Journal of Empirical Finance 23 (2013) 191–205

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Comoment risk and stock returns☆

M. Lambert [a,1], G. Hübner [a,b,c,*]

[a] HEC Management School, University of Liège, rue Louvrex, 14, B-4000 Liège, Belgium
[b] Faculty of Economics and Business Administration, Maastricht University, The Netherlands
[c] Gambit Financial Solutions, Belgium

ARTICLE INFO

Article history:
Received 27 June 2012
Received in revised form 26 June 2013
Accepted 2 July 2013
Available online 10 July 2013

JEL classification:
G11
G12

Keywords:
Comoment
Hedge portfolio
Equity risk
Fama–MacBeth test

ABSTRACT

We estimate investable comoment equity risk premiums for the US markets. The stock's contribution to the asymmetry and the fat tails of the market portfolio's payoff are priced into a coskewness premium and a cokurtosis premium. We construct zero-investment strategies that are long and short in coskewness and cokurtosis equity risks; we infer from the spread the returns attached to a unit exposure to US equity coskewness and cokurtosis. The coskewness and cokurtosis premiums present positive monthly average returns of 0.27% and 0.14% from January 1959 to December 2011. Comoment risks appear to be significantly priced within the US stock market and display significant explanatory power regarding the US size and book-to-market effects. The premiums do not subsume, but rather complement the empirical capital asset pricing model. Our analysis relies on data collected from CRSP (Chicago Research Center for Security Prices) over December 1955 to December 2011. To our knowledge, the paper is the first to propose investable higher-moment risk factors over such an extensive time period.

1. Introduction

The early asset pricing literature, starting with the Capital Asset Pricing Model (hereafter CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966), posits a linear market reward for any cross-sectional variation of returns. To induce a linear single factor relationship, the CAPM imposes strong assumptions regarding either the stock return distribution or the structure of investors' preferences (Samuelson, 1970). Such assumptions imply a spherical unconditional distribution of returns, or imply that the representative investor displays aversion towards only the second moment of the returns distribution.

Evidence of non-normality in the asset unconditional distributions and of a complex risk profile of the representative investor challenge the assumptions underlying the single factor model (e.g., Badrinath and Chatterjee, 1988; Kahneman and Tversky, 1979; Kimball, 1990, 1993; Peiro, 1999). An examination of investors' preferences in risky situations reveals that both the probability of experiencing a loss and the potential maximum loss amount are likely to influence their choice. This asymmetry in risk aversion has been related with preference directions for moments of an investment return distribution (e.g., Arditti, 1967, 1969; Benishay, 1992; Jean, 1971; Levy, 1969; Rubinstein, 1973; Scott and Horvath, 1980). As a result, the literature has progressively focused on the significance of adding distributional risk factors to asset pricing models like skewness (3rd moment of the return distribution capturing the downside risk probability) and kurtosis (4th moment of the return distribution capturing the extreme risk probability) factors. Especially, coskewness and cokurtosis parameters (measuring the contribution of an asset to

* Corresponding author at: HEC Management School, University of Liège, rue Louvrex, 14, B-4000 Liège, Belgium. Tel.: +32 42327428; fax: +32 4 2327240.
E-mail addresses: marie.lambert@ulg.ac.be (M. Lambert), G.Hubner@ulg.ac.be (G. Hübner).
[1] Tel.: +32 4 232 74 32.

0927-5398/$ – see front matter © 2013 The Authors. Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jempfin.2013.07.001

the skewness and kurtosis of a diversified portfolio) are informative about how the fund will behave in extreme market conditions.

The literature records many attempts to include higher-moment effects into asset pricing models. Most of them represent this effect through a simple nonlinear (polynomial) functional relation between the returns of each asset and the market index. A common approach is to use the square and the cube of the market index as proxies for (respectively) skewness, and kurtosis dependence. This is done, for instance, in the contexts of hedge funds (Chen and Passow, 2003; Spurgin, 2001), of international markets (Doan et al. (forthcoming) in Australia; Kostakis et al. (2012) in the U.K.), or of alternative model specifications including time-varying betas (Potì and Wang, 2010). Kat and Miffre (2006) question the relevance of these factors as they can be confused with the market timing measure of Treynor and Mazuy (1966). Besides, Ranaldo and Favre (2005) and Ding and Shawky (2007) also perform a four-moment model by regressing hedge fund returns on the levels of variance, skewness, and kurtosis observed in the market. The regression coefficients in both models correspond to the statistical measures of each asset's covariance, coskewness and cokurtosis with the market returns. Finally, using the approach of Bakshi et al. (2003), Conrad et al. (2013) and Hübner et al. (2013) derive factors from the higher moments of the risk-neutral asset return distributions implied in option prices.

Our objective here is different, because we do not simply supply estimates of the global levels of coskewness or cokurtosis in the US market. Rather, we aim to factor and isolate these fundamental risks into portfolio returns. Because they are meant to capture the global effect of each comoment, the previously defined factors may display colinearities with other risk factors which limit the power of the four-moment model (Hwang and Satchell, 1999). Our work thus differs from the research mentioned above by defining investable and pure higher-order moment factors. These factors could be interpreted as the premiums for a unit exposure to the risk, while the loadings in the four-moment model would be proxies for the exposure/allocation to this risk. The formation of this factor requires the specification of three particular portfolios: one that spans the marginal returns associated with a unit exposure to covariance, another one to coskewness, and a third one to cokurtosis. The first portfolio must express a perfect correlation with the market portfolio but zero with higher-order comoments. The returns on the second and third portfolios must have a unitary conditional covariance with, respectively, the squared and the cubed market returns and zero values for other comoments (see Jurczenko and Maillet, 2002; Samuelson, 1970). Unlike simple portfolios that merely reflect the total market impact of a higher moment on the return-generating process of securities, a portfolio identifying its marginal market impact is likely to better contribute to a proper multi-factor specification by not being prone to contamination effects.

In this paper, we provide a rigorous method to construct moment-based portfolios displaying these desired properties. In order to get market-based risk premiums, we adopt a framework similar to Fama and French's (1993) portfolio construction method. We work under an extension of the Capital Asset Pricing Model. In other words, our analysis is set up in the context of fully-diversified investors.[2] Therefore, because diversification erodes idiosyncratic skewness, our empirical analysis focuses on an asset's contribution to the skewness of a diversified portfolio, which is referred to as coskewness or systematic skewness. We follow Kraus and Litzenberger (1976) and Harvey and Siddique (2000) and consider the coskewness of a security with the market portfolio as a pricing factor. Similarly, we consider the cokurtosis of a security with the market portfolio as a second source of alternative risks. Assets that decrease (resp. increase) the portfolio's skewness (resp. kurtosis) should command higher expected returns in order to compensate for the undesirable sources of risks and vice versa. The combination of higher moment risk factors with the traditional size, value and momentum premiums reveals their complementary character.

Because our estimates of the higher-moment premiums are issued from a rigorous approach that gets rid of correlated variables when pricing one systematic moment, we expect our investable higher-moment risk premiums to represent sound benchmarks for systematic variance, skewness, and kurtosis risks in the US equity markets. Our analysis relies on data collected from CRSP (Chicago Research Center for Security Prices) over December 1955 to December 2011. To our knowledge, this is the first paper to propose and test higher-moment risk factors over such an extensive time period.

The paper is organized as follows. Section 2 presents the review of literature on the subject. In Section 3, we show how to construct the set of risk factors. The fourth section introduces the testing procedures. Section 5 presents and discusses the results, while a set of robustness checks are conducted and presented in Section 6. The seventh section concludes.

2. Literature review

Rubinstein (1973) and Kraus and Litzenberger (1976) extend Modern Portfolio Theory to the inclusion of nonlinear measures of risk. Their specific works relate the expected return of an asset to the weighted sum of the unconditional covariance and coskewness risk aversions by the asset systematic risks. Harvey and Siddique (2000) and Barone-Adesi et al. (2004), among others, demonstrate the likely misspecification of omitting the coskewness effect on the cross-sectional variation of asset returns. Subsequent work by Fang and Lai (1997), Satchell et al. (2000) and Dittmar (2002) for the most referenced studies suggests improving the significance of the market model by pricing more than the second and third asset covariations with the market portfolio. Under the hypothesis that investors display decreasing absolute prudence, they show that a stock's contribution to the market unconditional kurtosis (i.e. its cokurtosis or systematic kurtosis) could also be relevant for the evaluation of its relative

[2] Recent studies have shown that idiosyncratic skewness and not just coskewness have also strong asset pricing implications (Mitton and Vorkink, 2007). Because mean-variance diversification erodes skewness exposure, investors could decide to remain undiversified in order to capture a positive exposure to skewness. Therefore, for a less-diversified investor, lower returns are expected for stocks with high idiosyncratic skewness (Barberis and Huang, 2008; Boyer et al., 2010; Brunnermeier and Parker, 2005; Brunnermeier et al., 2007).

attractiveness. These empirical findings are justified from a utility-based perspective. Just as a covariance premium must reward the marginal contribution to the variance of the market portfolio returns, an asset return must be compensated if it contributes to the decrease in the market portfolio skewness (Moreno and Rodriguez, 2009) or to the increase in its kurtosis (Fang and Lai, 1997).

