

# Non-local multiscale analyzes of composite laminates based on a damage-enhanced mean-field homogenization formulation

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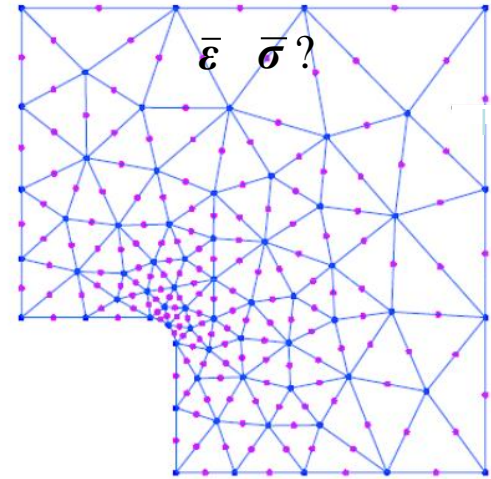
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- Introduction
  - Mean-field homogenization Multi-scale modelling
- Mean-Field-Homogenization with non-local damage
  - Incremental secant approach idea
  - Non-local damage-enhanced incremental secant approach
- Finite-element implementation
  - Direct resolution
  - Staggered resolution
- Applications
  - Laminate with unloading
  - Laminate with a hole
- Conclusions

- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point  $\bar{\epsilon}$  is known,  $\bar{\sigma}$  is sought

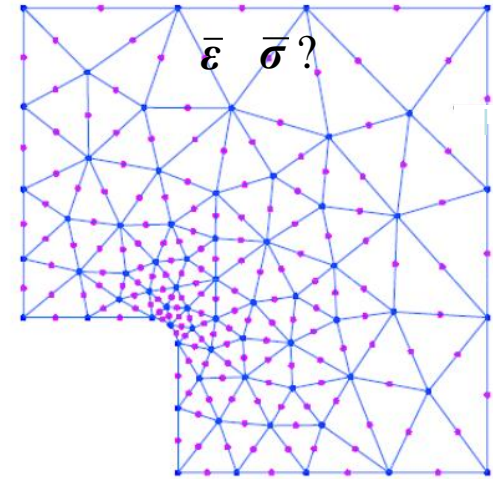


# Multi-scale modelling: How?

- Multiscale methods

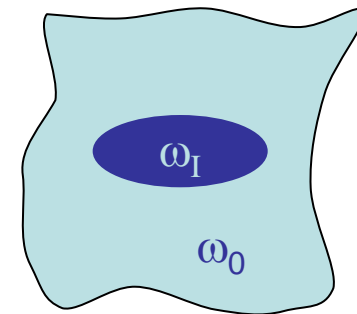
- Macro-scale

- FE model
    - At one integration point  $\bar{\epsilon}$  is known,  $\bar{\sigma}$  is sought



- Micro-scale

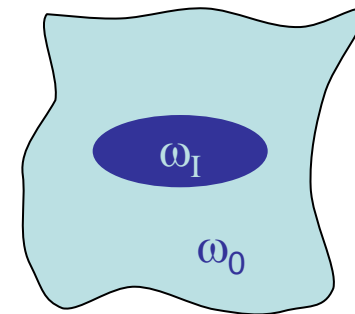
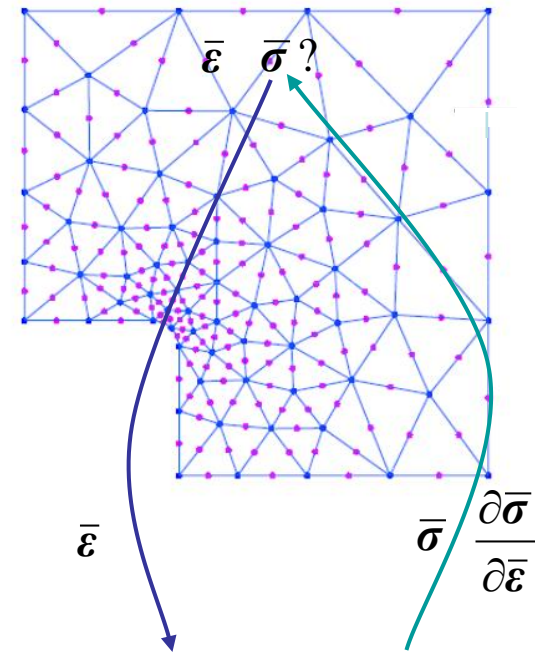
- Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



# Multi-scale modelling: How?

- Multiscale methods

- Macro-scale
  - FE model
  - At one integration point  $\bar{\epsilon}$  is known,  $\bar{\sigma}$  is sought
- Transition
  - Downscaling:  $\bar{\epsilon}$  is used as input of the MFH model
  - Upscaling:  $\bar{\sigma}$  is the output of the MFH model
- Micro-scale
  - Semi-analytical model
  - Predict composite meso-scale response
  - From components material models



**Assumptions:**

$$L_{\text{macro}} \gg L_{\text{RVE}} \gg L_{\text{micro}}$$

- Semi analytical Mean-Field Homogenization

- Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V} \int_V a(\mathbf{X}) dV$$

- Meso-response

- From the volume ratios ( $v_0 + v_I = 1$ )

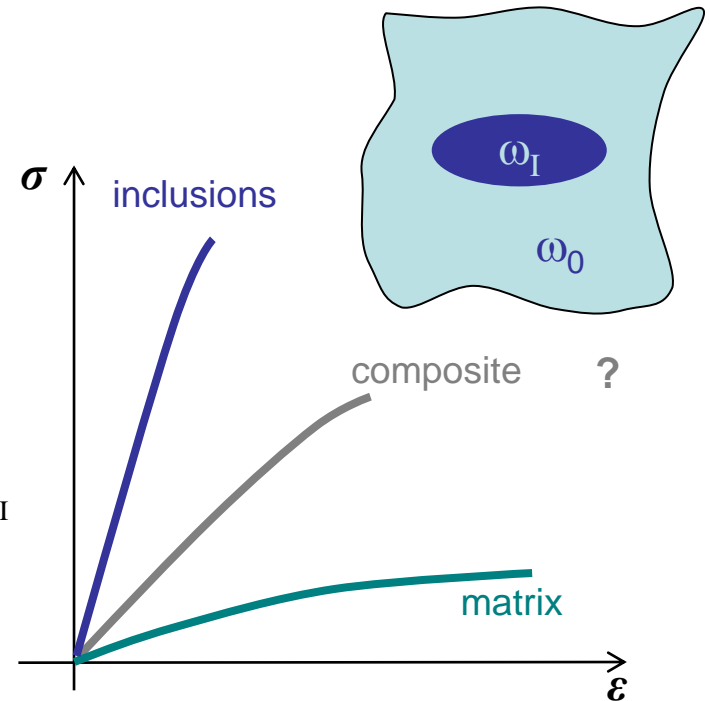
$$\begin{cases} \bar{\sigma} = \langle \sigma \rangle = v_0 \langle \sigma \rangle_{\omega_0} + v_I \langle \sigma \rangle_{\omega_I} = v_0 \sigma_0 + v_I \sigma_I \\ \bar{\varepsilon} = \langle \varepsilon \rangle = v_0 \langle \varepsilon \rangle_{\omega_0} + v_I \langle \varepsilon \rangle_{\omega_I} = v_0 \varepsilon_0 + v_I \varepsilon_I \end{cases}$$

