The Airline Container Loading Problem with Pickup and Delivery

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1 Motivation

2 Problem Description

3 Model
   - Main Parameters and Variables
   - Multi-criteria objective function
   - Constraints

4 Case studies

5 Conclusion and outlooks
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Context of the Research

⇒ Problem Statement:

“How to optimally load a set of containers/pallets (ULDs) into a cargo aircraft that has to serve multiple destinations under some safety, structural, economical and manoeuvrability constraints?”

- Transport of goods by air
- Airlines were among first industries to have used OR methods
- Still numerous challenges
  - Volatility and increasing trend in the oil prices
  - Increasing pressure for greater focus on environmental concerns
  - More attention to spendings
- Load planning has possibilities for costs cutting because it is still a manual task
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4. Conclusion and outlooks
A cargo aircraft has to deliver goods at two consecutive airports\textsuperscript{1}.

The optimal location for all ULDs into the aircraft in order to minimize:

- The fuel consumption during the entire trip
- The loading time at the intermediate destination

\textsuperscript{1}Generalization could be easily done to more than two destinations
Summary of the model

\textbf{Minimize} \quad \sum_{\forall k \in K} (\text{deviation most aft CG}) \text{ and } \# \text{ ULDs to unload}

\textbf{subject to:} \quad 
\begin{align*}
\text{Each ULD is loaded} \\
\text{Each ULD fits in a position} \\
\text{A position accepts only one ULD} \\
\text{Some positions are overlapping: not simultaneously used} \\
\text{Longitudinal stability: The CG is within certified limits} \\
\text{Lateral balance} \\
\text{Maximum weight per position} \\
\text{Combined load limits} \\
\text{Cumulative load limits} \\
\text{Regulations for hazardous goods} \\
\text{Two parts of larger ULDs in adjacent positions} \\
\text{P & D}
\end{align*}

\Rightarrow \text{“Assignment Problem / Combinatorial Problem”} \\
\Rightarrow \text{Integer Linear Problem}
Contribution

Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)
Some models already exist in the scientific and professional literature dealing with optimizing cargo load but...

- Those models are limited
- Most of the time, those models are specific
- They do not analyse the Economic and Ecological aspects
- They do not consider pick-up and delivery (multiple destinations)

Main references for the basic problem (CG)


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Summary of the model

Min $\alpha \times \sum_{i \in K} c_i + \beta \times \sum_{j \in P} n_j$

Subject to:

$CG_k - OCG_k - c_k < 0 \ \forall k \in K$
$CG_k - OCG_k + c_k \geq 0 \ \forall k \in K$
$\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} - \sum_{j : r_i \leq r_j} \tau_{ji} \leq |B_l| - |B_r| \ \forall j \in \mathbb{P}, \ \forall i \in U_k$

$\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} - \sum_{j : r_i \leq r_j} \tau_{ji} \leq 0 \ \forall j \in \mathbb{P}, \ \forall i \in U_k$

Objective Function:

- $\alpha \times \sum_{i \in K} c_i + \beta \times \sum_{j \in P} n_j$
  - OF: Cost consumption
  - OF: Loading time

- $D \leq \sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} - \sum_{j : r_i \leq r_j} \tau_{ji} \leq D$
  - Longitudinal stability

- $D \leq \sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} - \sum_{j : r_i \leq r_j} \tau_{ji} \leq 0$
  - Lateral stability

- $\sum_{i \in (U_l, U_h)} \tau_{ij} \leq 1 \ \forall i \in P$
  - Allowable positions

- $\sum_{i \in (U_l, U_h)} \tau_{ij} \leq 1 \ \forall i \in P$
  - Full load

- $w_i \times x_{ij} \leq W_i \ \forall i \in (U_l, U_h), \ \forall j \in P$
  - Weight restrictions
- $w_i \times x_{ij} \leq W_i \ \forall i \in (U_l, U_h), \ \forall j \in P$

- $\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} \leq Q_i \ \forall i \in \mathbb{P}$
  - P & D

- $\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} \leq Q_i \ \forall i \in \mathbb{P}$
  - Hazardous goods

- $\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} \leq T_i \ \forall i \in \mathbb{P}$
  - Larger ULDs

