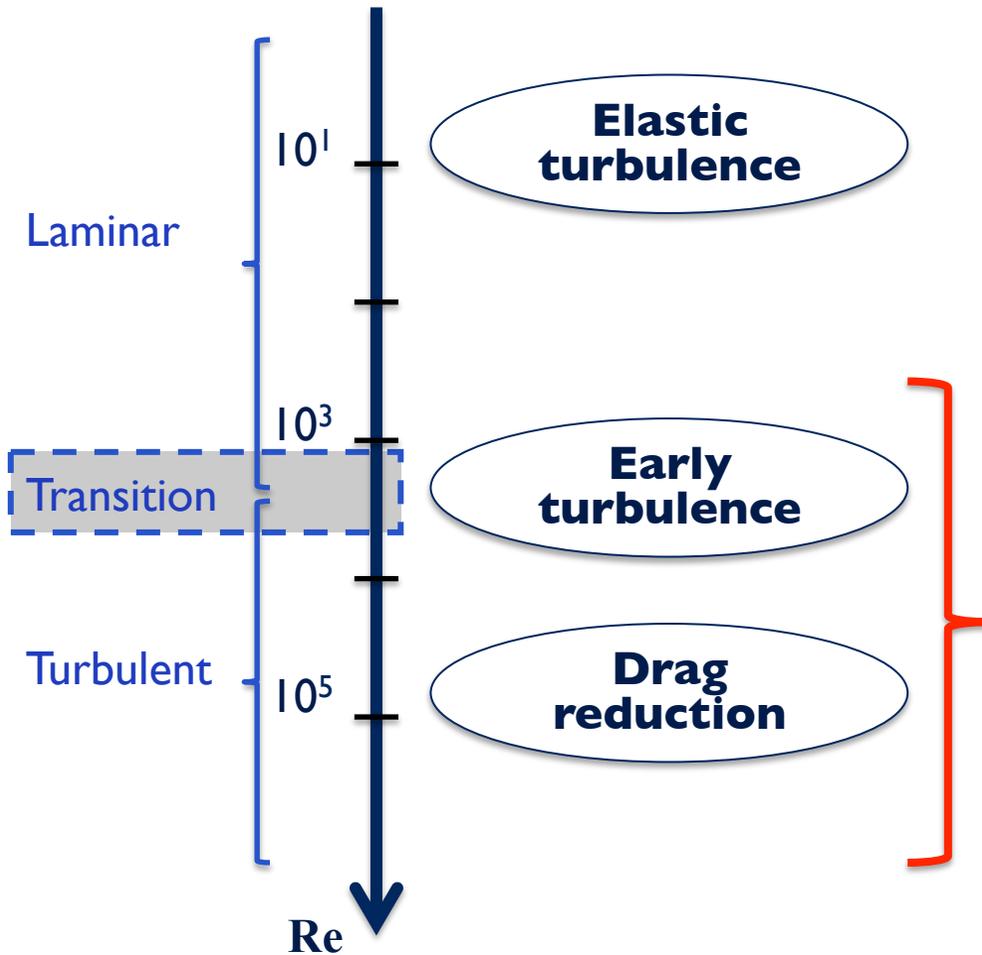
## Turbulent drag reduction



# Polymers and turbulence

Newtonian

Viscoelastic



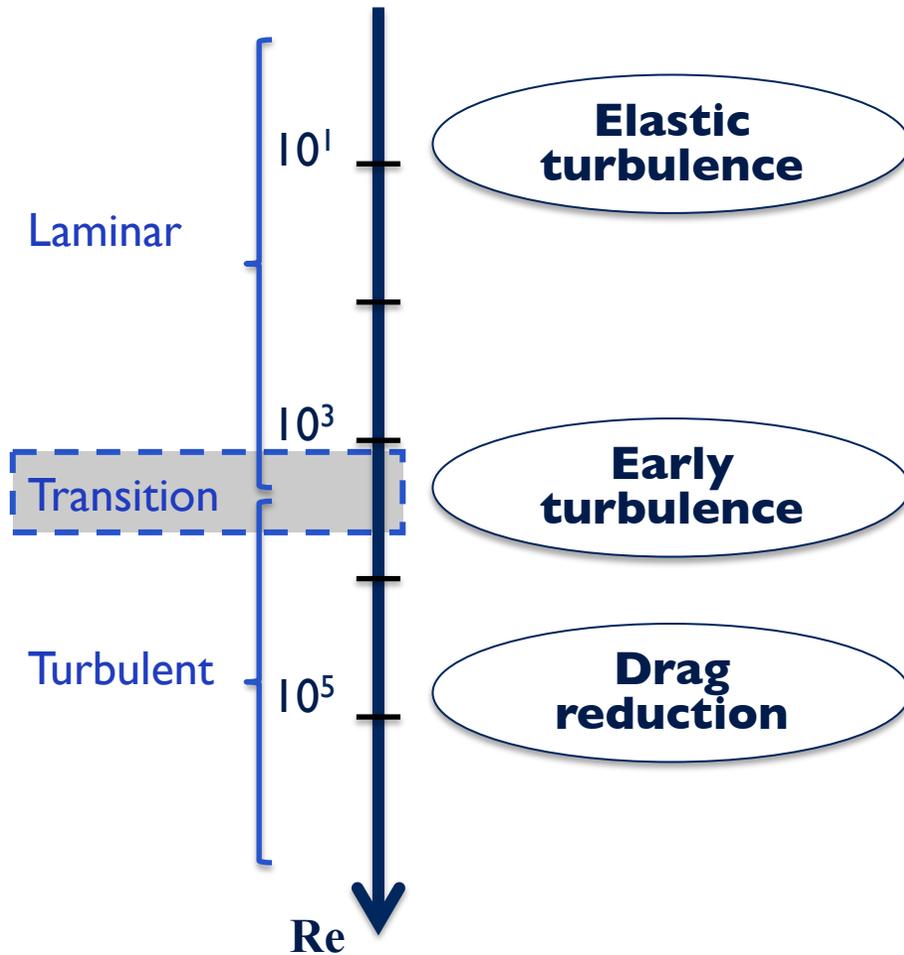
## Elasto-Inertial Turbulence (EIT)

- State of small-scale turbulence
- Contributions from both **elastic** and **inertial** instabilities
- Observed over a wide range of Reynolds numbers
- Possibly state characterizing MDR

# Polymers and turbulence

## Newtonian

## Viscoelastic



## Key questions

- Is drag reduction
  - a viscous and large-scale effect (Lumley)
  - an elastic and small-scale effect (de Gennes)
- What is the nature of EIT?
  - Relative contributions of elastic and inertial instabilities?
  - Characteristics of MDR?
  - Dynamical interactions between flow and polymers?