- One more equation required

$$\varepsilon_I = \mathbf{B}^\varepsilon : \varepsilon_0$$

- Difficulty: find the adequate relations

$$\begin{cases} \sigma_I = f(\varepsilon_I) \\ \sigma_0 = f(\varepsilon_0) \\ \varepsilon_I = \mathbf{B}^\varepsilon : \varepsilon_0 \end{cases} \quad \mathbf{B}^\varepsilon ?$$



- Mean-Field Homogenization for different materials

- Linear materials

- Materials behaviours

$$\begin{cases} \sigma_I = \bar{C}_I : \varepsilon_I \\ \sigma_0 = \bar{C}_0 : \varepsilon_0 \end{cases}$$

- Mori-Tanaka assumption  $\varepsilon^\infty = \varepsilon_0$

- Use Eshelby result

$$\varepsilon_I = \mathbf{B}^\varepsilon(\mathbf{I}, \bar{C}_0, \bar{C}_I) : \varepsilon_0$$

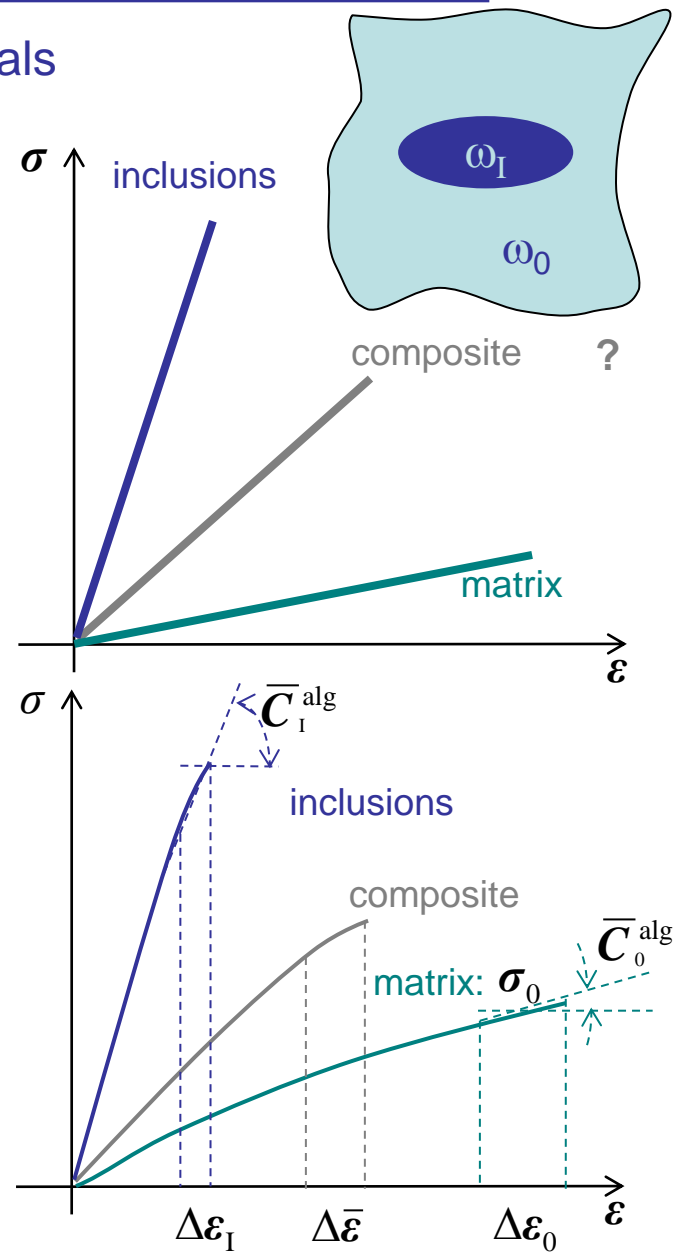
$$\text{with } \mathbf{B}^\varepsilon = [\mathbf{I} + \mathbf{S} : \bar{C}_0^{-1} : (\bar{C}_I - \bar{C}_0)]^{-1}$$

- Non-linear materials

- Define a Linear Comparison Composite

- Common approach: incremental tangent

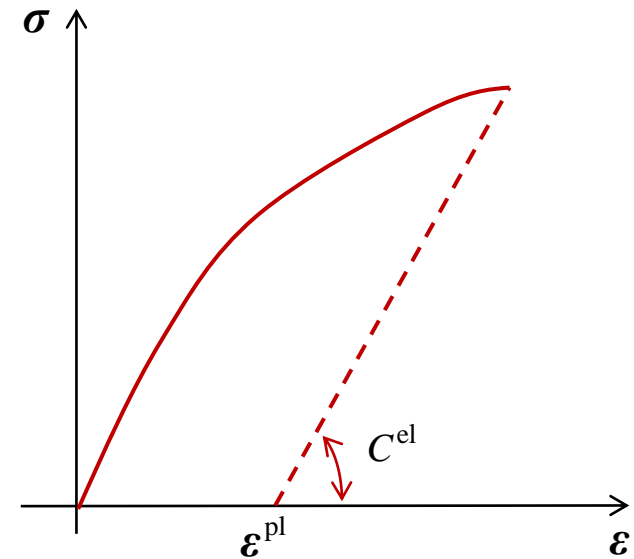
$$\Delta \varepsilon_I = \mathbf{B}^\varepsilon(\mathbf{I}, \bar{C}_0^{\text{alg}}, \bar{C}_I^{\text{alg}}) : \Delta \varepsilon_0$$



- Material models

- Elasto-plastic material

- Stress tensor  $\boldsymbol{\sigma} = \mathbf{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface  $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^Y - R(p) \leq 0$
    - Plastic flow  $\Delta \boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \mathbf{N} \quad \& \quad \mathbf{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization  $\delta \boldsymbol{\sigma} = \mathbf{C}^{\text{alg}} : \delta \boldsymbol{\varepsilon}$