Some efforts have been deployed in the estimation of moment-based investable risk factors. The first attempts to estimate such factors rely on one-dimensional portfolios. Two portfolios are constructed for each dimension to be priced: one portfolio with the 30% highest comoments and one portfolio with the 30% lowest comoments. The factor is constructed as the return differential between the high and the low scoring portfolios or inversely. Harvey and Siddique (2000) are the first authors who have constructed a coskewness factor in this fashion. Ajili (2005), Kole and Verbeek (2006), Moreno and Rodriguez (2009) and Kostakis et al. (2012) also factor coskewness and/or cokurtosis into returns by implementing a similar method. Such a technique does not however control for other moment-related fundamentals.

To overcome this drawback, Kat and Miffre (2006) construct two-dimensional mimicking portfolios for coskewness and cokurtosis in a way that is similar to the Fama and French (1993) technique for constructing empirical risk premiums. They rank stocks into two portfolios according to their covariance and allocate stocks to three portfolios according to their coskewness or their cokurtosis with the market portfolio. Six portfolios are formed at the intersection of the two rankings on respectively covariance and coskewness, or on covariance and cokurtosis. Each factor is then defined as the return differential between the average high and low scoring portfolios or inversely. However, due to moderate levels of correlation between risk fundamentals, performing two independent sorts could lead to unbalanced portfolios and sometimes even to empty ones.

To avoid such situations, Agarwal et al. (2008) perform a conditional (rather than independent) three-stage sort of higher-moment equity risks embedded in hedge funds into portfolios. They first sort funds in three portfolios according to covariance. Then, conditional on this first stage, they sort funds within each portfolio into three portfolios according to their level of coskewness, ending with a sort on the cokurtosis dimension. In this way, they control for the correlation among the rankings, but their method is sensitive to the ordering of the risk dimensions. As they use the cross-sections of hedge fund returns rather than the ones of the US equity stocks, their premiums translate the particular risk aversion of accredited investors who have a high risk profile. Besides, they are also exposed to the issue of the quality of and efficiency in hedge funds data.

All the above related premiums do not obey to the hypothesis that the expected return premiums should be a positive function of market beta and cokurtosis, and a negative function of coskewness. For instance, the cokurtosis factors of Kole and Verbeek (2006) and of Agarwal et al. (2008a,b) or the coskewness premium of Kat and Miffre (2006) is significantly negative over their respective period of analysis.

3. Construction of moment-related factors

This section describes the data we use and the methodology applied for constructing the second, third and fourth moment-related risk premiums. It presents some summary statistics about the estimated outputs.

3.1. Data

We collect all the NYSE, AMEX, and NASDAQ stocks available on CRSP (Center for Research in Security Prices) US stock database and for which the following information is available[3]: the official monthly closing price adjusted for subsequent capital actions, and the monthly market value. Monthly returns and market values[4] are then recorded for observations where the stock return does not exceed 100% and where market values are strictly positive. This enables us to avoid outliers that could result from errors in the data collection process.

Our sample covers 26,339 stocks for the period ranging from December 1955 to December 2011. The market risk premium inferred from this space corresponds to the value-weighted return on all US stocks minus the one-month Treasury Bill rate.

3.2. Methodology

We construct three hedge portfolios that mimic the required rewards implied by a high covariance, a high cokurtosis, and a low coskewness with the market portfolio.

Estimations of statistical comoments are consistent with the approach of Kraus and Litzenberger (1976), Harvey and Siddique (2000), Dittmar (2002), and Barone-Adesi et al. (2004). We rely on an extension of the traditional market model of Sharpe (1964) into a nonlinear return-generating process including the square and cube of the market returns as additional factors, i.e.:

$$R_{i,t}-r_{f,t} = c_{0,i} + c_{1,i}\left[R_{M,t}-r_{f,t}\right] + c_{2,i}\left[R_{M,t}-\overline{R}_{M,t}\right]^2 + c_{3,i}\left[R_{M,t}-\overline{R}_{M,t}\right]^3 + v_{i,t} \qquad (1)$$

where $R_{i,t}$, $r_{f,t}$ and $R_{M,t}$ are the monthly return on security i, on the one-month T-Bill and on the market portfolio, respectively; $\overline{R}_{M,t}$ is the average market return in the 36 preceding months. Proxies for beta, coskewness and cokurtosis coefficients correspond to

[3] Temporary data non-availability excludes the stock from the analysis at that time.

[4] We designate by market value at month t, the quoted share price multiplied by the number of ordinary shares of common stock outstanding at that moment.

the loadings on respectively the market premium ($c_{1,i}$), on the square of the market excess returns ($c_{2,i}$) and on their cube ($c_{3,i}$). These are estimated on a monthly basis, using a 36-month rolling window.

The moment-related premiums constitute backward-looking estimates of the returns attached to a unitary covariance, coskewness, or cokurtosis risk exposure. One drawback of using returns history for measuring covariance, coskewness and cokurtosis exposures lies in the tradeoff between long time-series for precise estimation and a short time window to allow for variation in higher-moments over time. Kole and Verbeek (2006) discuss the choice of the estimation window when forming hedge portfolios. They demonstrate that a coskewness risk premium formed along a 120-month window will present lower level of (and also less significant) average return than a 60-month window estimate. Halfway between the works of Kole and Verbeek (2006), Agarwal et al. (2008) and Moreno and Rodriguez (2009), we follow the work of Ajili (2005) and use a 36-month window for reaching statistical convergence of the estimates. Besides, the fact that skewness and kurtosis of returns are not stable over time forces us to reduce the window length. Another possibility would have been to extract from option prices some information about the skewness and the kurtosis of the US market index. In this case, however, we would not have defined tradable factors.

We consider three degrees of risk for each risk fundamental and define the hedge portfolios as the return differential between the highly scored and lowly scored portfolio, or inversely for odd moments. In order to control the correlation among the risk fundamentals, we perform three sorts within a sort. The first two sorts are operated on the "control risk" dimensions. The third sort is conducted on the risk dimension to be priced. We argue that such a conditional way of ranking stocks is closer to the way investors are approaching an asset allocation problem. Investors generally deal with one problem at a time, and we therefore expect them to consider sequentially the different dimensions of risk. This approach contrasts with direct attempts to construct factors reflecting higher moment risk on the basis of the direct rankings of the securities' comoments of order k (covariances, coskewness and cokurtosis) with the market portfolio without controlling for the comoments of different order from k, as done for instance by Kostakis et al. (2012).

We illustrate our technique in the case of the cokurtosis premium. Each month, we break up the NYSE, AMEX, and NASDAQ stocks into three groups according to the covariance criterion. We then successively scale stocks within each covariance portfolio into three classes according to their coskewness fundamentals. These nine portfolios are in turn split into three new portfolios according to their cokurtosis statistics. We end up with 27 value-weighted portfolios. The rebalancing is made on a monthly basis. For each month t, every stock is ranked on the selected risk dimensions. Its specific return in the following month is then related to the reward of the risks incurred in the portfolio.

The aim of the procedure is to feed multi-factor specifications for the return generating process of financial securities with three new self-financing portfolios. To create such a risk factor, we thus consider, among the 27 portfolios, the 18 portfolios that score at a high or a low level on the risk dimension. Nine hedge portfolios are then constituted from the difference between high and low scored portfolios, which display the same rankings on the other two risks (used as control variables). Finally, the risk factor is computed as the arithmetic average of these nine portfolios. We repeat the same technique for estimating the covariance and the coskewness portfolios.

