# Envelope reconstruction problem with static loadings

## Principal Static Wind Loads

N. Blaise & V. Denoël

University of Liège (Belgium)

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European-African Conference on Wind Engineering

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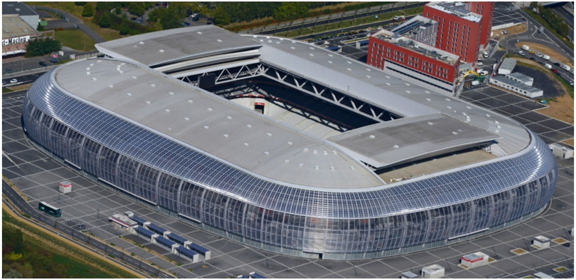
# Equivalent static design of structures under wind excitations

- Assumptions
  - Linear structures
  - Gaussian wind excitations
- Any type of structures
  - large roof structure



Stadium in Lille, France

CSTB le futur en construction

Nantes, France

greisch

Liège, Belgium

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## Buffeting analysis

Wind actions → Structural linear analysis → Structural Gaussian responses

Wind actions:
- Time/Space evolution
- Power Spectral Density
- Coherence function
- ...

Structural linear analysis:
- Step-by-step method
- Spectral analysis
- Nodal/Modal basis
- Background/Resonant decomposition

Structural Gaussian responses:
- $\mathbf{r}^d(t) = \boldsymbol{\mu}_r + \mathbf{r}(t)$

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# Buffeting analysis

Wind actions → Structural linear analysis → Structural Gaussian responses

**Wind actions**

- Time/Space evolution
- Power Spectral Density
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- ...

**Structural linear analysis**

- Step-by-step method
- $\boxed{\text{Spectral analysis}}$
- Nodal/Modal basis
- Background/Resonant decomposition

**Structural Gaussian responses**

- $\mathbf{r}^d(t) = \boldsymbol{\mu}_r + \mathbf{r}(t)$
- Envelope $(\mathbf{r}^{min}; \mathbf{r}^{max})$

$$\boxed{\mathbf{r}^{min} = -g\boldsymbol{\sigma}_r \ ; \ \mathbf{r}^{max} = g\boldsymbol{\sigma}_r}$$

- Design envelope $(\mathbf{r}^{d,min}; \mathbf{r}^{d,max})$

$$\mathbf{r}^{d,min} = \boldsymbol{\mu}_r + \mathbf{r}^{min}$$
$$\mathbf{r}^{d,max} = \boldsymbol{\mu}_r + \mathbf{r}^{max}$$

**Focus on zero-mean envelope**

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# Equivalent Static Design

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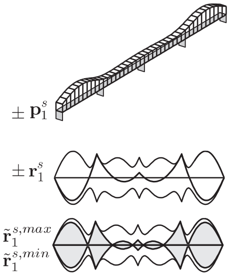
**Are there static loadings that reproduce efficiently this envelope ?**

# Equivalent Static Design


**Are there static loadings that reproduce efficiently this envelope ?**




$$\tilde{\mathbf{r}}_k^{s,min} = min\left(\tilde{\mathbf{r}}_{(k-1)}^{s,min}; \mathbf{r}^s; -\mathbf{r}^s; 0\right) \quad ; \quad \tilde{\mathbf{r}}_k^{s,max} = max\left(\tilde{\mathbf{r}}_{(k-1)}^{s,max}; \mathbf{r}^s; -\mathbf{r}^s; 0\right)$$

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# Equivalent Static Design


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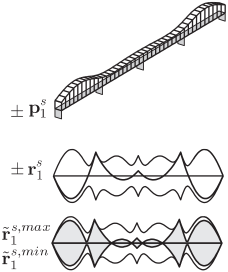


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**Envelope reconstruction problem**

# Equivalent Static Design


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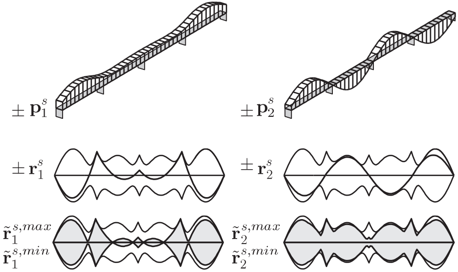