# Viscoelastic NSE – FENE-P model

**Continuity**  $\nabla \cdot \mathbf{u} = 0$

**Momentum** 
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{\beta}{\text{Re}} \nabla^2 \mathbf{u} + \frac{1-\beta}{\text{Re}} \nabla \cdot \mathbf{T} + \frac{dP}{dx} \mathbf{e}_x$$

**Polymer stress** 
$$\mathbf{T} = \frac{1}{\text{Wi}} \left( \frac{\mathbf{C}}{1 - \text{tr} \mathbf{C} / L^2} - \mathbf{I} \right)$$

**Conformation tensor** 
$$\frac{\partial \mathbf{C}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{C} = \nabla \mathbf{u} \cdot \mathbf{C} + \mathbf{C} \cdot \nabla \mathbf{u}^T - \mathbf{T}$$

$\beta$  Ratio of solvent viscosity to zero-shear viscosity of solution

$L$  Maximum polymer extension

$\text{Re}$  Reynolds number

$\text{Wi}$  Weissenberg number

$$\left. \begin{array}{l} \text{Re} \\ \text{Wi} \end{array} \right\} E = \frac{\text{Wi}}{\text{Re}} \quad \text{Elasticity}$$

# Numerical approach

## Space

- Structured grid
- 2<sup>nd</sup> order FD for velocity
- Non-dissipative 4<sup>th</sup> order compact scheme for polymer stress
- Compact upwind scheme for advection terms of conformation tensor

## Time

- Semi-implicit fractional step
- 2<sup>nd</sup> order Crank-Nicolson/3<sup>rd</sup> order Runge-Kutta
- Implicit scheme for trace of  $\mathbf{C}$  to ensure bounded trace

## Artificial dissipation

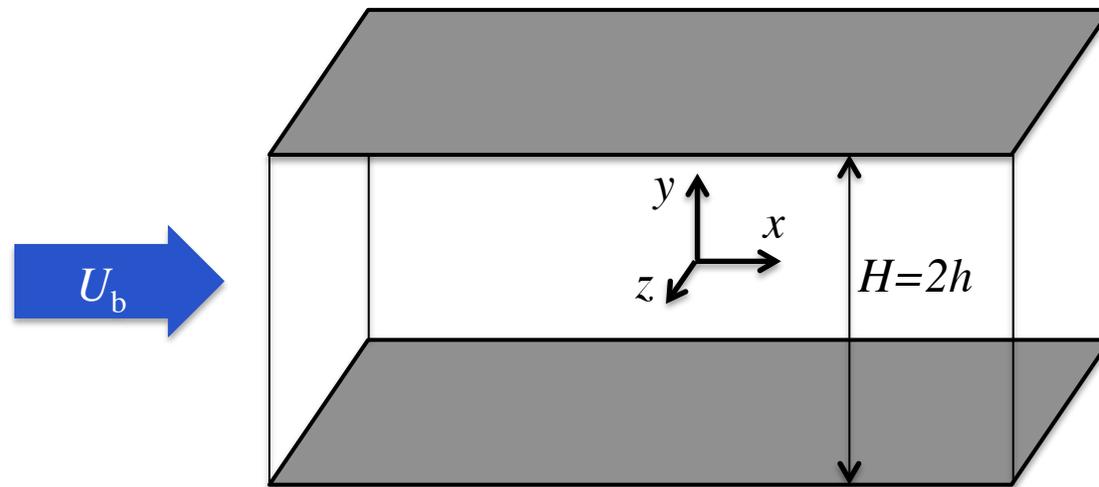
- Local artificial dissipation (LAD)
- Only used when determinant of tensor  $\mathbf{C}$  becomes negative

- Important to rely on accurate numerical method
- Global dissipation ( $Sc_{\text{eff}} \sim 1$ ) damps all small scales
- Capturing small polymer scales is critical to represent the correct physics

Min *et al.* (2001), Vaithianathan & Collins (2003), Dubief *et al.* (2005), Dallas *et al.* (2010)

# Configuration

## Periodic channel flow



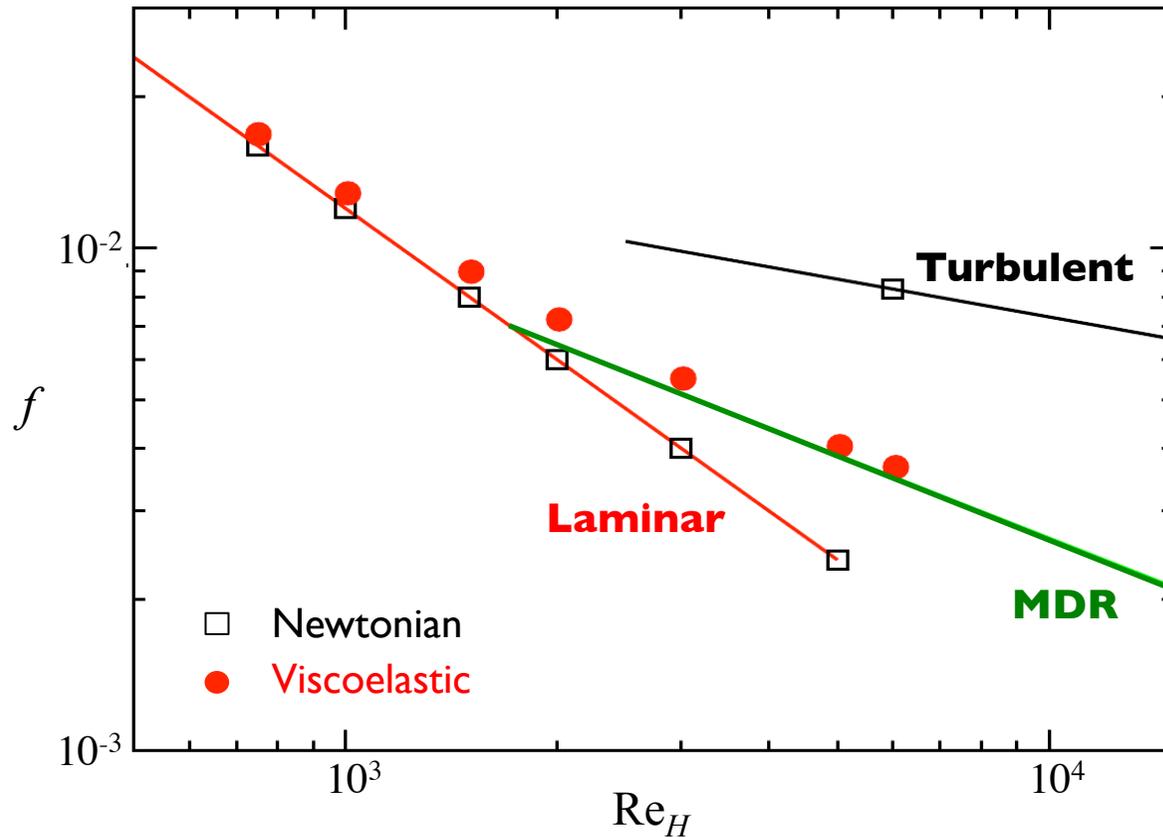
- Mean pressure gradient in  $x$
- Periodic in  $x$  and  $z$
- Wall (no-slip) at  $y=\pm h$
- Size:  $10h \times 2h \times 5h$
- Grid:  $256 \times 151 \times 256$