# Mean-Field-Homogenization with non-local damage

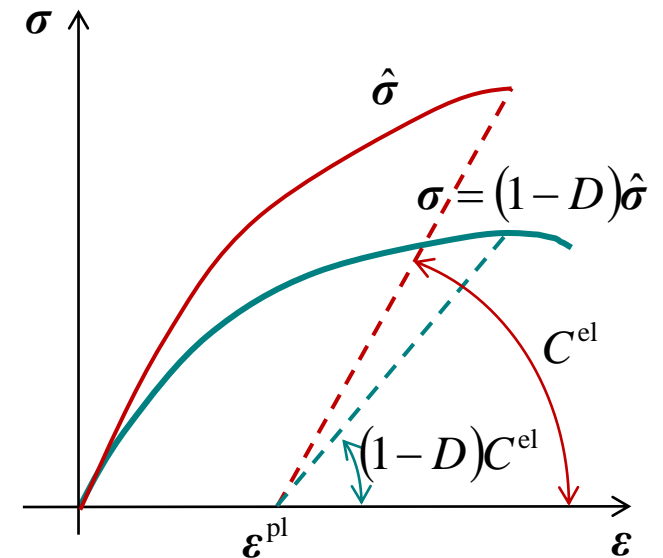
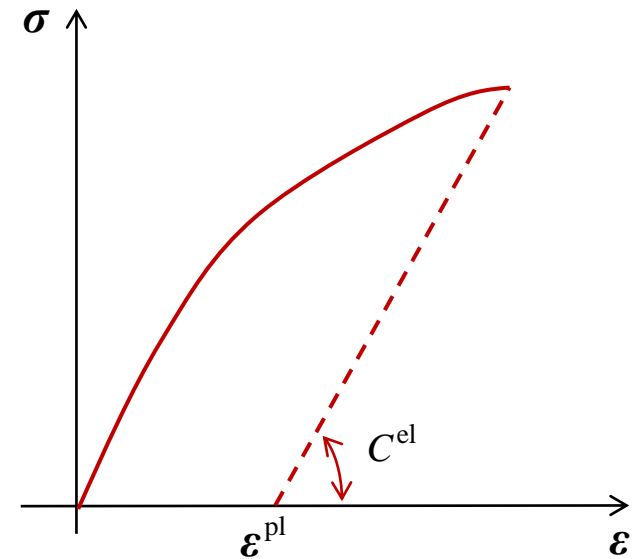
- Material models

- Elasto-plastic material

- Stress tensor  $\boldsymbol{\sigma} = \mathbf{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface  $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \boldsymbol{\sigma}^Y - R(p) \leq 0$
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    - Linearization  $\delta \boldsymbol{\sigma} = \mathbf{C}^{\text{alg}} : \delta \boldsymbol{\varepsilon}$

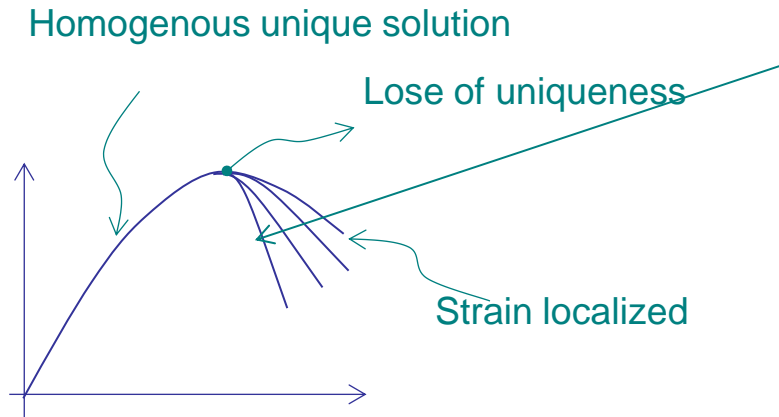
- Local damage model

- Apparent-effective stress tensors  $\boldsymbol{\sigma} = (1 - D) \hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution  $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$

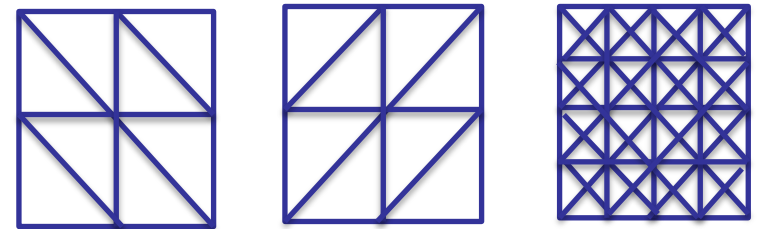


# Mean-Field-Homogenization with non-local damage

- Finite element solutions for strain softening problems suffer from:
  - The loss the uniqueness and strain localization
  - Mesh dependence



The numerical results change with the size of mesh and direction of mesh



The numerical results change without convergence

- **Implicit non-local approach** [Peerlings et al 96, Geers et al 97, ...]
  - A state variable is replaced by a non-local value reflecting the interaction between neighboring material points

$$\tilde{a}(\mathbf{x}) = \frac{1}{V_C} \int_{V_C} a(\mathbf{y}) w(\mathbf{y}; \mathbf{x}) dV$$

- Use Green functions as weight  $w(\mathbf{y}; \mathbf{x})$

➡  $\tilde{a} - c \nabla^2 \tilde{a} = a$  ➡ New degrees of freedom

# Mean-Field-Homogenization with non-local damage

## Material models

### – Elasto-plastic material

- Stress tensor  $\boldsymbol{\sigma} = \mathbf{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
- Yield surface  $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \boldsymbol{\sigma}^Y - R(p) \leq 0$
- Plastic flow  $\Delta \boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \mathbf{N} \quad \& \quad \mathbf{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
- Linearization  $\delta \boldsymbol{\sigma} = \mathbf{C}^{\text{alg}} : \delta \boldsymbol{\varepsilon}$

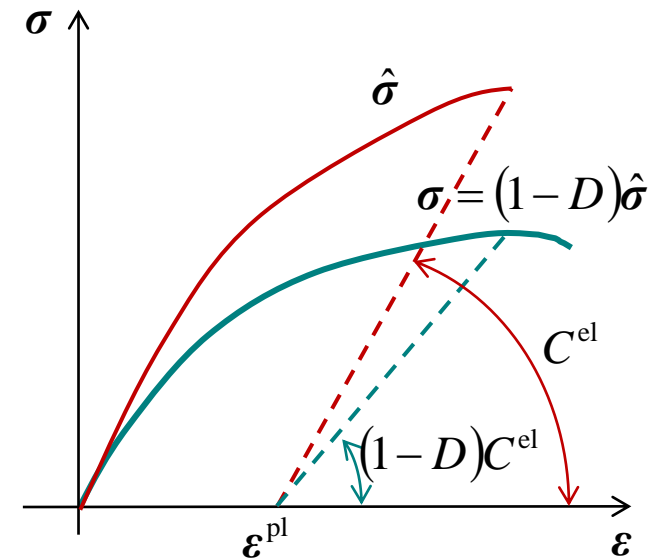
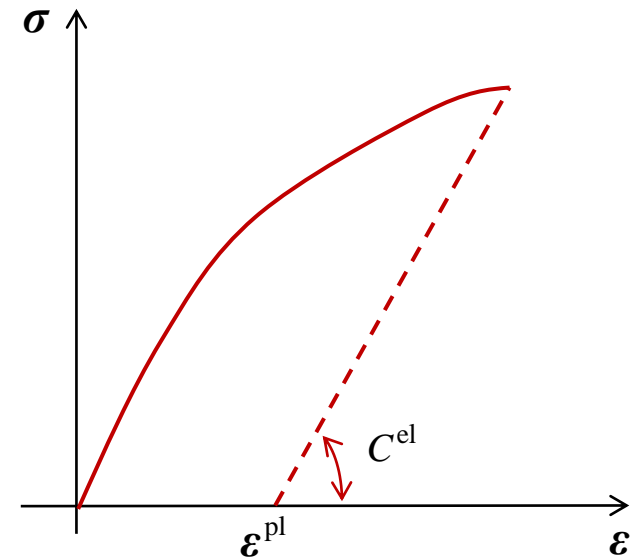
### – Local damage model