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**Envelope reconstruction problem**

# Equivalent Static Design


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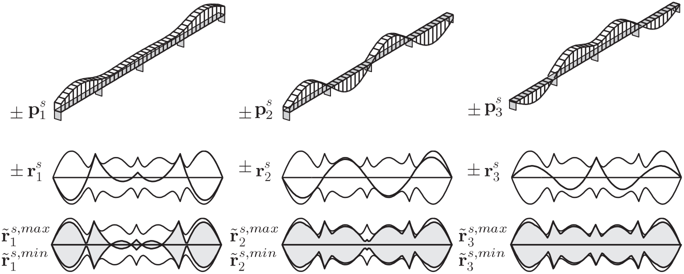


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**Envelope reconstruction problem**

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# Envelope reconstruction problem

- **Solution for specific structures**
  - Vertical ones - Global loading technique (Repetto & Solari, 2004)[1]

[1] Repetto M.P., Solari G. (2004). Equivalent static wind actions on vertical structures. *Journal of Wind Engineering and Industrial Aerodynamics* 92, 335-357.

# Envelope reconstruction problem

- **Solution for specific structures**
  - Vertical ones - Global loading technique (Repetto & Solari, 2004)[1]
- **Proposed basis established with**
  - CPT - Universal loads (Katsumura et al., 2007)[2]
  - SPT (Fiore & Monaco, 2009)[3]
  - ESWL - Least-squares fitting (Zhou et al., 2011)[4]

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[1] Repetto M.P., Solari G. (2004). Equivalent static wind actions on vertical structures. *Journal of Wind Engineering and Industrial Aerodynamics* 92, 335-357.

[2] Katsumura A., Tamura Y., Nakamura O. (2007). Universal wind load distribution simultaneously reproducing largest load effects in all subject members on large-span cantilevered roof. *Int. J. Wind Eng. Ind. Aerod.* 95 (9-11), pp. 1145-1165

[3] Fiore A., Monaco P. (2009). Pod-based representation of the alongwind equivalent static force for long-span bridges. *Wind and Structures*, 12 (3), pp. 239-257.

[4] Zhou X., Gu M., Li G. (2011). Application research of constrained least-squares method in computing equivalent static wind loads. *In : Proceeding of the 13th International Conference on Wind Engineering.*

# Covariance proper transformation

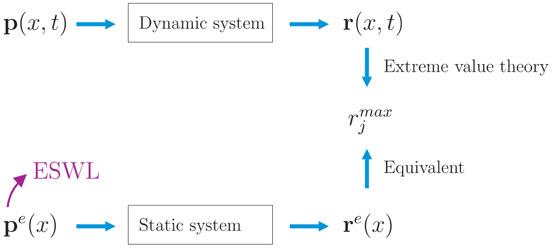
## CPT loading modes

$$\underbrace{\mathbf{C^{(p)}}}_{\text{Covariance matrix of external forces}} (\mathbf{C^{(p)}} - \mathbf{C^{(c)}}\mathbf{I})\underbrace{\mathbf{P}^c}_{\text{CPT loading modes}} = 0$$

- Automatic procedure
- Global loadings
- Do not take into account the resonant behavior of the structure

# Equivalent Static Wind Loads

- An **Equivalent** Static Wind Load (ESWL) ⇔ **one specific** extreme structural response




- Nodal background analysis : Load-response-correlation method (Kasperski, 1991)[1]
- Nodal background and modal resonant analysis (Chen & Kareem, 2001)[2]
- Full nodal analysis (Blaise & Denoël, 2012)[3]

[1] Kasperski M. (1992). Extreme wind load distributions for linear and nonlinear design *Engineering Structures* 14, 27-34.
[2] Chen X.Z., Kareem A. (2001). Equivalent static wind loads for buffeting response of bridges. *Journal of Structural Engineering-Asce* 127, 1467-1475.
[3] Blaise N., Denoël V. (2013). Principal Static Wind Loads. *Int. J. Wind Eng. Ind. Aerod.*, 113, 29-39.