	$Re = \frac{U_b H}{\nu}$	$Wi = \frac{\lambda U_b}{h}$	$Wi^+ = \lambda \dot{\gamma}$	$\beta$	$L$	$h^+$	$\Delta x^+$	DM
▶	1000	8	24			40	1.5	+7.0%
	1000	60	180			40	1.4	+3.5%
▶	6000	8	96	0.9	200	130	5.0	-56%
	6000	60	720			120	4.6	-61%

# Transitional viscoelastic flows

## Channel flow simulations

Friction factor

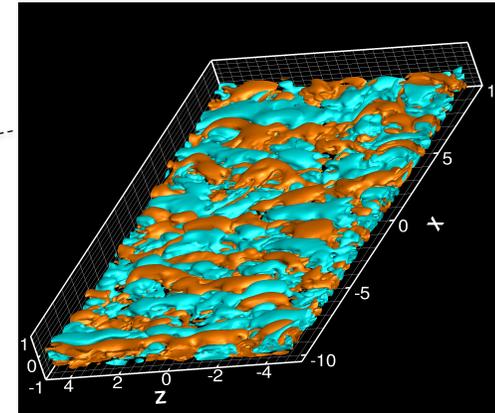
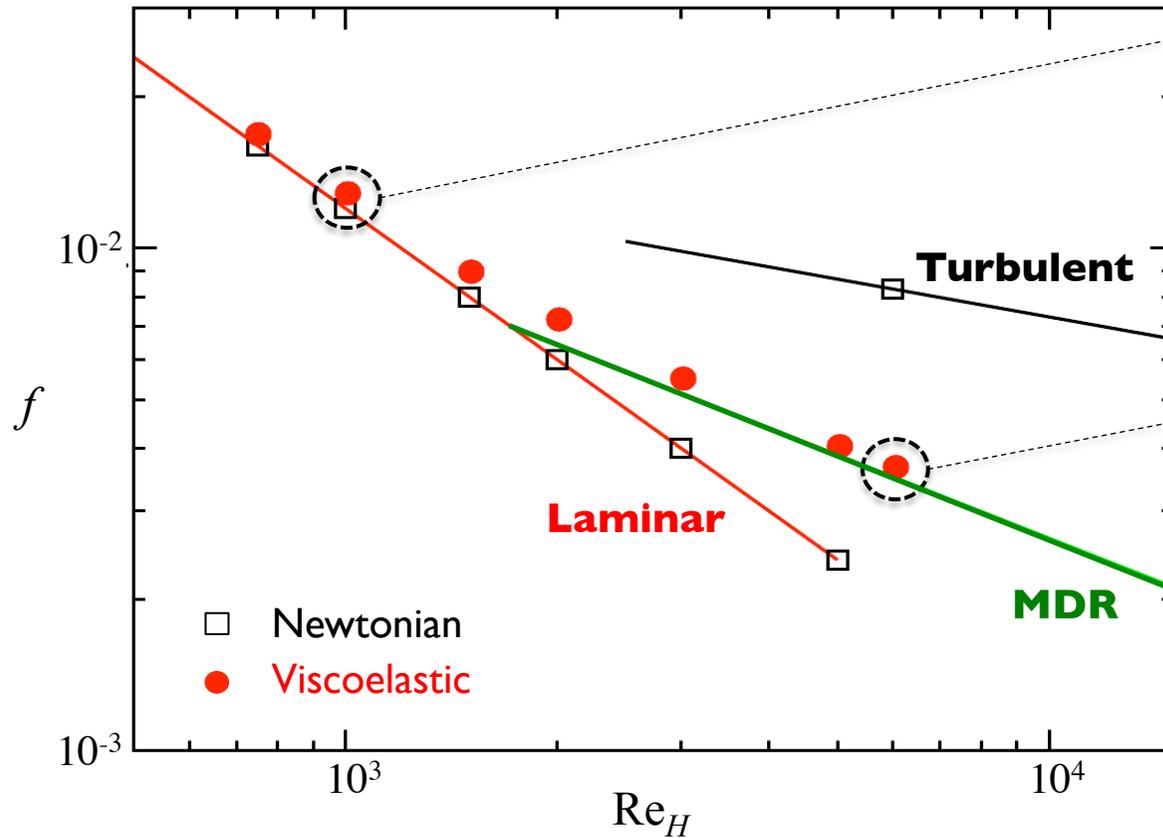


- Departure from laminar state at  $Re \sim 800$
- Smooth transition from laminar to MDR state
- Flow dynamics controlled by elastic and inertial instabilities