Each self-financing and tradable portfolio, corresponding to the second, third and fourth comoment risk premiums, represents the outcome of the sort. For instance, if we look for the risk premium corresponding to dimension X after sequentially controlling for dimensions Y and Z, the factor will be computed as:

$$X_{Y,Z,t} = \frac{1}{9}\left[\sum_{b=H,M,L}\ \sum_{c=H,M,L} R_t(HX|bY|cZ) - \sum_{b=H,M,L}\ \sum_{c=H,M,L} R_t(LX|bY|cZ)\right] \quad (2)$$

where $R_t(aX|bY|cZ)$ represents the return of a portfolio constituted with stocks ranked a on dimension X, among the portfolio of stocks ranked b on dimension Y, themselves among the portfolio of stocks ranked c on dimension Z. Dimensions X, Y and Z stand for covariance, coskewness and cokurtosis, in any possible order, while H, M and L stand for high, medium and low, respectively.

By ending with the risk dimension to be priced, we effectively control for the influence of other higher-order moments. Suppose for instance that one is pricing coskewness risk. By performing a unique sequential sorting — i.e. "covariance–coskewness–cokurtosis" like in Agarwal et al. (2008) — the levels of cokurtosis of portfolios scoring respectively high on cokurtosis and high on coskewness would be different from the levels of cokurtosis of portfolios scoring high on cokurtosis but low on coskewness. If it happens, one could not optimally diversify cokurtosis risks within the coskewness premium. By performing our specific conditional sort i.e. "*covariance–cokurtosis–coskewness*" or equivalently "*cokurtosis–covariance–coskewness*", one ensures that the levels of cokurtosis in both coskewness portfolios are equivalent. We refer to these factors as the *cubic* moment-related factors by reference to the three-stage sequential and triple sorting methodology.

Our technique implies that each premium can be defined using two different suites in the sorting of the risk controls. For the sake of simplicity, we only retain one possible set of the three factors (among the eight possible permutations using both sets). We only consider the explanatory power of the premiums whose ordering start with the lowest order comoment, while ending with the comoment to be priced, that is $V_{S,K}$, $S_{V,K}$, and $K_{V,S}$ (i.e.). Initials "V", "S" and "K" correspond to covariance, coskewness and cokurtosis, respectively. The sequence is reflected in the order of the indices.

We name them henceforth *Cov*, *Skew*, and *Kurt* for brevity.[5]

[5] Both sets of premiums have been compared over the period March 1986–June 2008 and deliver very similar descriptive statistics.

3.3. Descriptive analysis

Table 1 presents summary statistics for our covariance, coskewness, and cokurtosis premiums. Our final sample for the moment-related risk premiums ranges from January 1959 to December 2011. The table describes the moment-related factors over the total sample period and the particular period of 2007–2011. A 36-month rolling window is used to estimate the higher-order moments of the joint density of the stock return distributions with the one of the US market portfolio.

The covariance, coskewness and cokurtosis premiums display positive return over the studied period, i.e. January 1959 to December 2011. Downside risk is particularly important over the sample: the average coskewness premium is highly significant (at 1% confidence level) over the period. The following changes are however noted for the recent financial crisis: while the coskewness premium is becoming non-significant, the cokurtosis premium becomes positively significant at 10% confidence level.

Table 2 reports the correlations among the moment-related factors over the testing period.

Cross-correlations between the three moment-related risk premiums range from 10.19% to 47.63%. These levels are very low given the strong correlation found across these risk fundamentals (Scott and Horvath, 1980). The last table of this subsection provides a descriptive analysis of the nine return spreads composing the skewness and kurtosis factors.

The two risk factors are computed as an arithmetic average of nine difference portfolios issued from the triple sorting method. The distribution of the returns on the nine difference portfolios could affect the relevance of the premiums. Therefore, in Table 3, we check whether the premiums are not overly influenced by the higher or the lower sorts. The return spread related to coskewness exposures does not seem to be driven by one particular sorting. If we refer to the median, the returns spread across the nine portfolio differences stay mainly stable; only three out of the nine portfolios display relatively lower median return. Especially, the average and the median return on high covariance/high cokurtosis portfolios is negative over the period as these portfolios do concentrate stocks with high levels of coskewness due to the positive correlation between the fundamentals. The return spreads related to cokurtosis exposures however greatly vary according to the levels of covariance and coskewness. The highest returns are found amongst portfolios displaying high coskewness or low downside risk. For median to high level of covariance, significant returns are hard to find except for high level of coskewness. The non-significance of the cokurtosis premium could therefore be explained through the dispersion in return between the premium's components.

4. Fama and MacBeth tests

Characteristic-sorted portfolios on market capitalization and book-to-market present strong nonlinearities in their return distribution (Barone-Adesi et al., 2004; Chung et al., 2006; Harvey and Siddique, 2000; Hung, 2007; Hung et al., 2004; Nguyen and Puri, 2009). Therefore, an empirical condition for the multi-moment model to hold would be that the moment-related factors could be able to explain the cross-sections of these particular portfolios. We perform a Fama–MacBeth two-pass cross-sectional procedure for testing the significance of the moment-related factors on those portfolios.

4.1. The method

The Fama and MacBeth (1973) methodology (referred hereafter as the FMB method) consists in month-to-month cross-sectional regressions of the different asset returns (i) from time $t-1$ to time t on their betas estimated up to time $t-1$.

Table 1
Summary statistics for the moment-related risk premiums.

	Panel A		Panel B		Panel C	
	Covariance risk premiums		Coskewness risk premiums		Cokurtosis risk premiums	
	1959–2011	2007–2011	1959–2011	2007–2011	1959–2011	2007–2011
Mean	0.18%	0.48%	0.27%	0.05%	0.14%	0.54%
Median	0.061%	0.78%	0.21%	−0.00%	0.06%	0.62%
Maximum	18.02%	18.03%	11.29%	7.17%	19.77%	8.03%
Minimum	−17.00%	−12.14%	−6.81%	−4.79%	−9.29%	−7.77%
Std. dev.	3.89%	4.88%	2.10%	2.24%	2.45%	2.37%
Skewness	0.2618	0.2653	0.1563	0.6430	0.9111	−0.3523
Kurtosis	2.4777	5.2079	1.5525	4.7433	7.4686	5.5909
t-Stat	1.16	0.76	3.23	0.16	1.43	1.76
Jarque–Bera	169.96***	12.89***	66.46***	11.73***	15.61***	18.02***
#Observations	636	60	636	60	636	60

Notes. This table displays summary statistics about the covariance, the coskewness, and the cokurtosis premiums over the period January 1959–December 2011 and over the period January 2007–December 2011 (covering the crisis period). "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. * stands for statistically significant at 10%, ** at 5% and *** at 1% confidence level.

Table 2
Matrix of correlations across the moment-related premiums.

	$V_{S,K}$	$S_{V,K}$	$K_{V,S}$
$V_{S,K}$	100		
$S_{V,K}$	16.97	100	
$K_{V,S}$	47.63	10.19	100

Notes. This table displays the correlations (in %) among the moment-related premiums over the period January 1959–December 2011. "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices.

We consider the following three models, i.e.

$$\text{M.1} \quad R_{it} = \gamma_{0t+}\gamma_{1t}\widehat{\beta}_{iM} + \gamma_{2t}\widehat{\beta}_{iSMB} + \gamma_{3t}\widehat{\beta}_{iHML} + \gamma_{4t}\widehat{\beta}_{iUMD} + \gamma_{5t}\widehat{s}_{it} + \eta_{it} \tag{3}$$

$$\text{M.2} \quad R_{it} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_{iM} + \gamma_{2t}\widehat{\beta}_{iC} + \gamma_{3t}\widehat{\beta}_{iS} + \gamma_{4t}\widehat{\beta}_{iK} + \gamma_{8t}\widehat{s}_{it} + \eta_{it} \tag{4}$$

$$\text{M.3} \quad R_{it} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_{iM} + \gamma_{2t}\widehat{\beta}_{iC} + \gamma_{3t}\widehat{\beta}_{iS} + \gamma_{4t}\widehat{\beta}_{iK} + \gamma_{5t}\widehat{\beta}_{iSMB} + \gamma_{6t}\widehat{\beta}_{iHML} + \gamma_{7t}\widehat{\beta}_{iUMD} + \gamma_{8t}\widehat{s}_{it} + \eta_{it} \tag{5}$$

(for $i = 1,\ldots,N$) to infer the resulting premiums estimates $\widehat{\gamma}_{kt}$ and to analyze their significance.

The variable $\widehat{s}_{it}$ reflects the residual volatility of the time-series regression estimating the different betas. It is informative about the specification errors related to the two-pass FMB method. We consider jointly two proxies for covariance risk in M.2 and M.3, namely the market return and the covariance mimicking portfolio. Therefore, we re-define our covariance premium as the residuals of the regression of the covariance factor over the market portfolio. We refer to this indistinctly as the covariance premium.