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# Principal Static Wind Loads

- **Key-idea**[1] : Singular value decomposition of the **ESWL matrix** $\mathbf{P}^e$

$$
\underbrace{\overset{\mathbf{p}^e}{\begin{pmatrix} p^e_{11} & \cdots & p^e_{1N} \\ \vdots & \ddots & \\ p^e_{l1} & & p^e_{lN} \end{pmatrix}}}_{l \times N}
=
\underbrace{\overset{\mathbf{p}^p}{\begin{pmatrix} p^p_{11} & \cdots & p^p_{1M} \\ \vdots & \ddots & \\ p^p_{l1} & & p^p_{mM} \end{pmatrix}}}_{l \times M}
\underbrace{\overset{\mathbf{s}}{\begin{pmatrix} s_{11} & & 0 \\ & \ddots & \\ 0 & & s_{MM} \end{pmatrix}}}_{M \times M}
\underbrace{\overset{\mathbf{v}'}{\begin{pmatrix} v_{11} & \cdots & v_{1N} \\ \vdots & \ddots & \\ v_{M1} & & v_{MN} \end{pmatrix}}}_{M \times N}
$$

where $\mathbf{P}^p$ collects the **Principal Static Wind Load** (PSWL) basis[1].

$\boxed{\text{Convergence of the decomposition} \rightarrow \mathbf{M} \lll \mathbf{N}}$

[1] Blaise N., Denoël V. (2013). Principal Static Wind Loads. *Int. J. Wind Eng. Ind. Aerod.*, **113**, 29–39

# Principal Static Wind Loads

## Properties

- PSWLs are not associated with specific structural responses (global responses)
- PSWLs take into account the resonant behavior of the structure
- PSWLs are well-suited for combinations $\mathbf{P}^s = \mathbf{P}^p \mathbf{q}^p$ [1]
- PSWL basis is built with an automatic procedure

[1] Blaise, N. and Hamra, L. and Denoel, V. (2012). Principal Static Wind Loads on a large roof structure. *Proceedings of the 12th ANIV conference of wind engineering In Vento*

# Objective

- Comparison of three basis for the envelope reconstruction problem
  - CPT : Covariance Proper Transformation
  - ESWL : Equivalent Static Wind Loads
  - PSWL : Principal Static Wind Loads
- Loadings are scaled such that there is no-overestimation of the real envelope
  - Tangency condition

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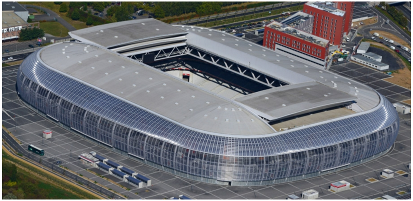
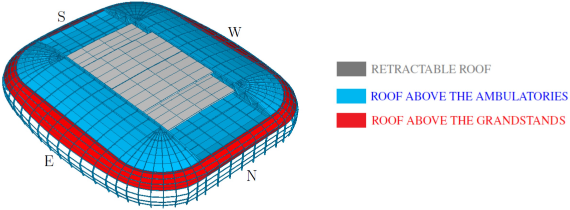
# Description of the structure



Lille's stadium, France[1]

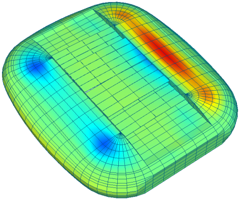



Structural finite element model (Greisch, Liège)[2]

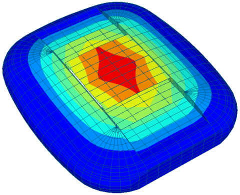
[1] http ://www.grandstade-lillemetropole.com/ [2] http ://www.greisch.com/

# Description of the structure

- Modal characteristics (FineLg[1])



mode 1 : 0.47 Hz (Z global)



mode 3 : 0.52 Hz (Z global)

- uncoupled equations of motions (proportional damping)
- the first **21** modes are kept
- modes 1-11 : $f_{nat} < 1Hz$ ; mode 21 : $f_{nat} = 1.41Hz$

[1] FineLg. (2003), Non linear finite element analysis program : User's Manual, Unveristy of Liège, ArGEnCo and Greisch Ingènierie.