# Transitional viscoelastic flows

## Channel flow simulations

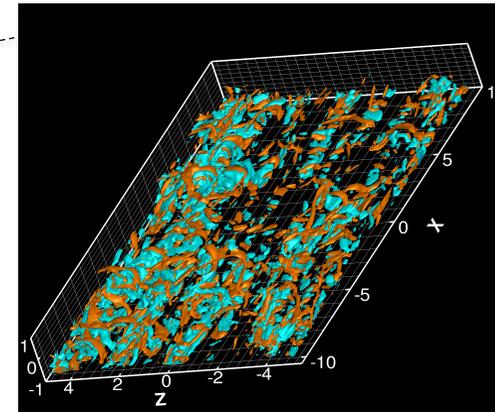
Friction factor



Re=1000, Wi+=24

- Not laminar
- Elastic contributions

Isosurface of  $Q_a$  invariant



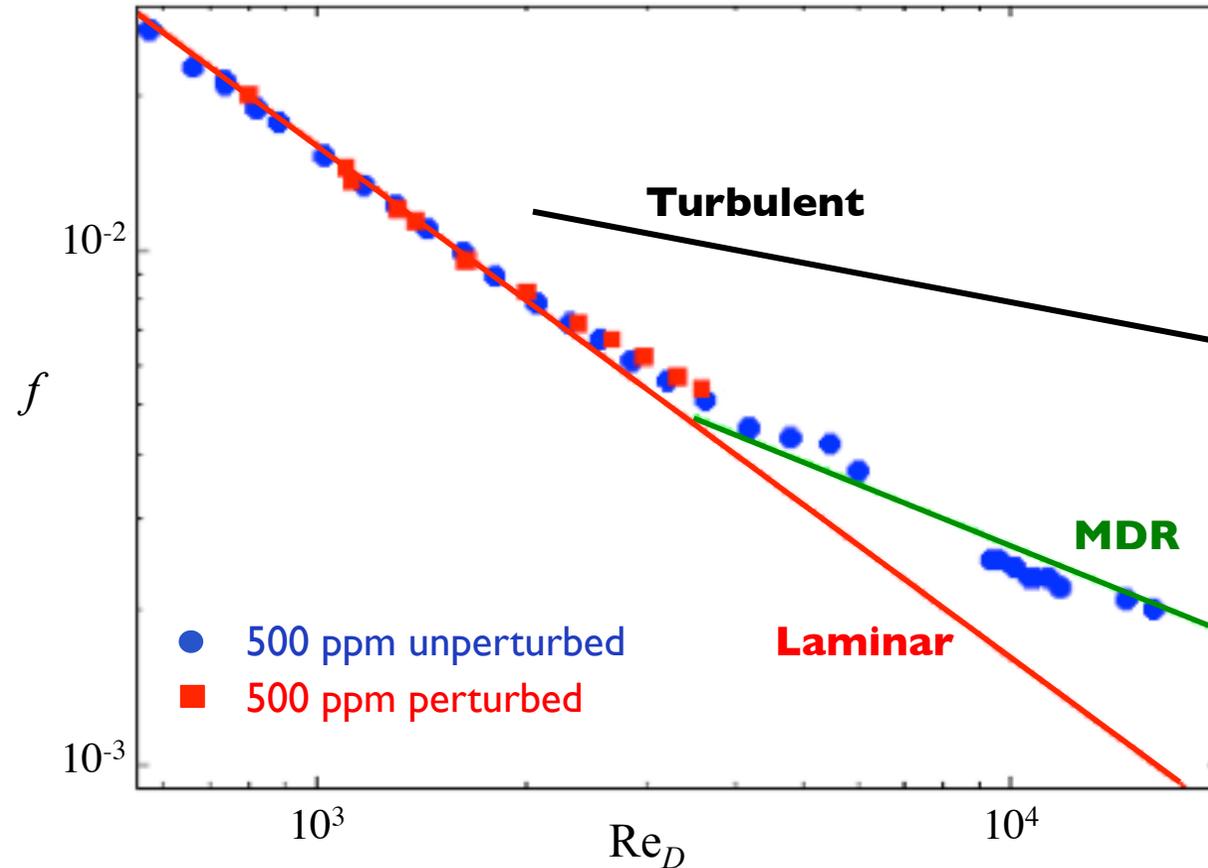
Re=6000, Wi+=96

- Inertial & elastic contributions
- Turbulent?
- New state?

# Transitional viscoelastic flows

## Pipe flow experiment with PAAm solution

Friction factor

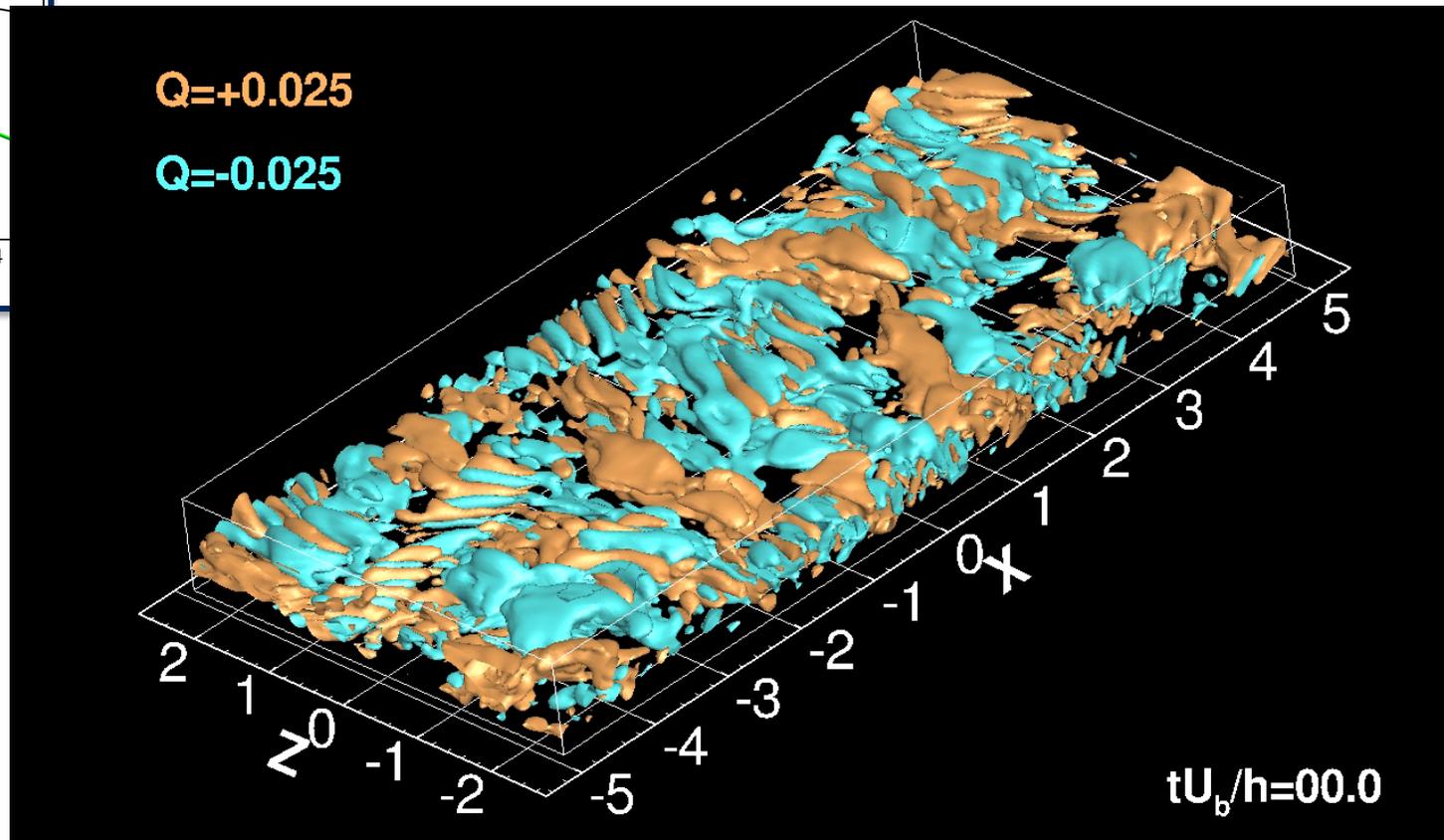
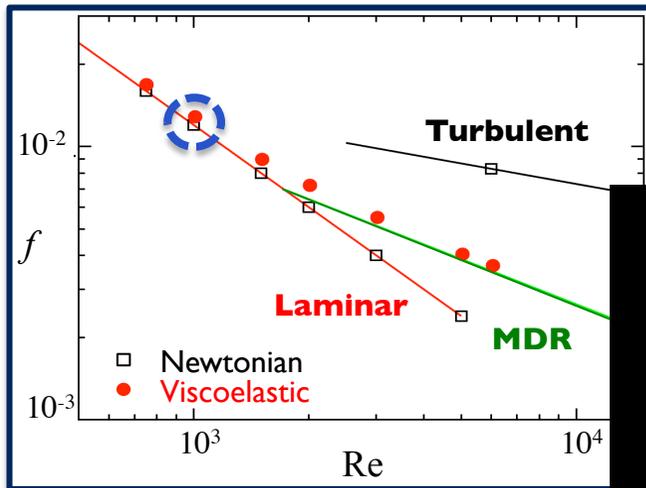