Model 1 uses the Fama and French risk factors[6] as benchmarks for empirical risk premiums. *SMB* stands for "*Small minus Big*" and corresponds to the size factor. *HML* stands for "*High minus Low*" and corresponds to the book-to-market factor. *UMD* stands for "*Up minus Down*" and corresponds to the momentum factor.

Model 2 performs a Four-Moment Asset Pricing Model. On the hypothesis that empirical risk premiums can proxy for higher-order moments, we compare the results of M.2 to the cross-sectional regression including the spread returns related to size, book-to-market and momentum (M.1). We also test the significance of the moment-related factors when the squared terms are included in the model. The model M.2* could be represented as:

$$\text{M.2}^* \quad R_{it} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_{iM} + \gamma_{2t}\widehat{\beta}_{iC} + \gamma_{3t}\widehat{\beta}^2_{iC} + \gamma_{4t}\widehat{\beta}_{iS} + \gamma_{5t}\widehat{\beta}^2_{iS} + \gamma_{6t}\widehat{\beta}_{iK} + \gamma_{7t}\widehat{\beta}^2_{iK} + \gamma_{8t}\widehat{s}_{it} + \eta_{it}. \tag{6}$$

The β_i^2s reflect the nonlinear exposures to the 2nd, 3rd, and 4th moment-related premiums.

Model 3 analyzes the complementarities between the empirical and the moment-related factors.

4.1.1. Conditional versus unconditional test

The traditional Fama–MacBeth cross-sectional procedure implies the test of an *unconditional* systematic relationship between the different beta loadings and returns. This fourth section will analyze the significance of the premiums related to both empirical and moment-related characteristics issued from the second stage of the Fama–MacBeth test. To complete our analysis, we also perform a modified Fama–MacBeth in order to test for a systematic *conditional* relationship between betas and realized returns. We follow the generalization of the Fama–MacBeth method developed by Pettengill et al. (1995) for testing the CAPM and extended by Hung et al. (2004) for testing a multi-moment empirical CAPM. Especially, a dummy variable is used to separate the months into up and down markets according to the sign of the market premium for the related month.

This methodological choice is guided by the following two reasons.

First, the papers of Pettengill et al. (1995) and Hung et al. (2004) recognize the impact of using realized market returns to proxy for expected market returns on the results of the Fama–MacBeth two-stage procedure. On average (expected) market returns must be greater than the risk-free rates. However, there must be some cases where the risk-free return exceeds the market return, otherwise nobody would accept to hold the risk-free rate. In this case, we observe a reverse relationship between market beta and returns. It is thus necessary to distinguish between up and down markets in the Fama–MacBeth setup in order to take into account such a "realization bias". If not, the results of the second-stage of a simple Fama–MacBeth procedure would be combined across the different cross-sectional regressions by averaging the estimated risk premiums and could lead to an insignificant relationship between beta risk and returns. Second, it could be interesting to investigate the investor's aversion to the higher-order US equity risks by types of markets (ups or downs) in order to see if the premiums differ according to the market regime. Coskewness and cokurtosis risks reflect the risk of extreme events. Investments with high cokurtosis and low coskewness are rewarded because of the likely extreme losses they might involve. In down markets, a liquidity squeeze could occur and lead to negative realizations for these premiums.

[6] Made available on the K. French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 3
Summary statistics for the return spread portfolios.

	L/L	L/M	L/H	M/L	M/M	M/H	H/L	H/M	H/H
Panel A: Return spreads composing the coskewness premium									
Mean (%)	0.773	0.373	0.017	0.345	0.181	−0.1541	0.978	0.288	−0.381
Median (%)	0.555	0.462	0.182	0.265	0.341	0.028	0.3903	0.246	−0.227
Maximum (%)	31.01	15.66	25.67	41.29	17.96	14.53	73.37	17.84	23.32
Minimum (%)	−18.01	−16.82	−29.29	−19.78	−11.75	−29.99	−15.01	−15.52	−58.97
Std. dev. (%)	4.64	3.58	4.80	4.50	3.34	3.96	6.00	3.86	4.93
Skewness	0.701	−0.046	−0.179	1.183	0.226	−0.720	3.149	0.219	−2.793
Kurtosis	7.670	4.847	7.415	15.256	4.950	9.599	36.304	4.715	35.970
Jarque–Bera	630***	91***	520***	4129***	106***	1209***	30,444***	83***	29,632***
# obs.	636	636	636	636	636	636	636	636	636
Panel B: Return spreads composing the cokurtosis premium									
Mean (%)	0.243	0.306	0.822	−0.138	−0.018	0.374	−0.650	−0.088	0.407
Median (%)	0.165	0.187	0.296	−0.122	−0.063	0.150	−0.441	0.063	0.372
Maximum (%)	17.02	47.48	42.28	17.63	12.36	35.20	14.42	13.29	48.43
Minimum (%)	−17.55	−18.92	−33.37	−40.55	−14.44	−19.04	−72.43	−15.20	−28.94
Std. dev. (%)	4.52	4.41	5.74	4.56	3.58	4.50	5.71	3.85	5.17
Skewness	0.015	1.819	1.236	−1.098	−0.261	0.798	−3.674	−0.264	1.155
Kurtosis	4.497	24.917	13.77	13.75	4.256	10.398	43.765	4.097	16.939
Jarque–Bera	59***	13,081***	3238***	3189***	49***	1518***	45,467***	39***	5290***
# obs.	636	636	636	636	636	636	636	636	636

Notes. This table presents some summary statistics about the nine return spread portfolios composing the $S_{V,K}$ factor (Panel A) and the $K_{V,S}$ factor (Panel B). The two first letters designate the levels on the control variables. 'L' stands for Low, 'M' for Mid, and 'H' for High. The table displays the mean, median, max., min., standard deviation, skewness, kurtosis and the Jarque–Bera statistics for the return spread portfolios. * stands for statistically significant at 10%, ** at 5% and *** at 1% confidence level.

Therefore, tests for an unconditional positive correlation between beta and realized returns are considered together with two conditional tests that take into account the segmented relationship between the up and down markets.

4.1.2. Test hypotheses

Tests are conducted on the values of the different $\widehat{\gamma}_{kt}$. The following hypotheses are to be tested.

Hypothesis I. The Four-Moment Asset Pricing Model predicts positive return as a positive function of market beta and cokurtosis, and a positive function of negative coskewness. Higher risk in one of the betas is associated with higher returns,

$$\text{i.e. for M.2, M.2}^* \text{ and M.3}: \quad E(\gamma_{1t}+\gamma_{2t}) > 0,\ E(\gamma_{3t}) > 0,\ E(\gamma_{4t}) > 0. \tag{7}$$

Hypothesis II. Betas are complete measures of risk in the efficient market portfolio,

$$\text{i.e. for M.1}: \quad E(\gamma_{5t}) = 0 \text{ and for M.2, M.2}^* \text{ and M.3}: \quad E(\gamma_{8t}) = 0. \tag{8}$$

Hypothesis III. The three moment-related premiums capture all nonlinear risk,

$$\text{i.e. for M.2}^*: \quad E(\gamma_{3t}) = 0,\ E(\gamma_{5t}) = 0,\ E(\gamma_{7t}) = 0. \tag{9}$$

The cross-sectional tests should be performed in two steps. First, stock returns must be regressed onto the different risk premiums, and betas are evaluated on a 48-month moving window time-series regression. Second, each month, the stock returns are regressed on lagged beta loadings (with and without the dummy variables) through cross-sectional regressions.

4.2. Data

The estimates of the cross-sectional regressions correspond to one observation in the time-series of the related risk premiums. However, the use of the estimates $\widehat{\beta}_{ik}$ instead of the true β_{ik} inevitably introduces an errors-in-variables problem when conducting the cross-sectional regressions. To solve this problem, Fama and MacBeth (1973) group stocks into portfolios so that the estimation error in betas can be averaged. Unfortunately, the type of portfolio used for the construction of the premiums in the previous section does not constitute a satisfactory dataset for two reasons: (i) the number of portfolios is insufficient to derive meaningful results, and (ii) the sample comoments of the portfolio constituents with the market portfolio are not stable over time. Hence, one of the key principles of the FMB procedure, namely the high correlation of the risk sensitivities measured during

Table 4
Fama–MacBeth regressions results for M.1, M.2, and M.3 on the 5 × 5 BTM/size portfolios.