# Wind tunnel simulation

- Aerodynamic loading characterization

1/200-scaled model (rigid) of the stadium[1]

- Measurement characteristics
  - 24 tested wind directions
  - number of sensors : ∼500
  - sampling frequency of 2.92 Hz ! (full scale model)
  - measurement period : 105 min (full scale model)

[1]Wind tunnel simulations at the Centre Scientifique et Technique du Bâtiment (CSTB) at Nantes, France

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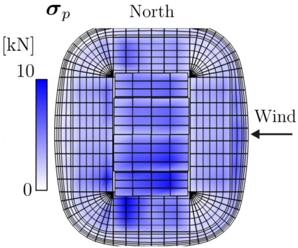
# Buffeting wind analysis

- Studied wind direction : 75°




- Nodal Background/Modal Resonant Spectral analysis[1]

[1] Blaise, N. and Grillaud, G. and De Ville de Goyet, V. and Denoël, V. (2011). Application of deterministic and stochastic analysis to calculate a stadium with pressure measurements in wind tunnel. *Proceedings of 8th International Conference on Structural Dynamics*, Leuven, Belgium

Introduction

Studied structure

Envelope reconstruction problem

Results

Conclusions

# Definition of the target envelope

- **Target envelope** collects the six internal forces for each beam element (2542) :
  - Axial force ; two bending moments ; two shear forces ; torque.
  - Number of structural responses : 30504

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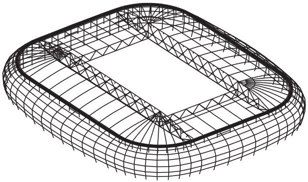
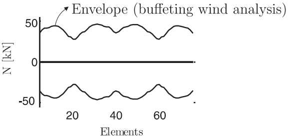
- Illustration of the envelope reconstruction
  - Axial force of 66 beam elements






  - Reconstruction of the full envelope is illustrated in the paper

## Definition of the indicator of convergence

- Design purposes : finite number of representative design load cases
- Selection in the available set of loading modes
- Maximization of a choosen indicator of convergence

$l^{th}$ structural response in the reconstructed envelope with $k$ loading modes

$$\Psi_k = \frac{1}{N^r} \sum_{l}^{N^r} \left( \frac{\tilde{r}_{lk}^{max} - r_l^{max}}{r_l^{max}} \right)$$

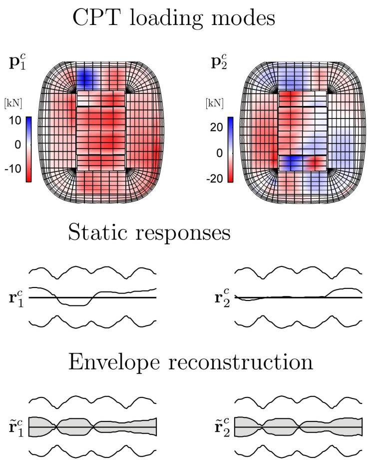
$l^{th}$ structural response

# CPT loading modes

Covariance matrix of external forces

$$(\mathbf{C}^{(\mathbf{p})} - \mathbf{C}^{(\mathbf{c})}\mathbf{I})\mathbf{P}^c = 0$$

CPT loading modes

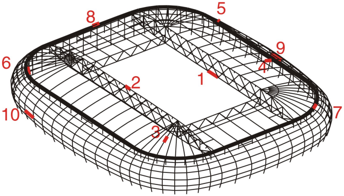


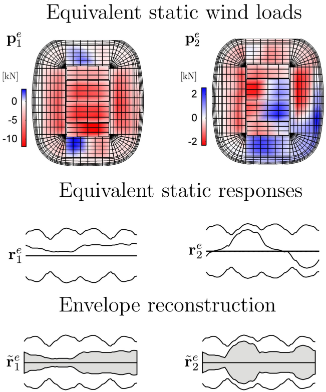
# Equivalent Static Wind Loads

- **ESWL matrix $\mathbf{P}^e$**
  - All investigated structural responses $(N = 30504)$
  - ESWLs are derived with the method by Chen & Kareem