Results of numerical simulations are confirmed by experimental measurements

Samanta et al., PNAS 110(26), 2013

# Qualitative flow behavior

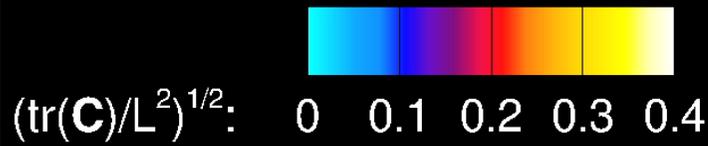
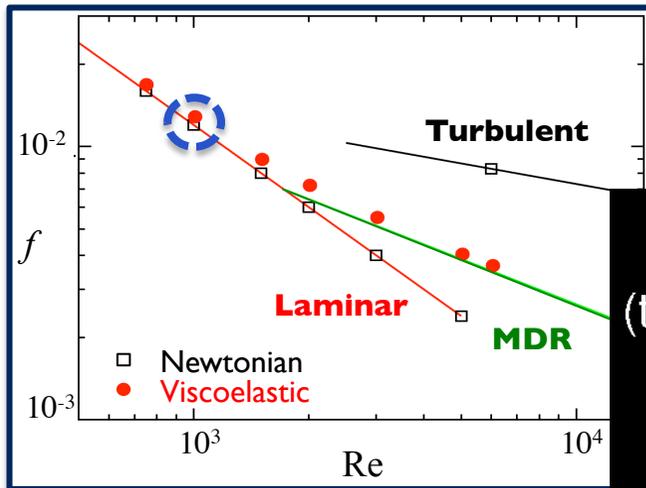
Re = 1000  
Wi<sup>+</sup> = 24



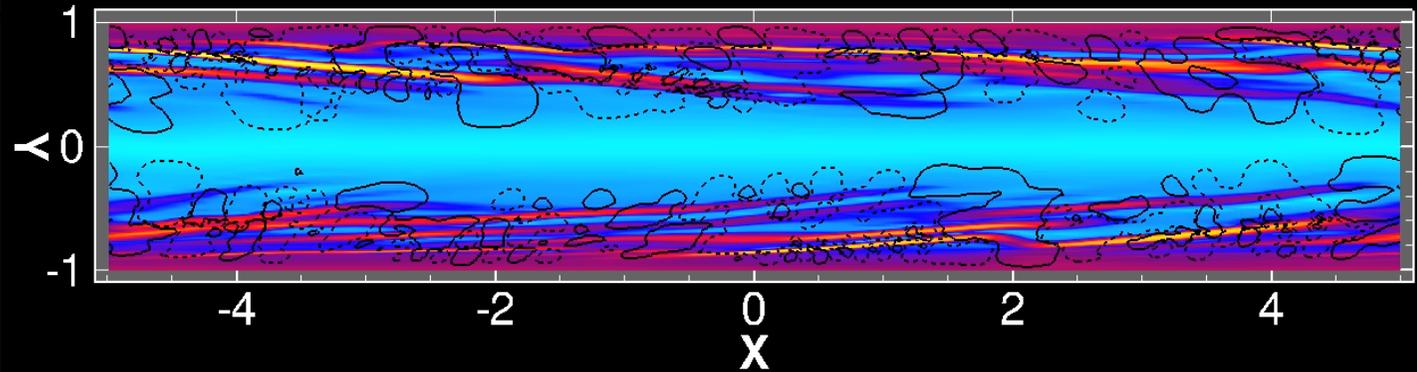
Second invariant of the velocity gradient tensor:  $Q_a = \frac{1}{2} (\Omega^2 - S^2)$