	M.1 *F&F* 4-factor CAPM			M.2 4-moment CAPM/M.2* 4-moment CAPM						M.3 4-moment empirical CAPM		
	Total	Up	Down	Total		Up		Down		Total	Up	Down
α	0.94***	1.76***	−0.18	0.93***	0.99***	1.99***	1.95***	−0.51	−0.32	0.95***	1.94	0.41
M	−0.16	1.80***	−2.84***	−0.36	−0.39	1.51***	1.51***	−2.92***	−2.99***	−0.19	1.55***	−2.58***
$V_{S,K}$				0.14	0.28	0.41*	0.58**	−0.24	−0.13	0.02	0.48**	−0.60**
$S_{V,K}$				0.11	0.17	0.27	0.31	−0.12	−0.03	0.03	0.04	0.02
$K_{V,S}$				0.12	0.22	0.74***	0.89***	−0.73***	−0.70***	0.15	0.71***	−0.61***
$V^2_{S,K}$					0.03*		0.03		0.03			
$S^2_{V,K}$					0.00		0.02***		−0.02**			
$K^2_{V,S}$					−0.01		0.00		−0.03*			
SMB	0.23*	0.89***	−0.69***							0.25*	0.91***	−0.73***
HML	0.43***	−0.23	1.33***							0.43***	−0.23	1.14***
UMD	0.38*	0.22	0.58*							0.11	−0.09	−0.41
s	−24.45***	−8.44	−46.41***	−4.47	−7.83	−0.94	−0.46	−9.31	−17.93*	−26***	−6.15	−52.42***
R^2	59.14%	57.65%	61.18%	54.94%	65.39%	57.65%	63.66%	57.66%	67.76%	68.31%	67.35%	69.63%
# obs.	588	340	248	588	588	340	340	248	248	588	340	248

Notes. This table conducts Fama–MacBeth 2-step cross-sectional regressions for the Model 1 (*F&F* empirical CAPM), Model 2/Model 2* (*cubic* 4-moment CAPM), and Model 3 (*cubic* 4-moment empirical CAPM) on the 25 BTM/size portfolios. Portfolios are first regressed on the risk premiums to infer estimates of betas. Second, each month, we perform a cross-sectional regression of the portfolio returns on the beta estimates. The analysis is conducted on the total sample period, i.e. 588 months. Dummy variables are then introduced in the cross-sectional regressions in order to separate premium realizations in up and down markets. For each regime (total period, up market, and down market), the table reports the average value of the different premiums time-series (in %) and their significance. The average adjusted R^2s over each period of time are also displayed. *, ** and *** stand for statistically significant at 10%, 5%, and 1% confidence level, respectively. (*) stand for significant at 20%. *SMB*, *HML*, and *UMD* correspond to the size, the book-to-market, and the momentum risk factors of Fama–French and Carhart. *M* stands for the market premium. "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices. *s* is the residual volatility of the time-series regression estimating the different betas. The premiums squared reflect the premiums attached to nonlinear exposures to the 2nd, 3rd, 4th moment-related premiums.

the formation and the estimation period, is not respected. As the portfolio characteristics do not remain homogeneous, a key condition for the validity of the FMB approach is violated.

We test the significance of our moment-related factors for the Fama and French (1993) 25 (5 × 5) and the 100 (10 × 10) book-to-market/size-sorted portfolios. Size and book-to-market fundamentals have indeed been shown to act as proxies for higher-order comoment risks (Chung et al., 2006; Hung, 2007; Hung et al., 2004; Nguyen and Puri, 2009). According to this view, we can form independent portfolios, while maximizing the return variation related to the variables of interest between portfolios. Besides, we follow Cremers et al. (2008) and Hung et al. (2004) whose papers both show that Fama and French — *F&F* — premiums do not capture all risks related to portfolios sorted on size and book-to-market. Cremers et al. (2008) use the Fama and French 10 × 10 size/book-to-market-sorted portfolios in a two-step FMB cross-sectional regression-based analysis. Hung et al. (2004) form one-dimensional portfolios along the same fundamentals and according to the same rebalancing technique as in Fama and French (1993). Both sets of portfolios are rebalanced every June of year y according to the market capitalization and book-to-market at the end of year $y-1$. According to both studies, there is still residual risk to be priced in these portfolios next to the *F&F* premiums. We expect our moment-related premiums to be able to complement the 4-factor Carhart model for those portfolios.

4.3. Sample period

Our testing period ranges from January 1963 to December 2011. We divide our period into successive 3 non-overlapping sub-periods: a formation, an estimation period, and a testing period.

- The formation period (or SI) sorts individual stocks into portfolio according to their size and book-to-market fundamentals. The portfolios are rebalanced every June of year y according to the market capitalization and book-to-market of December of year $y-1$.
- The estimation period (or SII) estimates $\widehat{\beta}_i$ for each portfolio over a 4-year period — the premiums are estimated and tested over the subsequent month. The procedure is repeated every month. If estimation errors are not correlated among stocks, this should lead to more precise estimates of the true β_i.
- The testing period (or SIII) performs the regressions M.1, M.2, M.2* and M.3 on the portfolios and analyzes the significance of the premium estimates.

The testing period covers 588 months, divided into 340 months for the "up market" period and 248 for the "down market" period.

5. Results and discussion

This section performs the two-step Fama–MacBeth cross-sectional procedure. Results for Eqs. (3) to (6) are discussed.

First, Table 4 tests Hypotheses I, II, and III on the set of 5 × 5 book-to-market/size portfolios. We consider three different scenarios corresponding to the three market regimes, i.e. for the total period, for the up market period and for the down market period.

Table 4 displays high level of explanatory power for the 4-factor Fama and French and Carhart model. All three empirical factors are positive and significant over the total period. When we distinguish between the ups and downs in the market, the size factor keeps being significantly positive in good market conditions but significantly negative in down market conditions. On the contrary, the value and momentum premiums both follow a contrarian strategy.

The four-moment model displays very close levels of explanatory power to the empirical model. We could therefore assume at first sight that a kind of substitution effect is taking place between these two models. The alphas, which reflect the specification errors related to the model, are significant for both models except in the down market sub-period. The level of specification errors related to the two-step FMB procedure (s) are however significant in the case of the empirical approach while it mostly stays insignificant in the moment-related approach. This would indicate that the Four-Moment Asset Pricing Model outperforms the empirical CAPM.

In Model 2, no higher-comoment equity risk premiums are significant over the total period. Even the market factor is not significant over the period. Such perceived evidence should be challenged with a more detailed analysis of the sub-periods. When up and down markets are studied separately, we emphasize strong, but asymmetric influences of the various premiums. The average estimates of the market premiums are symmetrical in up and down markets showing a constant (in absolute value) market beta-return relationship. In down markets, moreover, next to the market portfolio, the cokurtosis-related returns are significantly negative at the 1% confidence level. When considering the up market regime however, the cokurtosis premium is shown to be significantly positive over the up periods at the 1% significance level. Finally, the covariance risk premium brings significant positive corrections to the market premium over the up market period; the market premium could underestimate beta risk. The table shows a great improvement in the levels of the adjusted R-squared when up and down periods are distinguished, supporting the evidence of a differential effect in these periods.

The results for Model 2* are consistent with our conclusions about Model 2. When the squared terms are included in M.2, the coskewness premium becomes significantly positive and negative in up and down markets respectively. To conclude on Models 2 and 2*, the market does price any linear and nonlinear effect related to moment risk premiums. Overall, Hypotheses I and II are

Table 5
Fama–MacBeth regressions results for M.1, M.2, and M.3 on the 10 × 10 BTM/size portfolios.