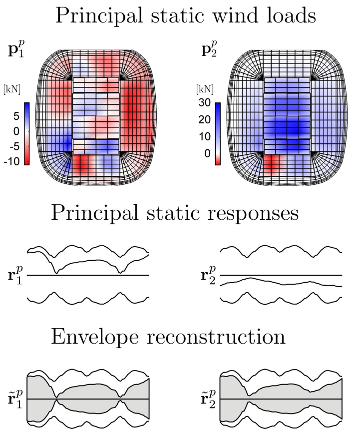
$$\mathbf{P}^e = \underbrace{\begin{pmatrix} p^e_{11} & \cdots & p^e_{1N} \\ \vdots & \ddots & \\ p^e_{m1} & & p^e_{mN} \end{pmatrix}}_{m \times N}$$

# Equivalent Static Wind Loads



# Principal Static Wind Loads



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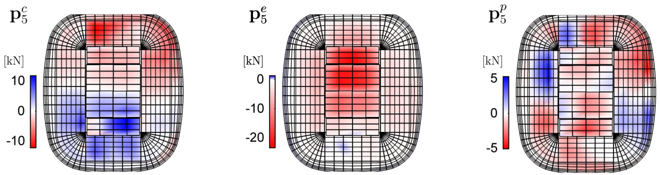
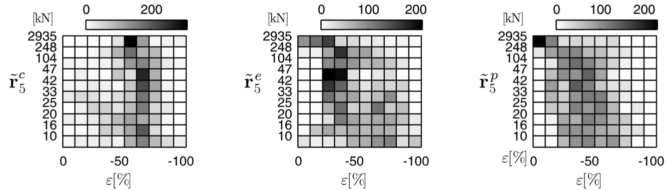
# Without combinations

- $5^{th}$ loading modes

CPT l.m. ESWLs PSWLs

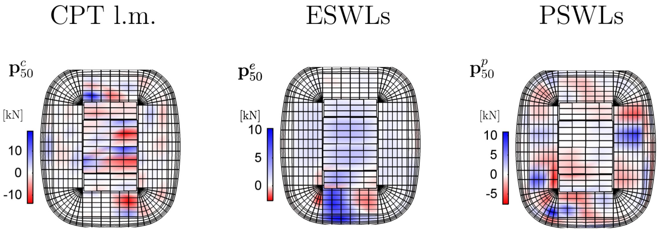



- $5^{th}$ reconstructed envelope (axial force)



# Without combinations

- $50^{th}$ loading modes

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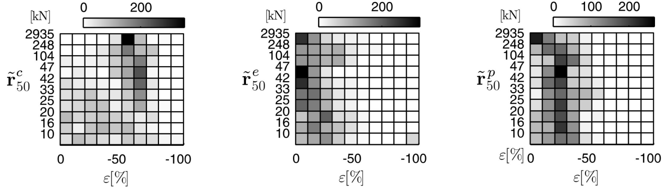

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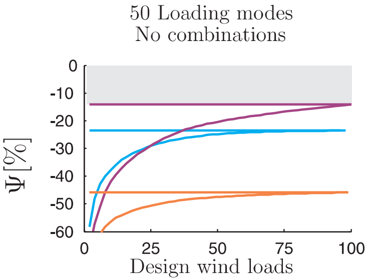



CPT Loading modes ESWLs PSWLs

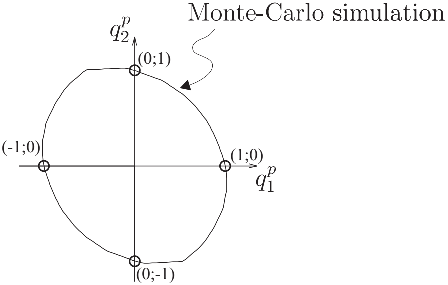
## Combinations

- Combinations $\mathbf{P}^s = \tilde{\mathbf{P}}\mathbf{q}$
  - $\tilde{\mathbf{P}}$ : Limited number of static loadings from the entire basis
  - Speed up the convergence to the real envelope
  - ESWLs : overcome the dependency to unique specific structural responses