# Qualitative flow behavior

**Re = 1000**  
**Wi<sup>+</sup> = 24**



○ **Q=+0.01**  
○ **Q=-0.01**

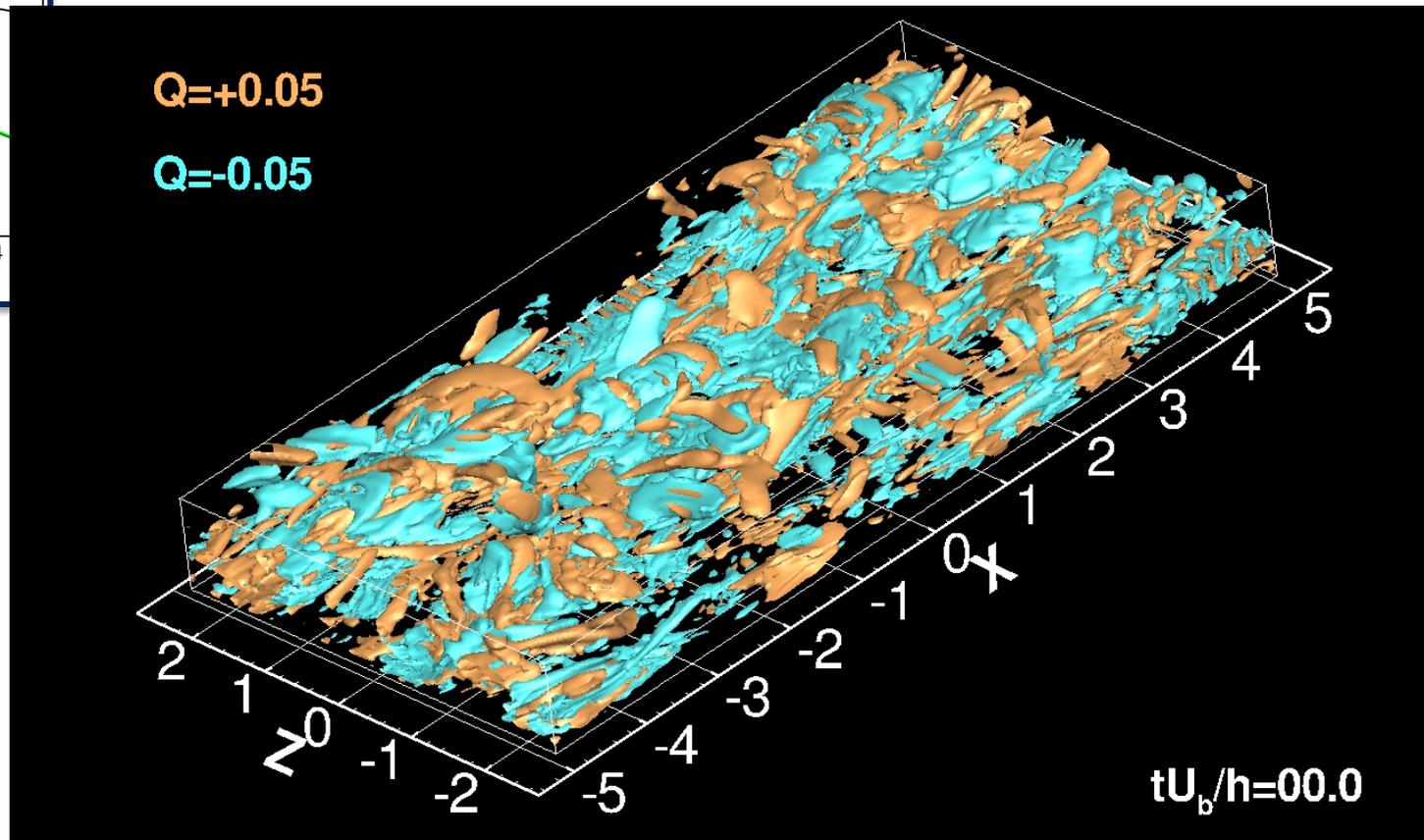
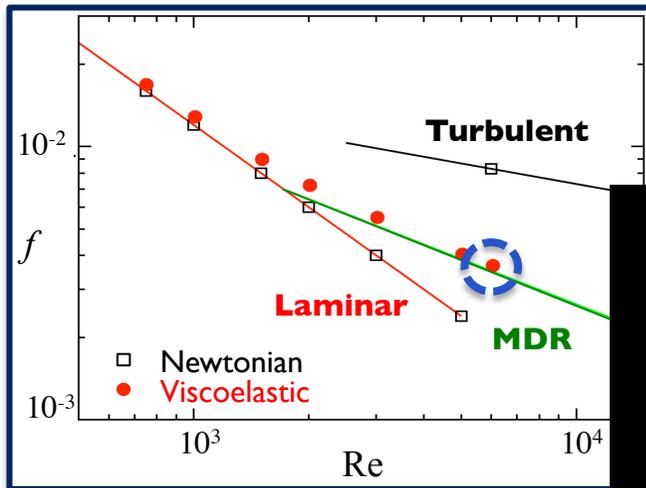


Polymer extension  $(C_{ii} / L^2)^{1/2}$

**tU<sub>b</sub>/h=00.0**

# Qualitative flow behavior

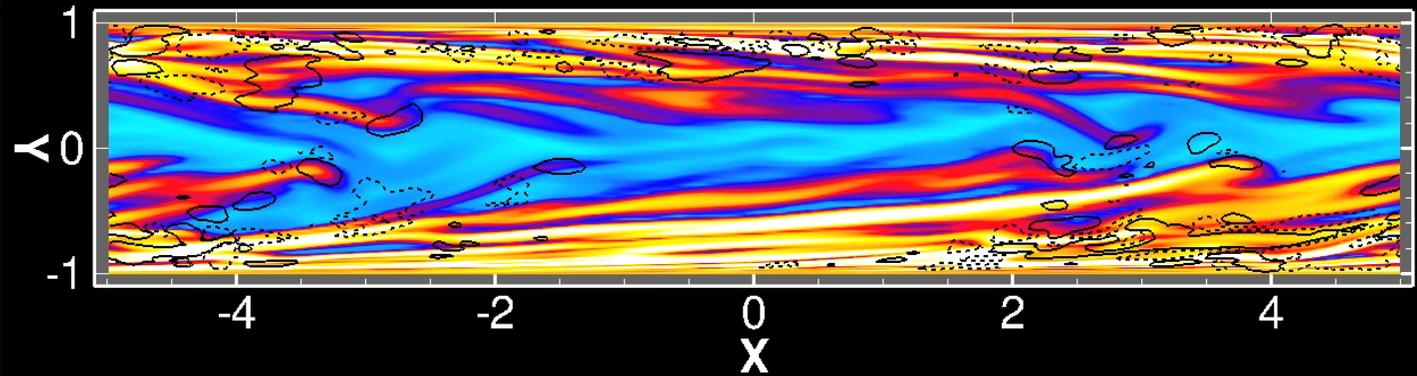
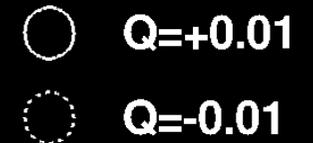
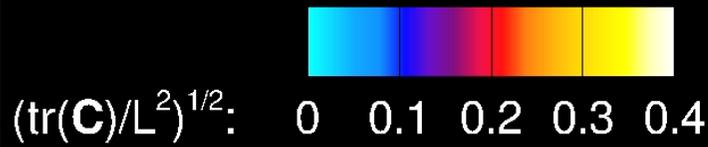
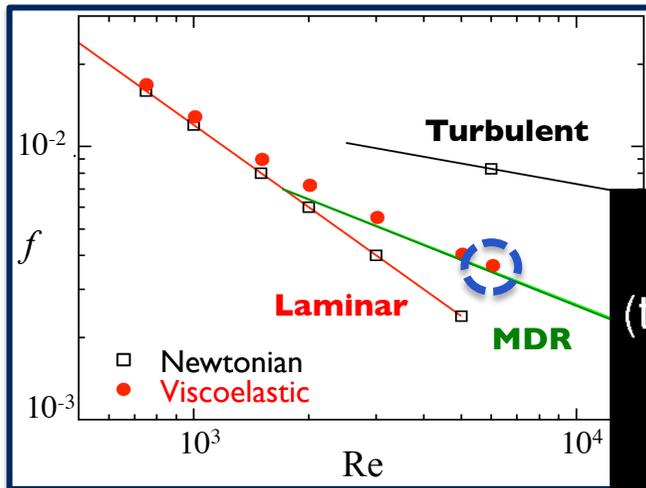
Re = 6000  
Wi<sup>+</sup> = 96



Second invariant of the velocity gradient tensor:  $Q_a = \frac{1}{2} (\Omega^2 - S^2)$

