Understanding variable importances in forests of randomized trees

Gilles Louppe, Louis Wehenkel, Antonio Sutera and Pierre Geurts [Sun88]

Random Forests are a well-tested, efficient and versatile tool. Yet, they are still not fully theoretically understood.
Variable importances were first proposed as a heuristic to assess the influence of input variables.

\[ \text{Imp}(X_m) = \frac{1}{N_T} \sum_T \sum_{t \in T : v(t) = X_m} p(t) \Delta i(t) \]

In the case of totally randomized trees, variable importances actually show sound and desirable theoretical properties.
Variable importances provide a three-level decomposition of the information jointly provided by all the input variables about the output, accounting for all interaction terms in a fair and exhaustive way.

**Thm. 1:**

\[
\text{Imp}(X_m) = \sum_{k=0}^{p-1} \frac{1}{C_p^k} \frac{1}{p-k} \sum_{B \in \mathcal{P}_k(V^{-m})} I(X_m; Y|B)
\]

i) Decomposition in terms of the MDI importance of each input variable

ii) Decomposition along the degrees \(k\) of interaction with the other variables

iii) Decomposition along all interaction terms \(B\) of a given degree \(k\)

**Thm. 2:**

\[
\sum_{m=1}^{p} \text{Imp}(X_m) = I(X_1, \ldots, X_p; Y)
\]

i) Decomposition in terms of the MDI importance of each input variable

Information jointly provided by all input variables about the output
Variable importances depend only on the relevant variables.

**Thm. 3** : A variable is irrelevant if and only if its importance is 0.

**Thm. 5** : The importance of a relevant variable is insensitive to the addition or the removal of irrelevant variables.