- Apparent-effective stress tensors  $\boldsymbol{\sigma} = (1 - D) \hat{\boldsymbol{\sigma}}$
- Plastic flow in the effective stress space
- Damage evolution  $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$

### – Non-Local damage model

- Damage evolution  $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta \tilde{p})$
- Anisotropic governing equation  $\tilde{p} - \nabla \cdot (\mathbf{c}_g \cdot \nabla \tilde{p}) = p$
- Linearization

$$\delta \boldsymbol{\sigma} = \left[ (1 - D) \mathbf{C}^{\text{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}} \right] : \delta \boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta \tilde{p}$$



# Mean-Field-Homogenization with non-local damage

- Problem

- We want the fibres to get unloaded during the matrix damaging process

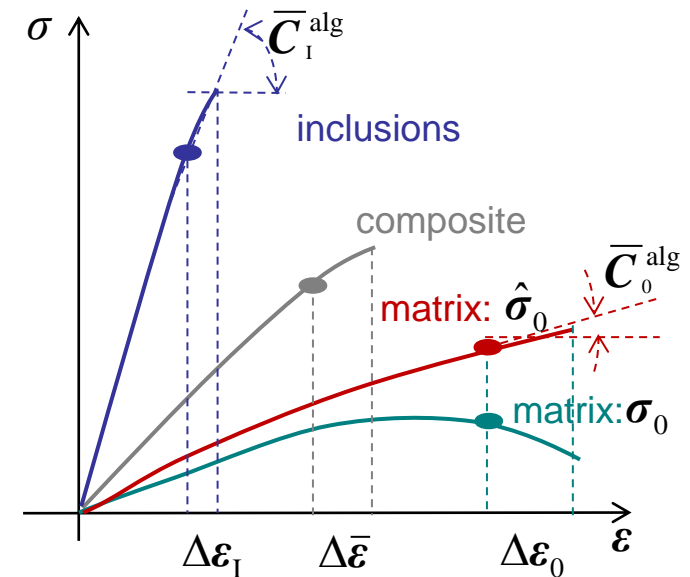
- For the incremental-tangent approach

$$\Delta \boldsymbol{\varepsilon}_I = \mathbf{B}^\varepsilon \left( \mathbf{I}, (1-D) \bar{\mathbf{C}}_0^{\text{alg}}, \bar{\mathbf{C}}_I^{\text{alg}} \right) : \Delta \boldsymbol{\varepsilon}_0$$

- To unload the fibres (  $\boldsymbol{\varepsilon}_I < 0$  ) with such approach would require  $\bar{\mathbf{C}}_I^{\text{alg}} < 0$

- We cannot use the incremental tangent MFH

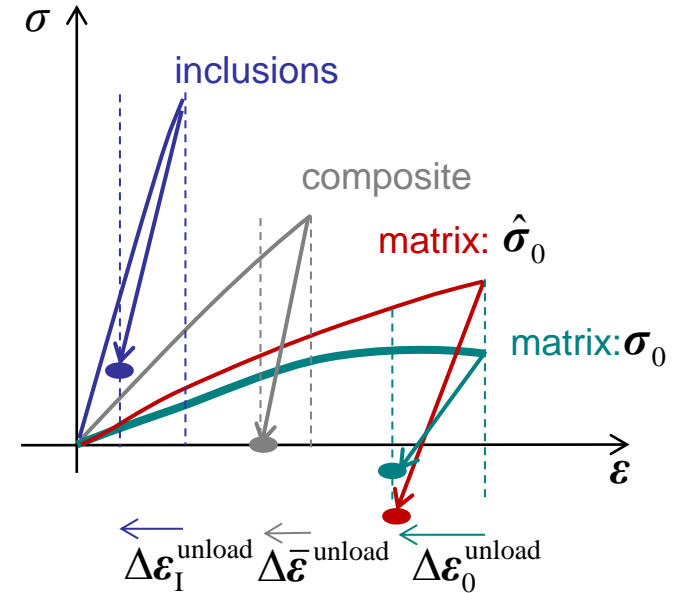
- We need to define the LCC from another stress state



# Mean-Field-Homogenization with non-local damage

- Idea

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components



# Mean-Field-Homogenization with non-local damage

- Idea

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

- Apply MFH from unloaded state
  - New strain increments ( $>0$ )

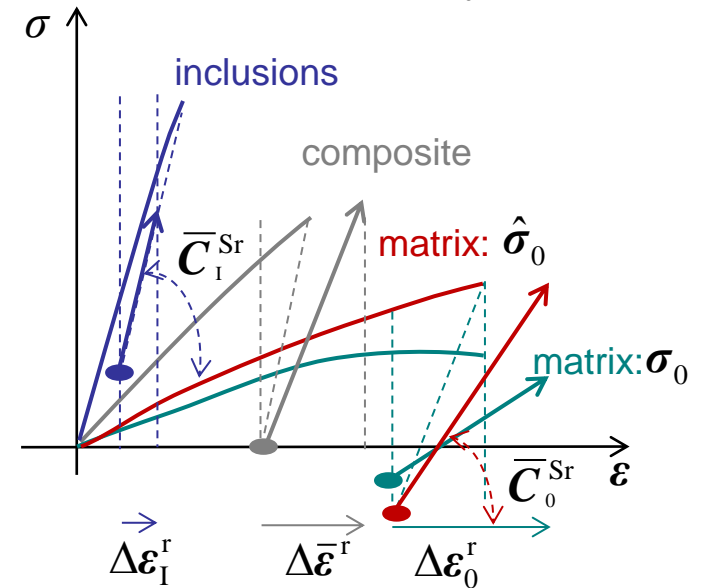
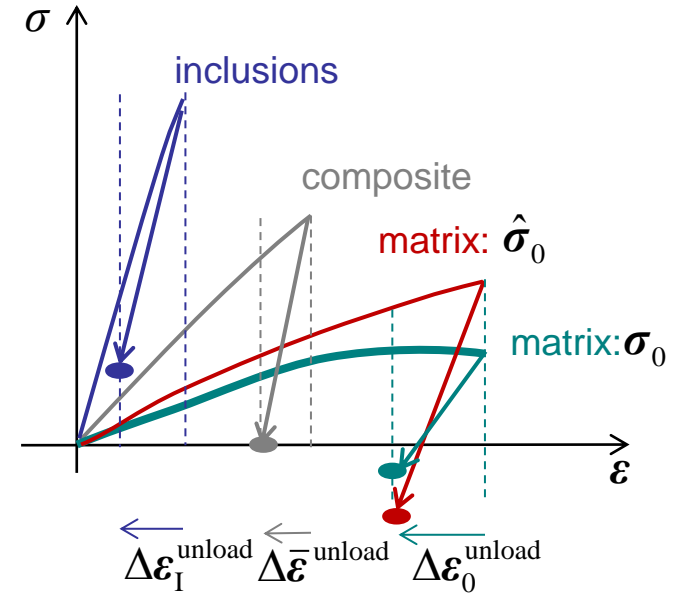
$$\Delta \boldsymbol{\varepsilon}_{I/0}^r = \Delta \boldsymbol{\varepsilon}_{I/0} + \Delta \boldsymbol{\varepsilon}_{I/0}^{\text{unload}}$$