# Qualitative flow behavior

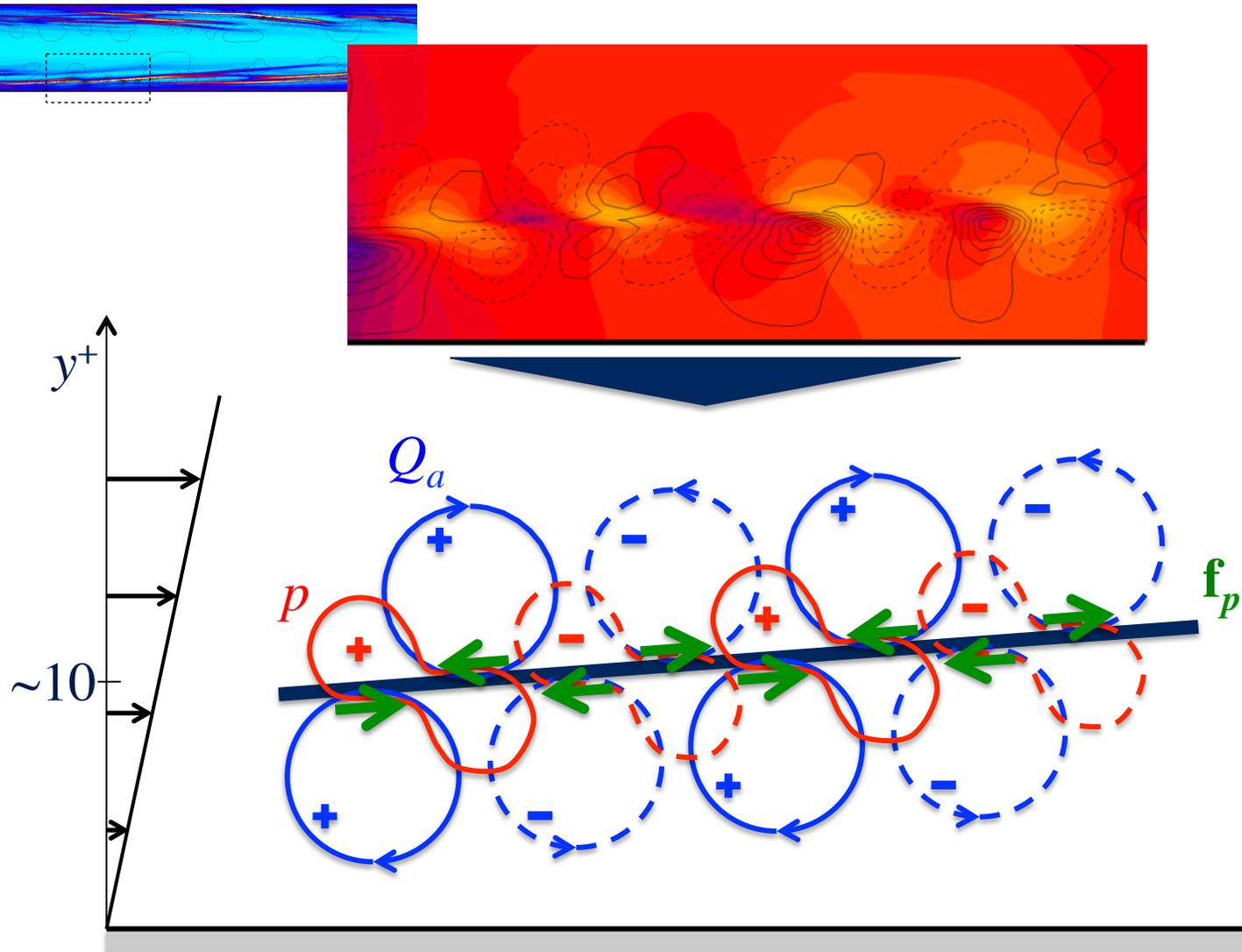
**Re = 6000**  
**Wi<sup>+</sup> = 96**



Polymer extension  $(C_{ii}/L^2)^{1/2}$

**tU<sub>b</sub>/h=00.0**

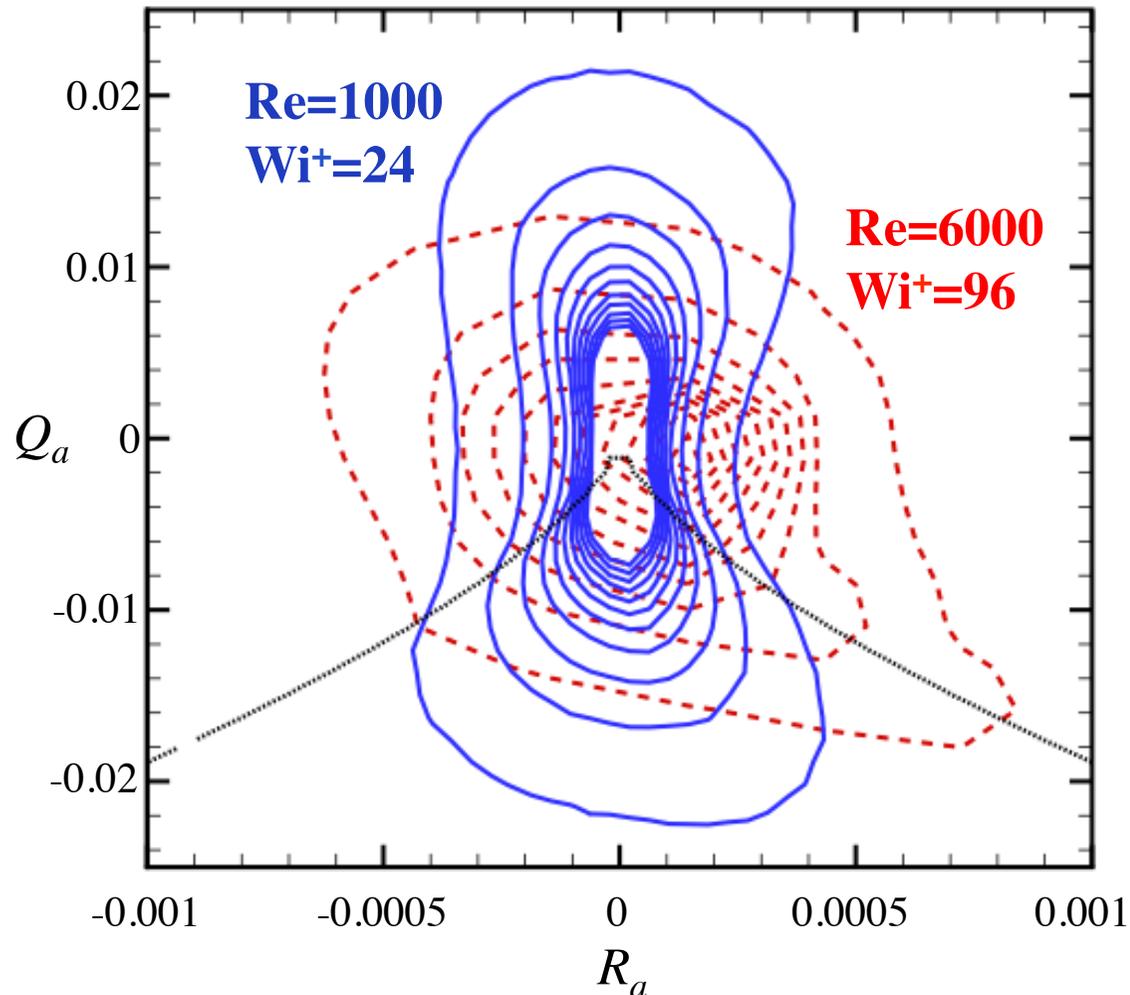
# Typical structures



- Train of cylindrical  $Q_a$  structures of alternating sign
- On each side of sheet
- Associated with polymeric part of pressure
- Correlated with polymer body force  $\mathbf{f}_p$