	M.1 *F&F* 4-factor CAPM			M.2 4-moment CAPM/M.2* 4-moment CAPM						M.3 4-moment empirical CAPM		
	Total	Up	Down	Total		Up		Down		Total	Up	Down
α	0.97***	2.18***	−0.70**	1.02***	0.71***	2.46***	2.15***	−0.95***	−1.26***	0.95***	2.29***	−0.89***
M	0.16***	1.69***	−1.95***	0.14	0.07	1.61***	1.59***	−1.87***	−2.02***	0.06	1.49***	−1.90***
$V_{S,K}$				−0.08	−0.05	0.33*	0.33*	−0.65***	−0.56***	−0.12	0.25*	−0.63***
$S_{V,K}$				−0.29**	−0.18*	0.07	0.16	−0.78***	−0.65***	−0.18**	0.11	−0.57***
$K_{V,S}$				0.32**	0.20*	0.89***	0.79***	−0.48**	−0.60***	0.10	0.62***	−0.60***
$V^2_{S,K}$					0.00		−0.00		0.01			
$S^2_{V,K}$					−0.02***		−0.01*		−0.03**			
$K^2_{V,S}$					−0.01		0.00		−0.03***			
SMB	0.23*	0.83***	−0.60***							0.24**	0.84***	−0.64***
HML	0.30***	−0.32**	1.15***							0.34***	−0.28*	1.05***
UMD	0.40**	0.35	0.48*							0.31**	0.25	−0.33
s	−25.36***	−11.83**	−43.90***	−18.95***	−7.31*	−11.71**	−2.04	−28.88***	−14.54*	−23.69***	−10.11*	−42.30***
R^2	34.13%	32.42%	36.46	29.65%	34.96%	28.30%	33.37%	31.49%	37.13%	38.09%	36.53%	40.23%
# obs.	588	340	248	588	588	340	340	248	248	588	340	248

Notes. This table conducts Fama–MacBeth 2-step cross-sectional regressions for the Model 1 (*F&F* empirical CAPM), Model 2/Model 2* (*cubic* 4-moment CAPM), and Model 3 (*cubic* 4-moment empirical CAPM) on the 100 BTM/size portfolios. Portfolios are first regressed on the risk premiums to infer estimates of betas. Second, we perform each month a cross-sectional regression of the portfolio returns on the beta estimates. The analysis is conducted on the total sample period, i.e. 588 months. Dummy variables are then introduced in the cross-sectional regressions in order to separate premium realizations in up and down markets. For each regime (total period, up market, and down market), the table reports the average value of the different premiums time-series (in %) and their significance. The average adjusted R^2s over each period of time are also displayed. *, ** and *** stand for significant at 10%, 5%, and 1% confidence level, respectively. (*) stand for significant at 20%. *SMB*, *HML*, and *UMD* correspond to the size, the book-to-market, and the momentum risk factors of Fama–French and Carhart. *M* stands for the market premium. "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices. *s* is the residual volatility of the time-series regression estimating the different betas. The premiums squared reflect the premiums attached to nonlinear exposures to the 2nd, 3rd, 4th moment-related premiums.

Table 6
Economic significance of the M.2 and M.3 factors for the 10 × 10 portfolios.

	Total	Up	Down	Total	Up	Down
Panel A: Model 2						
$E(\overline{\beta}\mu)$	*M*			$V_{S,K}$		
	0.082	1.544***	−1.923***	0.000	−0.008	0.013
$E(\overline{\beta}\mu)$	$S_{V,K}$			$K_{V,S}$		
	0.022***	0.014**	0.033***	−0.016*	−0.020*	−0.010
Panel B: Model 3						
$E(\overline{\beta}\mu)$	*M*			$V_{S,K}$		
	0.151	1.073***	−2.001***	0.179***	0.121	0.258***
$E(\overline{\beta}\mu)$	$S_{V,K}$			$K_{V,S}$		
	0.068**	0.090***	0.038	−0.090*	−0.034	−0.167**

Notes. This table conducts statistical (*t*-)tests on the average statistics (in %) of the time-series made of the product of the moment premium inferred from the second pass of the FMB procedure by the average of the related beta coefficient (estimated by the first pass of the FMB procedure) for M.2 and M.3
*, ** and *** stand for significant at 10%, 5%, and 1%, respectively. "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices.

confirmed when the stock market rises. This joint acceptance implies that we can emphasize a set of linear and non-linear risk premiums corresponding to all three higher-comoments with the market returns.

The composite model or the 4-moment empirical CAPM demonstrates the relevance of adding moment-related risk premiums to the extensively used 4-factor model of Carhart. Adding moment-related factors to the *F&F* empirical factors or adding the *F&F* empirical risk factors to moment-related factors bring up to respectively 10% and 13% of additional R^2. The significance of the cokurtosis premium is not subsumed by empirical factors. It demonstrates the relevance of considering empirical and moment-related factors in one regression when evaluating the risk-return tradeoff for BTM/size portfolios. Contrary to the studies of Chung et al. (2006) and Nguyen and Puri (2009) — where the significance of the *F&F* premiums has been shown to vanish when higher-order comoments are considered in the regression-based analysis — we point out that the significance of both empirical premiums is not subsumed by the significant moment-related factors. This contradicts evidence documented by Heaney et al. (2011), whose results are based on direct estimates of the higher-moment betas as in Eq. (1), which indicates that the sequential procedure leading to tradable risk factors conveys information with clear economic significance. However, our results evidence a complementary character of both types of premiums.

Table 5 reproduces the same analysis on the set of 10 × 10 portfolios.

The levels of R^2 displayed by Table 5 are significantly lower than in Table 4, showing a decrease in the explanatory power of the models for explaining a large cross-section of returns. Specification errors related to the FMB procedure also become strongly significant over the period and sub-periods for all models considered. Nevertheless we still reach similar conclusion. The 4-moment empirical model delivers better explanatory power than the four-factor Carhart model and the moment-related

Table 7
Fama–MacBeth regressions results for M.2, and M.3 with extreme downside risk variables.

	M.2ᵈ 4-moment CAPM with downside risk			M.3ᵈ 4-moment empirical CAPM with downside risk		
	Total	Up	Down	Total	Up	Down
α	0.93***	2.27***	−0.91***	0.90***	2.27***	−0.97***
M	0.10	1.62***	−1.98***	0.06	1.48***	−1.88***
$V_{S,K}$	−0.21*	0.18	−0.74***	−0.07	0.32**	−0.60***
$S_{V,K}$	−0.217**	0.112	−0.67***	−0.13(*)	0.14	−0.50***
$K_{V,S}$	0.16(*)	0.68***	−0.54***	0.07	0.57***	−0.62***
VaR	0.82***	0.60(*)	1.12**	0.63**	0.43	0.90**
β_D	0.37***	1.11***	−0.64***	0.13	0.87***	−0.88***
SMB				0.24	0.85***	−0.59***
HML				0.34***	−0.27**	1.18***
UMD				0.24*	0.19	0.33
s	−16.85***	−8.59*	−28.18***	−22.17***	−9.07*	−40.14***
R^2	33.78%	32.25%	35.89%	40.49%	38.93%	42.63%
# obs.	588	340	248	588	340	248

Notes. This table conducts Fama–MacBeth 2-step cross-sectional regressions for Model 2ᵈ with downside risk measures and Model 3ᵈ with downside risk measures on the 100 BTM/size portfolios. Portfolios are first regressed on the risk premiums to infer estimates of betas. Second, we perform each month a cross-sectional regression of the portfolio returns on the beta estimates. The analysis is conducted on the total sample period, i.e. 588 months. Dummy variables are then introduced in the cross-sectional regressions in order to separate premium realizations in up and down markets. For each regime (total period, up market, and down market), the table reports the average value of the different premiums time-series (in %) and their significance. The average adjusted R^2s over each period of time are also displayed. *, ** and *** stand for significant at 10%, 5%, and 1%, respectively. (*) stand for significant at 20%. *SMB*, *HML*, and *UMD* correspond to the size, the book-to-market, and the momentum risk factors of Fama–French and Carhart. *M* stands for the market premium. "*V*", "*S*" and "*K*" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices. *s* is the residual volatility of the time-series regression estimating the different betas.

model, demonstrating the complementarity between the premiums. Table 5 also confirms the main results drawn from Table 4. First, the *F&F* four-factor CAPM displays significant *SMB* and *HML* factors across the three periods. Second, the analysis evidences positive and significant returns related to extreme risk in stock market in good market state and significantly negative returns in down market period. Table 5 demonstrates a similar effect on the coskewness risk in down market. Significant negative return is associated with downside risk in down market. Finally, the conclusion drawn at Table 4 that the market portfolio underestimates covariance risk in ups and overestimates it in down is also supported by Table 5: the table indeed gives evidence of a significant and positive premium relative to covariance risk in up market and significant and negative premium related to covariance risk in down market. The results for the composite model M.3 support the findings of M.2 regarding the correcting role of the covariance risk premium over the market excess returns and the differential effects in the investor's risk aversion to moment related premiums. In the down market period, we also observe a negative and significant downside factor. To conclude, the coskewness and the cokurtosis risk premiums have to be considered separately in up and down markets. These risk premiums reward the probability of extreme losses. When the market does well, bearing such risks leads to an extra return. When markets go down, these extreme risks (represented by both the coskewness and cokurtosis coefficients and, to a lesser extent, by the nonlinear effects) lead to significant negative premiums.