- Use of secant operators

$$\Delta \boldsymbol{\varepsilon}_I^r = \mathbf{B}^\varepsilon \left( \mathbf{I}, (1-D) \bar{\mathbf{C}}_0^{\text{Sr}}, \bar{\mathbf{C}}_I^{\text{Sr}} \right) : \Delta \boldsymbol{\varepsilon}_0^r$$

- Possibility of have unloading

$$\begin{cases} \Delta \boldsymbol{\varepsilon}_I^r > 0 \\ \Delta \boldsymbol{\varepsilon}_I < 0 \end{cases}$$



# Mean-Field-Homogenization with non-local damage

- New incremental-secant approach

- Equations summary

- Inputs

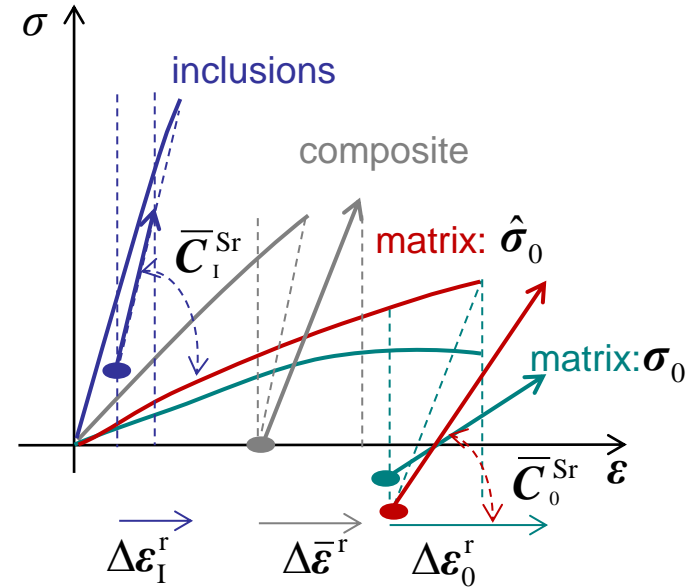
- Internal variable at last increment
- Residual tensor after virtual unloading
- $\Delta\bar{\boldsymbol{\varepsilon}}, \Delta\tilde{\boldsymbol{p}}$  from FE resolution

- Solve iteratively the system

$$\left\{ \begin{array}{l} \Delta\bar{\boldsymbol{\varepsilon}}^{(r)} = v_0\Delta\boldsymbol{\varepsilon}_0^{(r)} + v_I\Delta\boldsymbol{\varepsilon}_I^{(r)} \\ \Delta\boldsymbol{\varepsilon}_I^r = \Delta\boldsymbol{\varepsilon}_I + \Delta\boldsymbol{\varepsilon}_I^{\text{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^r = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\text{unload}} \\ \Delta\boldsymbol{\varepsilon}_I^r = \mathbf{B}^\varepsilon \left( \mathbf{I}, (1-D)\bar{\mathbf{C}}_0^{\text{Sr}}, \bar{\mathbf{C}}_I^{\text{Sr}} \right) : \Delta\boldsymbol{\varepsilon}_0^r \end{array} \right.$$

- With the stress tensors

$$\left\{ \begin{array}{l} \bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_I\boldsymbol{\sigma}_I \\ \boldsymbol{\sigma}_I = \boldsymbol{\sigma}_I^{\text{res}} + \bar{\mathbf{C}}_I^{\text{Sr}} : \Delta\boldsymbol{\varepsilon}_I^r \\ \boldsymbol{\sigma}_0 = (1-D)\hat{\boldsymbol{\sigma}}_0^{\text{res}} + (1-D)\bar{\mathbf{C}}_0^{\text{Sr}} : \Delta\boldsymbol{\varepsilon}_0^r \end{array} \right.$$



# Mean-Field-Homogenization with non-local damage

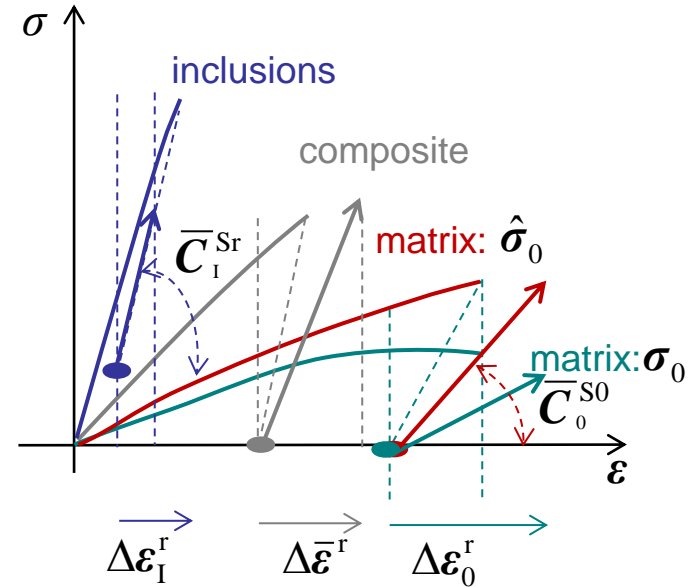
- New incremental-secant approach (2)
  - Alternative
    - For soft matrix response
      - Remove residual stress in matrix
      - Avoid adding spurious internal energy

- Solve iteratively the system

$$\left\{ \begin{array}{l} \Delta \bar{\boldsymbol{\varepsilon}}^{(r)} = v_0 \Delta \boldsymbol{\varepsilon}_0^{(r)} + v_I \Delta \boldsymbol{\varepsilon}_I^{(r)} \\ \Delta \boldsymbol{\varepsilon}_I^r = \Delta \boldsymbol{\varepsilon}_I + \Delta \boldsymbol{\varepsilon}_I^{\text{unload}} \\ \Delta \boldsymbol{\varepsilon}_0^r = \Delta \boldsymbol{\varepsilon}_0 + \Delta \boldsymbol{\varepsilon}_0^{\text{unload}} \\ \Delta \boldsymbol{\varepsilon}_I^r = \mathbf{B}^\varepsilon \left( \mathbf{I}, (1-D) \bar{\mathbf{C}}_0^{S0}, \bar{\mathbf{C}}_I^{\text{Sr}} \right) : \Delta \boldsymbol{\varepsilon}_0^r \end{array} \right.$$