- $\sum_{i \in (U_l, U_h)} \sum_{j : r_i \leq r_j} \tau_{ij} \leq T_i \ \forall i \in \mathbb{P}$
  - $x_{ij} = 0 \ \forall i \notin (U_l, U_h), \ \forall j \in P$
  - $x_{ij} = 0 \ \forall i \notin (U_l, U_h), \ \forall j \in P$

- $x_{ij} - x_{ji} \leq s_j \ \forall i \in U_k, \ \forall j \in P$

- $|x_{ij} - x_{ji}| \leq |B_l| - |B_r| \ \forall j \in \mathbb{P}, \ \forall i \in U_k$

- $x_{ij} \leq n_j \ \forall i \in U_k, \ \forall j \in P$
  - $x_{ij} \leq n_j \ \forall i \in U_k, \ \forall j \in P$

- $x_{ij} \leq |B_l| - |B_r| \ \forall j \in \mathbb{P}, \ \forall i \in U_k$
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Main Parameters and Variables

- **Set of routes** $K$
- **Set of ULDs** $U$
  - According to their origin and destination: three subsets of ULDs: $U_1$, $U_2$, $U_3$
- **Set of positions** $P$
  - There is only one central door situated at the extremity of the aircraft

Binary Variables

$$x_{ijk} = \begin{cases} 
1 & \text{if ULD } i \text{ is in position } j \text{ during the route } k \\
0 & \text{otherwise}
\end{cases}$$
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Objective function: Most Aft CG

In terms of fuel consumption, the optimal location for the CG is the most aft within certified limits.

In mathematical terms, it gives:

$$\text{Min} \sum_{\forall k \in K} \epsilon_k$$

Subject to:

$$\begin{align*}
CG_k - OCG_k - \epsilon_k & \leq 0 \\
CG_k - OCG_k + \epsilon_k & \geq 0
\end{align*} \ \forall k \in K$$

where:

- $CG_k$ is the CG obtained after assignment of ULDs in the aircraft during the route $k$
- $OCG_k$ is the optimal CG, i.e. most aft CG on the route $k$
Objective function: minimize $\# \text{ ULDs to Unload}$

The loading time is function of the $\#$ of ULDs to be unloaded. We want to prevent the unnecessary unloads at the intermediate destination.

In mathematical terms, it gives:

$$\text{Min } \sum_{j \in P} n_j$$

Subject to:

$$\sum_{i' \in U_1} \sum_{j' \in B_j} x_{i'j'1} - n_j \times |B_j| - (1 - x_{ij1}) \times |B_j| \leq 0 \quad \forall j \in P, \forall i \in U_3$$

where:

- $n_j$ is a binary variable
- $B_j$ is the set of all position situated behind $j$ relative to the door
In definitive, both objectives have to be considered together:

\[
\text{Min } E(\alpha) \times \sum_{\forall k \in K} r_k + E(\beta) \times \sum_{\forall j \in P} n_j
\]

Subject to:

\[
\sum_{\forall i' \in U_1} \sum_{\forall j' \in B_j} x_{i'j'1} - n_j \times |B_j| - (1 - x_{ij1}) \times |B_j| \leq 0 \quad \forall j \in P, \quad \forall i \in U_3
\]

\[
\begin{align*}
CG_k - OCG_k - \epsilon_k & \leq 0 \\
CG_k - OCG_k + \epsilon_k & \geq 0
\end{align*}
\] \quad \forall k \in K

where:
- $\alpha$ is the additional cost (fuel + emissions) for a deviation of one inch from the most aft CG
- $\beta$ is the cost associated with the loading time of one ULD (in terms of wages, fees to the airport for the usage of the runway...)
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Cargo Load Planning

Model

Constraints

Standard Constraints

Stability

Longitudinal stability: CG within certified limits
Lateral balance of the aircraft

Full Load

Each ULD is loaded

Allowable positions

Each ULD fits in a position
A position accepts only one ULD
Overlapping positions: not simultaneously used

Weights restrictions

Maximum weight per position
Combined load limits
Cumulative load limits
Specific Constraints