In order to test the economic significance of these premiums, we analyze the total required return associated to each source of risk within the sets of size and book-to-market sorted portfolios. Namely, based on the results for the models M.2 and M.3, we construct a table with, for each moment-related premium, the product of its level (derived from the second pass of the FMB procedure) with the average beta coefficient (derived from the first pass of the FMB procedure). We then test the economic significance of each premium. We conduct bilateral tests over the average values taken by the different premiums. Table 6 presents the analysis on the 5 × 5 sets of BTM/size portfolios, on the 10 × 10 BTM/size portfolios. Like in the previous analyses, this table presents the results for the total period, but also over the up and down sub-periods.

Panels A and B present the results respectively for the higher-moment model M.2 and the empirical higher-moment model M.3. While the previous analysis extracts the premiums related to downside and extreme risks priced by the US stock markets, this table decomposes the return on size and book-to-market portfolios that could be explained by their exposures to these factors. Their exposure to downside risk is shown to drive positive returns, while their exposure to extreme risk drives significant negative returns.

6. Robustness checks

Section 6 conducts several robustness checks over the results discussed previously in the paper. First, since our comoment risk factors could capture extreme downside risk, we control the robustness of our results when proxies for downside risk and tail event risk are included in the model. Second, as our higher moment factors could drive the hedging properties of securities, we

Table 8
Fama–MacBeth regressions results for M.1, M.2, and M.3 on the 10 × 10 BTM/size portfolios over the financial crisis period.

	M.1	M.2			M.3	
	F&F 4-factor CAPM	4-moment CAPM			4-moment empirical CAPM	
	M.1	M.2	M.2*	M.2^d	M.3	M.3^d
α	−0.65	−0.51	−0.49	−0.61	−0.71	−0.74
M	0.79	0.53	0.60	0.64	0.88	0.98
$V_{S,K}$		0.29	0.16	0.35	−0.01	0.84
$S_{V,K}$		−0.52*	−0.51*	−0.41(*)	−0.45*	−0.34(*)
$K_{V,S}$		−0.09	−0.05	−0.06	0.10	0.15
$V^2_{S,K}$			−0.02			
$K^2_{V,K}$			−0.01			
$K^2_{V,S}$			−0.00			
VaR				1.31		1.35
β_D				0.41		0.49
SMB	0.03				0.02	0.06
HML	0.05				0.06	0.06
UMD	0.95(*)				0.67	0.49
s	−3.67	2.78	−2.68	1.60	−5.48	−12.20
R^2	25.21%	20.18%	27.06%	23.93%	29.87%	32.98%
# obs.	54	54	54	54	54	54

Notes. This table conducts Fama–MacBeth 2-step cross-sectional regressions for the Model 1 (*F&F* empirical CAPM), Model 2/Model 2* (*cubic* 4-moment CAPM) and Model 2^d with downside risk measures, Model 3 (*cubic* 4-moment empirical CAPM) and Model 3^d with downside risk measures on the 100 BTM/size portfolios. Portfolios are first regressed on the risk premiums to infer estimates of betas. Second, we perform each month a cross-sectional regression of the portfolio returns on the beta estimates. The analysis is conducted on the crisis sample period, i.e. 54 months. The table reports the average value of the different premiums time-series (in %) and their significance. The average adjusted R^2s over each period of time are also displayed. *, ** and *** stand for statistically significant at 10%, 5%, and 1%, respectively. (*) stand for significant at 20% confidence level. *SMB*, *HML*, and *UMD* correspond to the size, the book-to-market, and the momentum risk factors of Fama–French and Carhart. *M* stands for the market premium. “*V*”, “*S*” and “*K*” correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices. *s* is the residual volatility of the time-series regression estimating the different betas. The premiums squared reflect the premiums attached to nonlinear exposures to the 2nd, 3rd, 4th moment-related premiums.

Table 9
Economic significance of the M.2 and M.3 factors for the 10 × 10 portfolios over the crisis period.

	Total	Up	Down	Total	Up	Down
Panel A: Model 2						
$E(\overline{\beta}\mu)$	M			$V_{S,K}$		
	0.583	3.729**	−3.422***	0.787(*)	0.009	0.165(*)
$E(\overline{\beta}\mu)$	$S_{V,K}$			$K_{V,S}$		
	−0.155 **	−0.134*	−0.145	0.381	0.015	0.066
Panel B: Model 3						
$E(\overline{\beta}\mu)$	M			$V_{S,K}$		
	0.875	3.179**	−2.061***	−0.013	−0.453	0.517
$E(\overline{\beta}\mu)$	$S_{V,K}$			$K_{V,S}$		
	−0.455*	−0.512(*)	−0.350	0.099	0.220	−0.018

Notes. This table conducts statistical (t-)tests on the average statistics (in %) of the time-series made of the product of the moment premium inferred from the second pass of the FMB procedure by the average of the related beta coefficient (estimated by the first pass of the FMB procedure) for M.2 and M.3. The analysis covers the crisis period, i.e. August 2007–December 2011.
*, ** and *** stand for significant at 10%, 5%, and 1%, respectively. (*) stand for significant at 20%. "V", "S" and "K" correspond to the covariance, coskewness and cokurtosis premiums, respectively. The ordering sequence is reflected in the order of the indices.

consider the specific sample period of 2007–2011 that encompasses a severe financial crisis involving extreme price movements, i.e. the materialization of risks underlying the comoment premiums.

6.1. Alternative downside risk variables

Table 7 conducts Fama–MacBeth 2-step cross-sectional regressions on the 100 size/BTM portfolios for Model 2 (four-moment CAPM) and Model 3 (four-moment empirical CAPM) when alternative downside risk factors are introduced in the model. First, we consider for each month the value-at-risk (at 5% confidence level) of the market portfolio distribution over the last 12 months. The VaR has been computed using daily data. This time series is introduced in the first stage of our Fama–MacBeth test. Second, next to the traditional CAPM beta, the first stage of the cross-sectional regression considers a downside beta computed as the loading on the minimum between the market portfolio returns and 0.

Table 7 confirms our conclusions drawn from Tables 4 and 5. Moment-related premiums keep to be priced when alternative measures of downside risk are introduced into the model. The explanatory power of the model is increased showing the complementarity between the two types of variables. Model 2 R-squares increase by about 4%, while Model 3 R-squares increase by about 2%. Especially, the risk premium related to downside beta follows the same sign pattern and magnitude as our cokurtosis and coskewness factors. The return levels of the coskewness and cokurtosis factors are reduced accordingly; this evidences a cannibalization effect between the two sets of premiums. On the contrary, the VaR risk premium factors the risk of extreme losses into return. Contrary to our downside and extreme risk measures, the premium stays significantly positive across the periods considered (the total period, up and down market periods). This premium still captures a risk feature alternative to the higher-moment factors.

6.2. Financial crisis period and the level of downside risk

We repeat the analysis conducted at Table 5 (M.1, M.2, M.2*, and M.3) and at Table 7 (M.2^d and M.3^d) over the financial crisis period, i.e. 54 months covering the period August 2007–December 2011.

The analysis conducted at Table 8 on the 54-month period only highlights the significance of the coskewness factors. The period is characterized by high instability and could explain why only an extreme downside risk factor is priced in the US market over this period.

Table 9 extends the analysis by analyzing the exposures of the characteristics-sorted portfolios on empirical sources of risk (size and book-to-market).

The market factor significantly explains the excess return on these portfolios. The market exposure drives the returns on size- and book-to-market-sorted portfolios down in downturn but drives it up in upturn. Exposure to downside risk drives however negative return for those portfolios as shown from the significantly negative total premium on the coskewness factor for Model 2.

7. Conclusions

Based on an extensive data sample from CRSP, this research paper derives a unique set of premiums factoring higher-moment risk into returns. The premiums capture the downside and extreme risk implied in the US stock market for the period January 1959 to December 2011.

Our study has shed new light over the complementary relationship between moment-related factors and the Fama and French (1993) empirical risk factors. It is also shown that our estimation guidelines for constructing fundamental risk factors compare favorably with the numerous attempts displayed in the literature.

Developing a consistent sequential methodology to construct investable risk factors based on market covariance, coskewness and cokurtosis, as we do in this paper, could be viewed as an incremental contribution to the literature on higher moment risk premiums. Evidence presented in this paper, however, opens the way to an alternative interpretation.

By applying the seminal FMB methodology within our framework, we have been able to directly assess the statistical and economic relevance of the new risk premiums. The flexibility and robustness of the empirical context have also enabled us to test the coexistence of the empirical risk factors of Fama and French (1993) and Carhart (1997) with moment-based premiums. The Fama and MacBeth (1973) cross-sectional regressions provide very consistent results: we show both premiums to be significant, even when empirical risk premiums are added to the multi-moment analysis.