- With the stress tensors

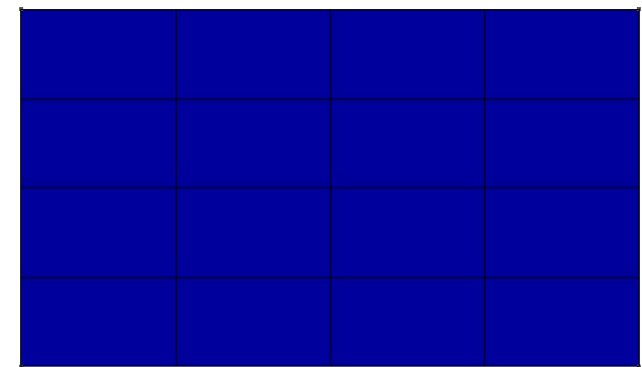
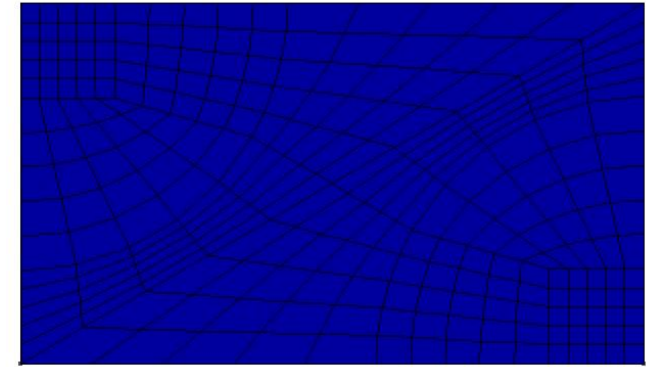
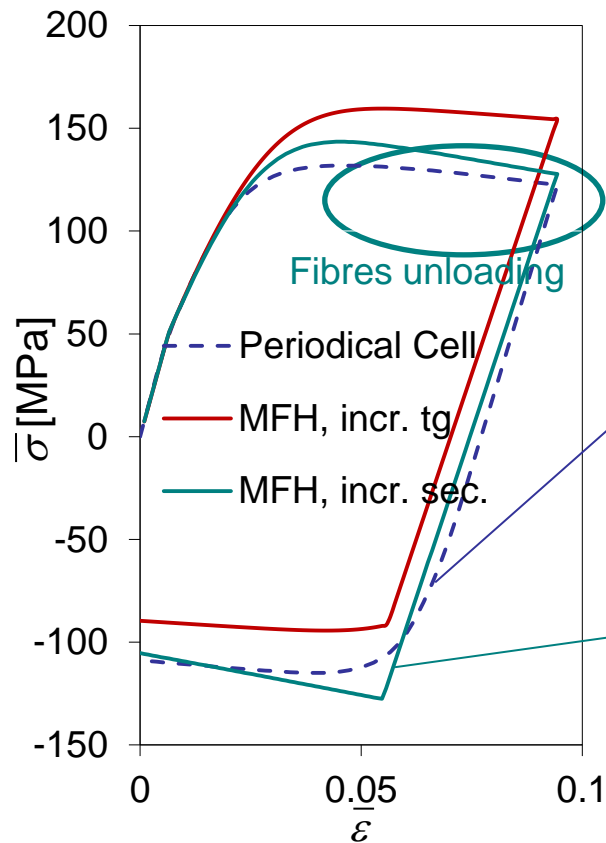
$$\left\{ \begin{array}{l} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_I \boldsymbol{\sigma}_I \\ \boldsymbol{\sigma}_I = \boldsymbol{\sigma}_I^{\text{res}} + \bar{\mathbf{C}}_I^{\text{Sr}} : \Delta \boldsymbol{\varepsilon}_I^r \\ \boldsymbol{\sigma}_0 = (1-D) \bar{\mathbf{C}}_0^{S0} : \Delta \boldsymbol{\varepsilon}_0^r \end{array} \right.$$





# Mean-Field-Homogenization with non-local damage

- New results for damage
  - Fictitious composite
    - 50%-UD fibres
    - Analyse phases behaviours



- Weak formulation

- Strong form

$$\begin{cases} \nabla \cdot \bar{\boldsymbol{\sigma}}^T + \mathbf{f} = \mathbf{0} & \text{for the homogenized composite material} \\ \tilde{p} - \nabla \cdot (\mathbf{c}_g \cdot \nabla \tilde{p}) = p & \text{for the matrix phase} \end{cases}$$

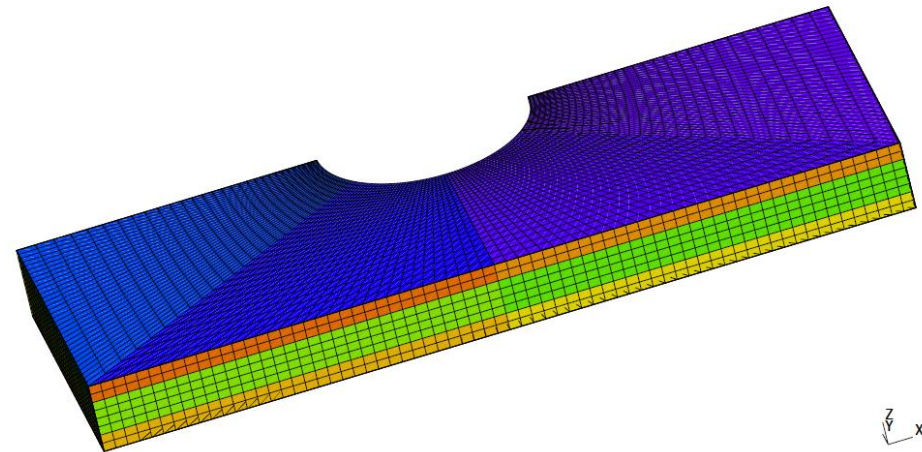
- Boundary conditions

$$\begin{cases} \boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{T} \\ \mathbf{n} \cdot (\mathbf{c}_g \cdot \nabla \tilde{p}) = 0 \end{cases}$$

- Finite-element discretization

$$\begin{cases} \tilde{p} = N_{\tilde{p}}^a \tilde{p}^a \\ \mathbf{u} = N_u^a \mathbf{u}^a \end{cases}$$

$$\Rightarrow \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\tilde{p}} \\ \mathbf{K}_{\tilde{p}u} & \mathbf{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\mathbf{u} \\ d\tilde{p} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\text{ext}} - \mathbf{F}_{\text{int}} \\ \mathbf{F}_p - \mathbf{F}_{\tilde{p}} \end{bmatrix}$$



- Resolution strategies
  - Fully coupled resolution

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\tilde{p}} \\ \mathbf{K}_{\tilde{p}u} & \mathbf{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\mathbf{u} \\ d\tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\text{ext}} - \mathbf{F}_{\text{int}} \\ \mathbf{F}_p - \mathbf{F}_{\tilde{p}} \end{bmatrix}$$