# Combinations

- Monte-Carlo simulation+ tangency condition

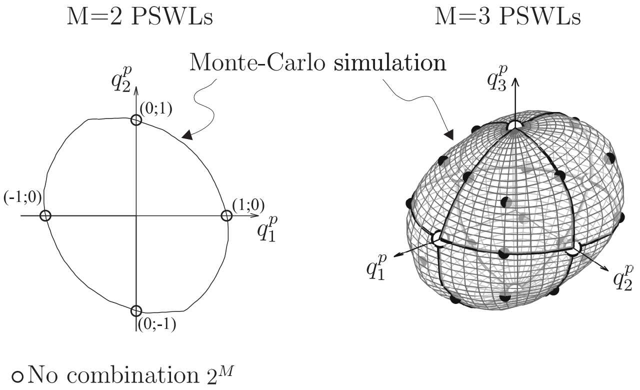
M=2 PSWLs




○ No combination $2^M$

# Combinations

- Extension : three first PSWLs



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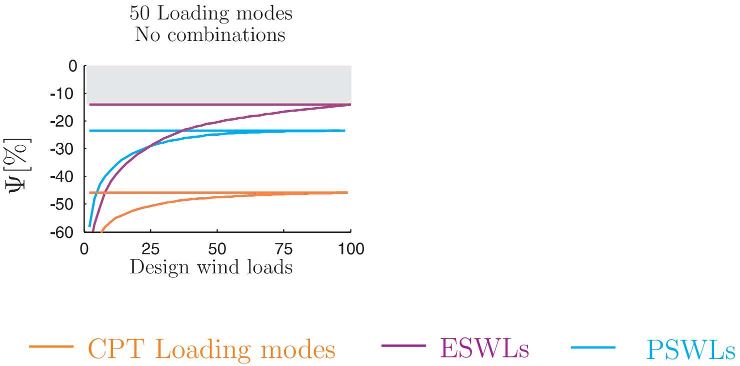

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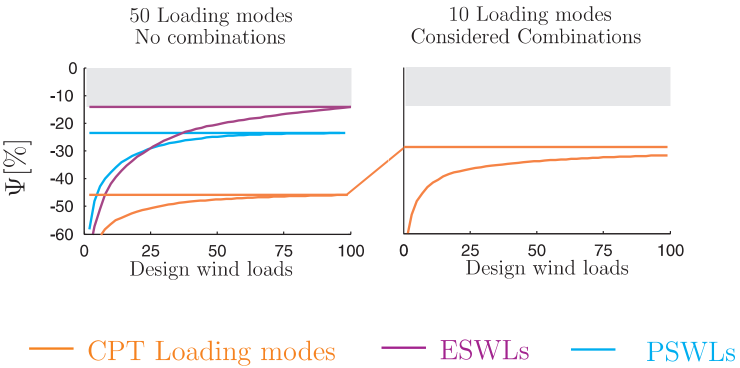

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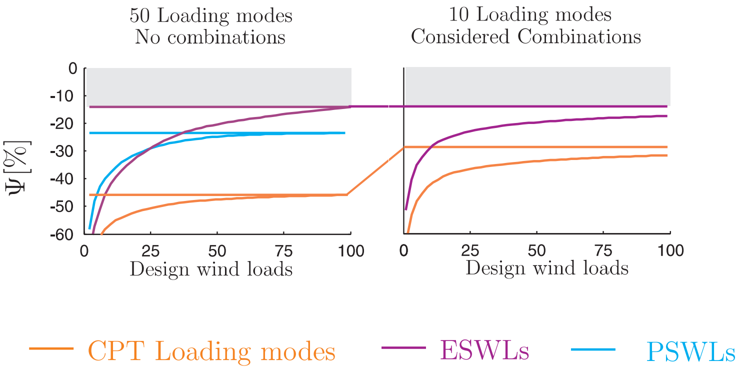