Routes constraints

\[ x_{ij1} = 0 \quad \forall i \notin (U_1 \cup U_3), \forall j \in P \]
\[ x_{ij2} = 0 \quad \forall i \notin (U_2 \cup U_3), \forall j \in P \]

Same position for ULDs not unloaded

\[ x_{ij1} - x_{ij2} \leq n_j \quad \forall i \in U_3, \forall j \in P \]
\[ x_{ij2} - x_{ij1} \leq n_j \quad \forall i \in U_3, \forall j \in P \]

Allowable positions for ULDs loaded at the intermediate destination

\[ \sum_{i' \in U_2} \sum_{j' \in B_j} x_{i'j'2} + (x_{ij1} - n_j)|B_j| \leq |B_j| \quad \forall j \in P, \forall i \in U_3 \]
Specific Constraints

Changing the position of an ULD with minimal unloading

\[ x_{ij'1} - n'_j + n_j \leq 1 \quad \forall i \in U_3, \forall j \in P, \forall j' \in P \mid j' \text{ is before } j \]

Hazardous goods

\[ x_{ij1} + x_{i'j'1} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (U_1 \cup U_3), \text{ and } \forall j, j' \in P \]
\[ x_{ij2} + x_{i'j'2} \leq 1 \quad \forall i, i', j, j' \mid d_{jj'} \leq e_{ii'}; \forall i, i' \in (U_2 \cup U_3), \text{ and } \forall j, j' \in P \]

ULDs of larger dimensions

\[ x_{ij1} \leq \sum_{\forall j' \in A_j} x_{i'j'1} \quad \forall i \in (U_1 \cup U_3), \forall i' \in L_i, \forall j \in P \]
\[ x_{ij2} \leq \sum_{\forall j' \in A_j} x_{i'j'2} \quad \forall i \in (U_2 \cup U_3), \forall i' \in L_i, \forall j \in P \]
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The case of a Boeing 747²

Data

- Parameters alpha and beta equal to 1
- 38 ULDs distributed as follows:

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Number of ULDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA</td>
<td>LGG</td>
<td>28</td>
</tr>
<tr>
<td>FRA</td>
<td>LHR</td>
<td>10</td>
</tr>
<tr>
<td>LGG</td>
<td>LHR</td>
<td>/</td>
</tr>
</tbody>
</table>

- 67 standard positions, plus 10 larger ones

Results

- All constraints satisfied
- Deviations CG’s: 0 and 0.0028
- # ULDs unloaded: 5
- Computation time: 18’21”

²Model implemented in Java using IBM ILOG CPLEX 12 and tested on real data with personal computer (Windows 7, Intel Core i5-2450M, 2.50GHz, 8.00 GB of RAM)
Additional results

- Small instances (15 ULDs) and Larger instances (35 ULDs)
- Same set of data
- Variations of the origin and destination (randomly)

Results (15 tests for each)

<table>
<thead>
<tr>
<th></th>
<th>Small instances</th>
<th>Large instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status of solution</td>
<td>Optimal</td>
<td>Optimal</td>
</tr>
<tr>
<td>Longest computation</td>
<td>17’</td>
<td>nearly 8 hours</td>
</tr>
</tbody>
</table>

General trends
- The computation time ↗ when more ULDs present on several routes
- But it also depends on the number of position accepted by each ULD
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Conclusion and outlooks

- The goal of this paper was the development of a mixed integer linear programming model for loading optimally a set of unit load devices into a cargo aircraft that visits successively two airports.

- This sequence of destinations had never been considered in the literature before us.

- We can summarize results of our research as follows:

<table>
<thead>
<tr>
<th>Objective</th>
<th>Implies</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min fuel consumption</td>
<td>CG’s close to the aft limit</td>
<td>√</td>
</tr>
<tr>
<td>Min loading time</td>
<td>Minimizing the unloads</td>
<td>√</td>
</tr>
</tbody>
</table>

- Better results in terms of both fuel consumption and loading time.
- Computation time ↑ when the distribution of ULDs becomes more balanced
Conclusion and outlooks

To do list

✓ Mathematical formulation of the model

✓ Implementation of the model

✓ Tests on real instances

□ Consideration of lateral doors

□ Complexity of the model

□ Development of Heuristics?
Contact me

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Thank you for your attention!