- Staggered dynamic resolution
  - Explicit resolution of the displacement dofs

$$\ddot{\mathbf{u}}^{n+1} = \frac{1}{1 - \alpha_M} \mathbf{M} \left[ \mathbf{F}_{\text{ext}}^n - \mathbf{F}_{\text{int}}^n \right] - \frac{\alpha_M}{1 - \alpha_M} \mathbf{u}^n$$

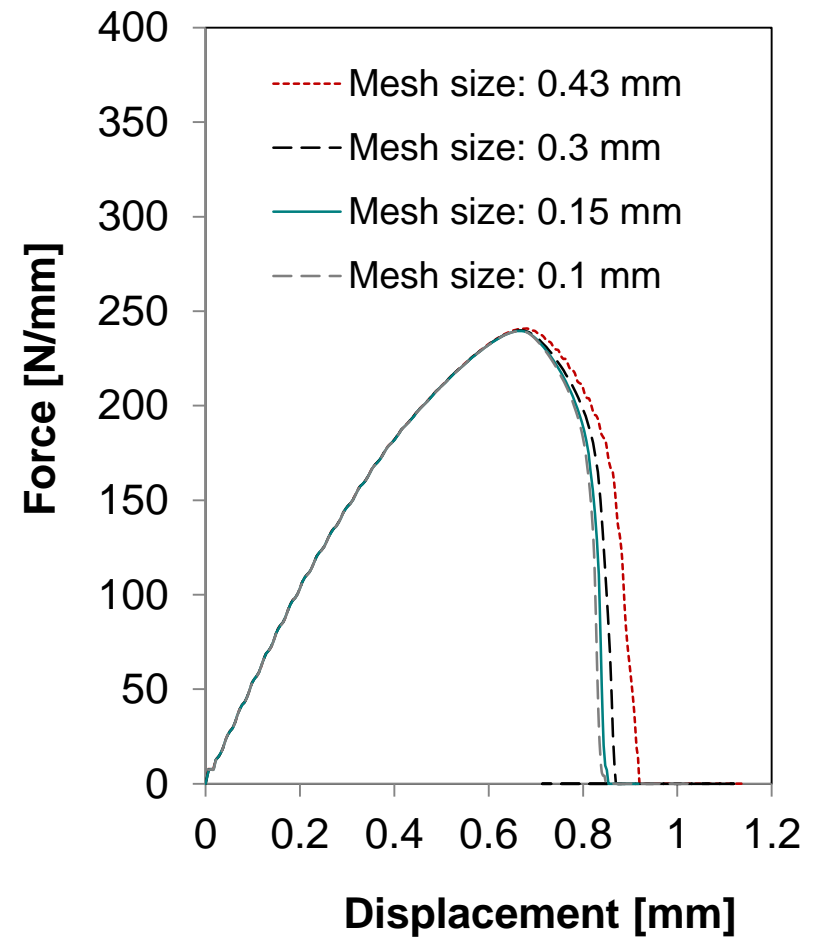
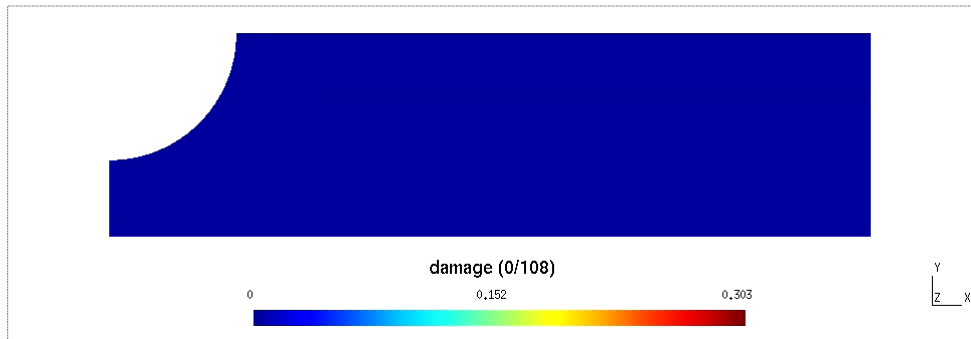
$$\dot{\mathbf{u}}^{n+1} = \dot{\mathbf{u}}^n + \Delta t [1 - \gamma_M] \ddot{\mathbf{u}}^n + \Delta t \gamma_M \ddot{\mathbf{u}}^{n+1}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^n + \Delta t \dot{\mathbf{u}}^n + \Delta t^2 \left[ \frac{1}{2} - \beta_M \right] \ddot{\mathbf{u}}^{n+1} + \Delta t^2 \beta_M \ddot{\mathbf{u}}^{n+1}$$

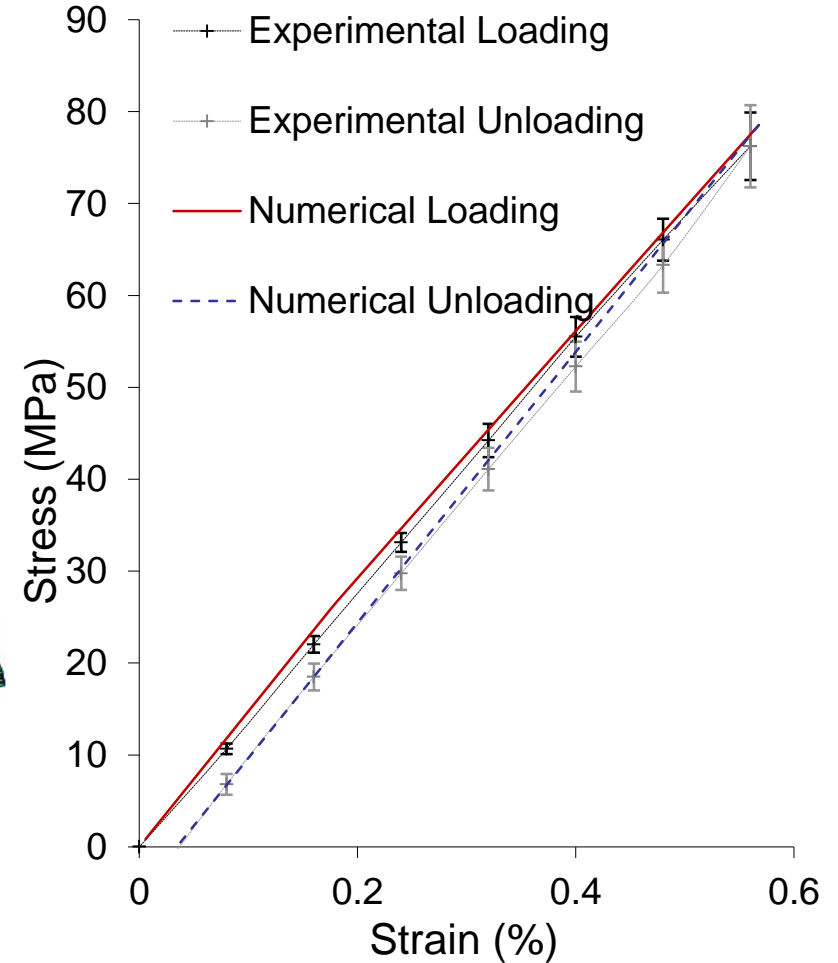
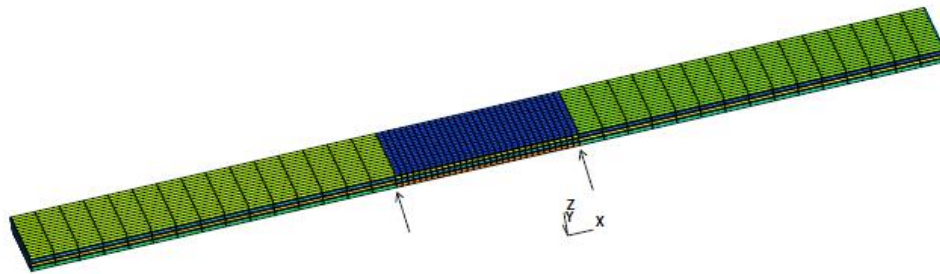
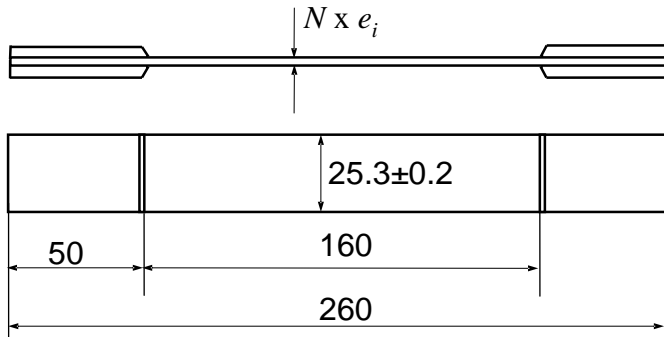
- Resolution of the non-local equation once every  $N$  steps