# Flow topology

## EIT flow: JPDF



- Change from shear flow ( $R_a=Q_a=0$ ) to mixed flow
- At low Re, symmetric distribution around 2D flow ( $R_a=0$ )
- At higher Re, “teardrop” shape similar to Newtonian turbulence

# Energy transfers

## Turbulent kinetic energy budget

$$\int_V P dV - \int_V \varepsilon dV - \int_V \Pi_e dV = 0$$

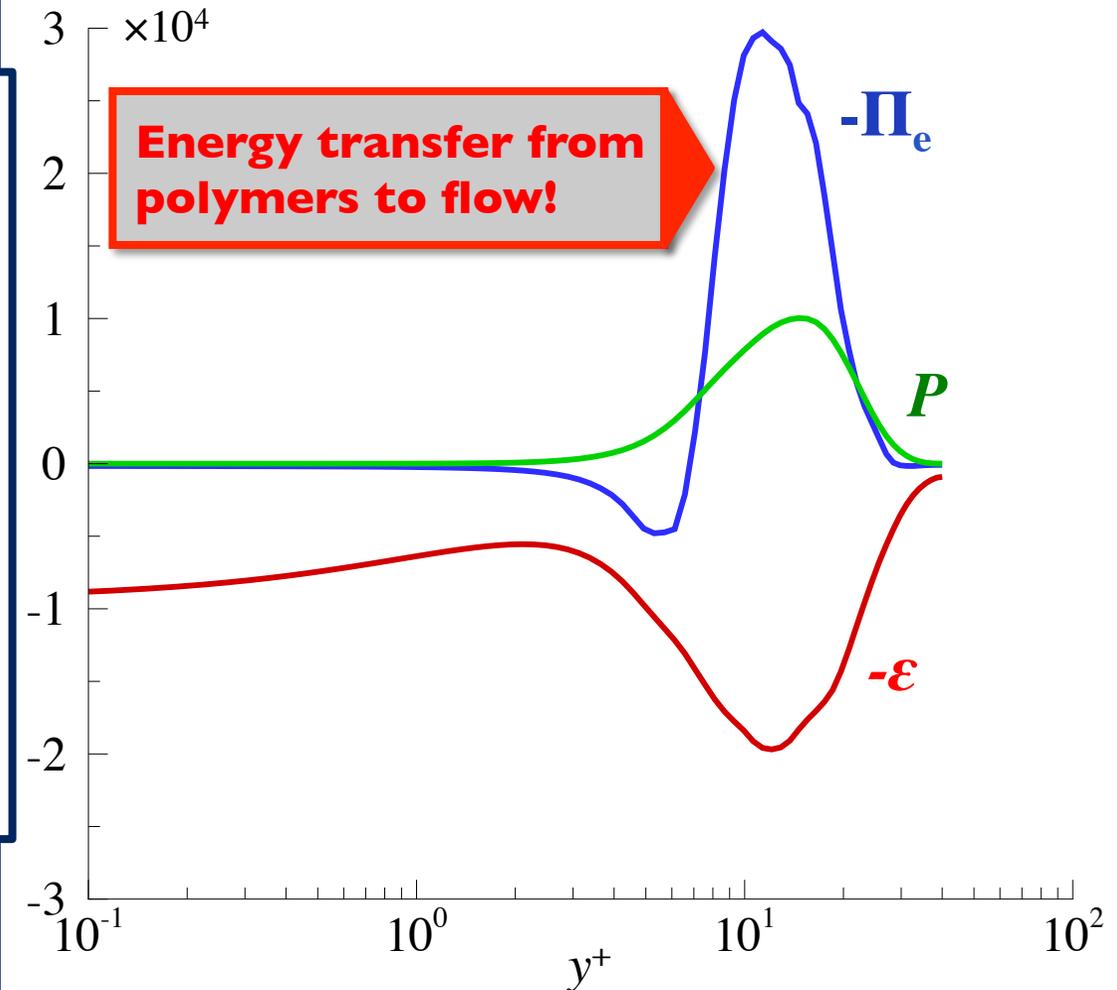
↓  
Production

↓  
Dissipation

↓  
Transfer between  
elastic energy and  
turbulent kinetic  
energy

## Transfers of turbulent kinetic energy

Re=1000, Wi+=24

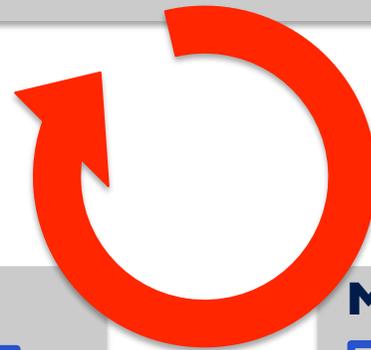


# Our current understanding

## Hyperbolic transport equation

$$\partial_t \mathbf{C} + (\mathbf{u} \cdot \nabla) \mathbf{C} = \dots$$

- Formation of very thin sheets
- Trains of cylindrical structures



**Self-sustained**

## Pressure Poisson equation

$$\nabla^2 p = 2Q_a - \frac{1-\beta}{\text{Re}} \nabla \cdot (\nabla \cdot \mathbf{T})$$

- Elliptical pressure redistribution of energy
- Excitation of extensional sheet flow

## Mixed extensional-shear flow

$$\dots = \mathbf{C}(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T \mathbf{C} - \mathbf{T}$$

- Increase of extensional viscosity (anisotropic)
- Anisotropic polymer body force

# Conclusion and future work

## Key take-away messages

- EIT is a new state of small-scale turbulence driven by both elastic and inertial instabilities
- EIT could characterize MDR regime
- EIT explains seemingly contradictory phenomena in viscoelastic turbulence
- EIT provides support to de Gennes' theory

## Next steps

- Further characterize EIT
  - Two-dimensionality
  - Energy transfers and backscatter
- Understand the exact mechanisms during transition process

Dubief, Terrapon & Soria, "On the mechanism of Elasto-inertial turbulence", *Phys. Fluids* 2013

Samanta *et al.*, "Elasto-inertial turbulence", *PNAS* **110**(26), 2013