We also address the intuitive interpretation of the coskewness and cokurtosis risk premiums through considering separately in up and down markets. These risk premiums reward the probability of extreme losses. Therefore, when the market does well, bearing such risks lead to extra return. In adverse market conditions, bearing these risks could result in negative realizations of the premiums. This finding is, to some extent, to be considered in light of the succession of crises following the 2008 collapse of Lehman Brothers. Since our risk premiums explicitly recognize extreme risks, the behavior of stocks loading differently to such factors is likely to differ most under highly stressed market conditions, and thus the higher moment betas could drive the hedging properties of some securities in crisis times. As shown by our robustness checks, the price of coskewness risk remains significant. This finding calls for further investigation about the specific hedging properties of this particular risk premium, as it is the only one that remains across any extent of negative market trends. This is part of our ongoing research agenda.

References

Agarwal, V., Bakshi, G., Huij, J., 2008. Higher-moment equity risk and the cross-section of hedge fund returns. Working Paper. Robert H. Smith School of Business, University of Maryland, US.

Ajili, S., 2005. Size and book to market effects vs. coskewness and cokurtosis in explaining stock returns. Working Paper. Université Paris IX Dauphine — CEREG, France.

Arditti, F.D., 1967. Risk and the required return on equity. J. Finance 22 (1), 19–36.

Arditti, F.D., 1969. A utility function depending on the first three moments: reply. J. Finance 24 (4), 720.

Badrinath, S.G., Chatterjee, S., 1988. On measuring skewness and elongation in common stock return distributions: the case of the market index. J. Bus. 61 (4), 451–472.

Bakshi, G., Kapadia, N., Madan, D., 2003. Stock return characteristics, skew laws, and the differential pricing of individual equity options. Rev. Financ. Stud. 16, 101–143.

Barberis, N., Huang, M., 2008. Stocks as lotteries: the implications of probability weighting for security prices. Am. Econ. Rev. 98 (5), 2066–2100.

Barone-Adesi, G., Gagliardini, P., Urga, G., 2004. Testing asset pricing models with coskewness. J. Bus. Econ. Stat. 22 (4), 474–485.

Benishay, H., 1992. The Pratt–Arrow requirement in a fourth degree polynomial utility function. J. Acc. Audit. Finance 7 (1), 97–112.

Boyer, B.H., Mitton, T., Vorkink, K., 2010. Expected idiosyncratic skewness. Rev. Financ. Stud. 23 (1), 169–202.

Brunnermeier, M., Parker, J., 2005. Optimal expectations. Am. Econ. Rev. 95 (4), 1092–1118.

Brunnermeier, M., Gollier, C., Parker, J., 2007. Optimal beliefs, asset prices and the preference for skewed returns. Am. Econ. Rev. 97 (2), 159–165.

Carhart, M.M., 1997. On persistence in mutual fund performance. J. Finance 52 (1), 57–82.

Chen, K., Passow, A., 2003. Quantitative selection of long-short hedge funds. Working Paper, FAME Research Paper. No. 94. Swiss Finance Institute, Switzerland.

Chung, Y.P., Johnson, H., Schill, M.J., 2006. Asset pricing when returns are nonnormal: Fama–French factors versus higher-order systematic comoments. J. Bus. 79 (2), 923–940.

Conrad, J., Dittmar, R.S., Ghysels, E., 2013. Ex ante skewness and expected stock returns. J. Finance 68 (1), 85–124.

Cremers, M., Petajusto, A., Zitzewitz, E., 2008. Should benchmark indices have alpha? Revisiting performance evaluation. Working Paper. Yale School of Management, NYC.

Ding, B., Shawky, H.A., 2007. The performance of hedge fund strategies and the asymmetry of return distributions. Eur. Financ. Manage. 13 (2), 309–331.

Dittmar, R.F., 2002. Non-linear pricing kernels, kurtosis preference, and evidence from the cross-section of equity returns. J. Finance 57 (1), 369–403.

Doan, M.P., Lin, C.-T., Chng, M., 2013. Higher moments and beta asymmetry: evidence from Australia. Accounting and Finance. (forthcoming).

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33 (1), 3–56.

Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: empirical tests. J. Polit. Econ. 81 (3), 607–636.

Fang, H., Lai, T.-Y., 1997. Cokurtosis and capital asset pricing. Financ. Rev. 32 (2), 293–307.

Harvey, C.R., Siddique, A., 2000. Conditional skewness in asset pricing tests. J. Finance 55 (3), 1263–1295.

Heaney, R., Lan, Y., Treepongkaruna, S., 2011. A test of coskewness and cokurtosis: the relevance of size, book to market and momentum in asset pricing. Working Paper. University of Western Australia.

Hübner, G., Lambert, M., Papageorgiou, N., 2013. Higher-moment risk exposures in hedge funds. Working Paper. HEC—University of Liège, Belgium.

Hung, D.C.-H., 2007. Momentum, size and value factors versus systematic comoments in stock returns. Working Paper. Durham University, UK.

Hung, D.C.-H., Shackleton, M., Xu, X., 2004. CAPM, higher comoment and factor models of UK Stock returns. J. Bus. Finance Acc. 31 (1–2), 87–112.

Hwang, S., Satchell, S.E., 1999. Modelling emerging market risk premiums using higher moments. Int. J. Finance Econ. 4 (4), 271–296.

Jean, W.H., 1971. The extension of portfolio analysis to three or more parameters. J. Financ. Quant. Anal. 6 (1), 505–515.

Jurczenko, E., Maillet, B., 2002. Multi-moment kernel asset pricing model (KAPM): some basic results. Working Paper. University of Paris I Panthéon-Sorbonne, France.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47 (2), 263–291.

Kat, H., Miffre, J., 2006. The impact of non-normality risks and tactical trading on hedge funds alphas. Working Paper. Cass Business School, City University, London Faculty of Finance, UK.

Kimball, M.S., 1990. Precautionary saving in the small and in the large. Econometrica 58 (1), 53–73.

Kimball, M.S., 1993. Standard risk aversion. Econometrica 61 (3), 589–611.

Kole, E., Verbeek, M., 2006. Crash risk in the cross section of stock returns. Working Paper. Erasmus University Rotterdam, Netherlands.

Kostakis, A., Muhammad, K., Siganos, A., 2012. Higher comoments and asset pricing on the London Stock Exchange. J. Bank. Finance 36 (3), 913–922.

Kraus, A., Litzenberger, R.H., 1976. Skewness preference and the valuation of risk assets. J. Finance 31 (4), 1085–1100.

Levy, H., 1969. Comment: a utility function depending on the first three moments. J. Finance 24 (4), 715–719.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Rev. Econ. Stat. 47 (1), 13–37.

Mitton, T., Vorkink, K., 2007. Equilibrium underdiversification and the preference for skewness. Rev. Financ. Stud. 20 (4), 1255–1288.

Moreno, D., Rodriguez, R., 2009. The coskewness factor: implications for performance evaluation. J. Bank. Finance 33 (9), 1664–1676.

Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica 34 (4), 768–783.

Nguyen, D., Puri, T.N., 2009. Higher order systematic comoments and asset pricing: new evidence. Financ. Rev. 44 (3), 345–369.

Peiro, A., 1999. Skewness in financial returns. J Bank. Finance 23 (6), 847–862.
Pettengill, G., Sundaram, S., Mathur, I., 1995. The conditional relation between beta and returns. J. Financ. Quant. Anal. 30 (1), 101–116.
Potì, V., Wang, D.L., 2010. The coskewness puzzle. J Bank. Finance 34 (8), 1827–1838.
Ranaldo, A., Favre, L., 2005. How to price hedge funds: from two- to four-moment CAPM. Working Paper. EDHEC Business School, France.
Rubinstein, M.E., 1973. The fundamental theorem of parameter-preference security valuation. J. Financ. Quant. Anal. 8 (1), 61–69.
Samuelson, P.A., 1970. The fundamental approximation theorem of portfolio analysis in terms of means, variances, and higher moments. Rev. Econ. Stud. 37 (112), 537–542.
Satchell, S.E., Damant, D.C., Hwang, S., 2000. Exponential risk measure with application to UK asset allocation. Appl. Math. Finance 7 (2), 127–152.
Scott, R.C., Horvath, P.A., 1980. On the direction of preference for moments of higher order than the variance. J. Finance 35 (4), 915–919.
Sharpe, W.F., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. J. Finance 19 (3), 425–442.
Spurgin, R.B., 2001. How to game your sharpe ratio? J. Altern. Invest. 4 (3), 38–46.
Treynor, J., Mazuy, K., 1966. Can mutual funds outguess the market? Harv. Bus. Rev. 44, 131–136.