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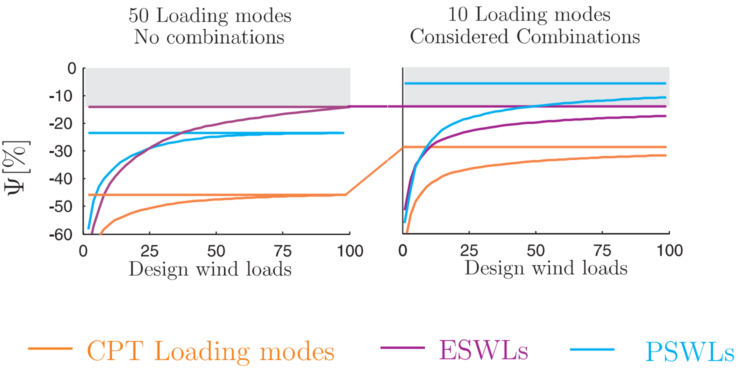
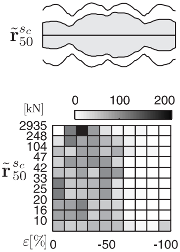
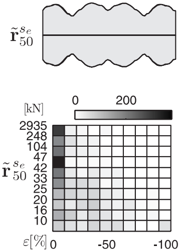


# Combinations

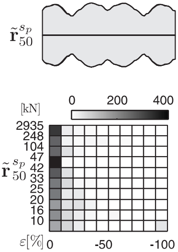
## CPT Loading modes




## ESWLs




## PSWLs



# Combinations

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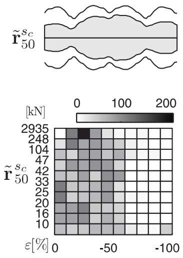


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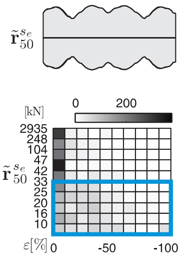


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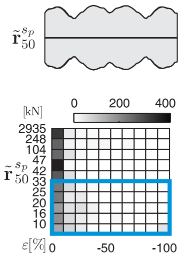


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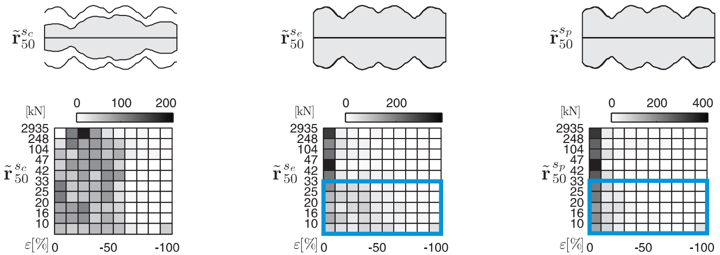
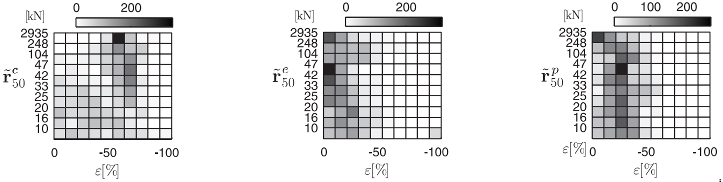
# Combinations

CPT Loading modes | ESWLs | PSWLs




## Without combinations

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- **Topic :** Envelope reconstruction problem
- **Objective** : Comparison of three basis[1]
  - CPT l.m., ESWLs, PSWLs

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Recommend the use of PSWLs

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$\boxed{\text{Recommend the use of PSWLs}}$

- **Main characteristics**
  - Obtained by an automatic procedure (**SVD**) → Robust
  - Minimum number of principal loadings is necessary
  - Represent global loadings
  - Well-suited for combinations
  - Possible codification

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  - Obtained by an automatic procedure (**SVD**) $\rightarrow$ Robust
  - Minimum number of principal loadings is necessary
  - Represent global loadings
  - Well-suited for combinations
  - Possible codification
- **Assumptions** : linear analysis and gaussian responses
- **Perspective** : non gaussian responses

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Thank you for your kind attention.

Questions ?

Read out more about me on : www.orbi.ulg.ac.be

Contact me at : N.Blaise@ulg.ac.be