$$\mathbf{K}_{\tilde{p}\tilde{p}} d\tilde{\mathbf{p}} = \mathbf{F}_p - \mathbf{F}_{\tilde{p}}$$

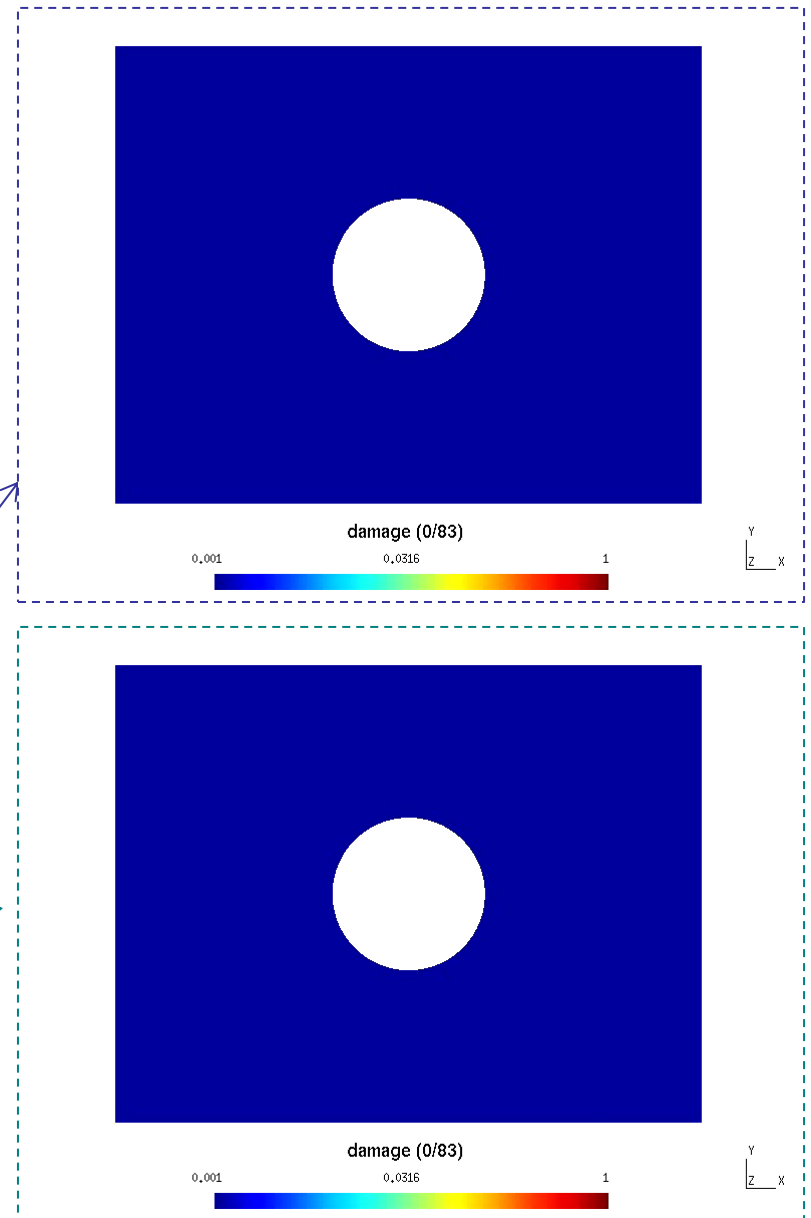
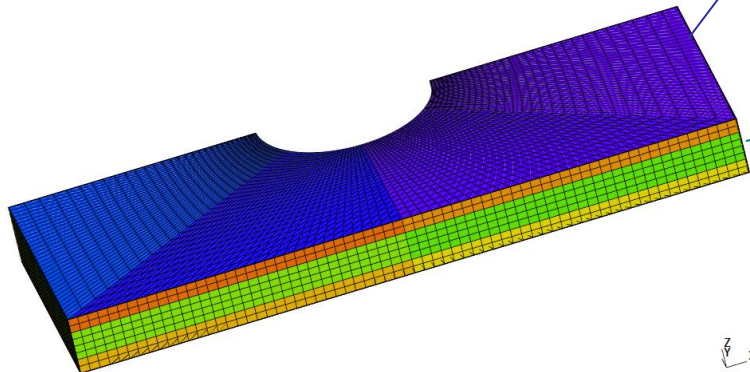
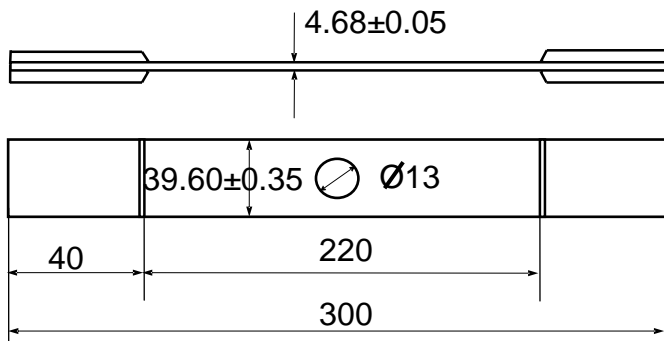
- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply



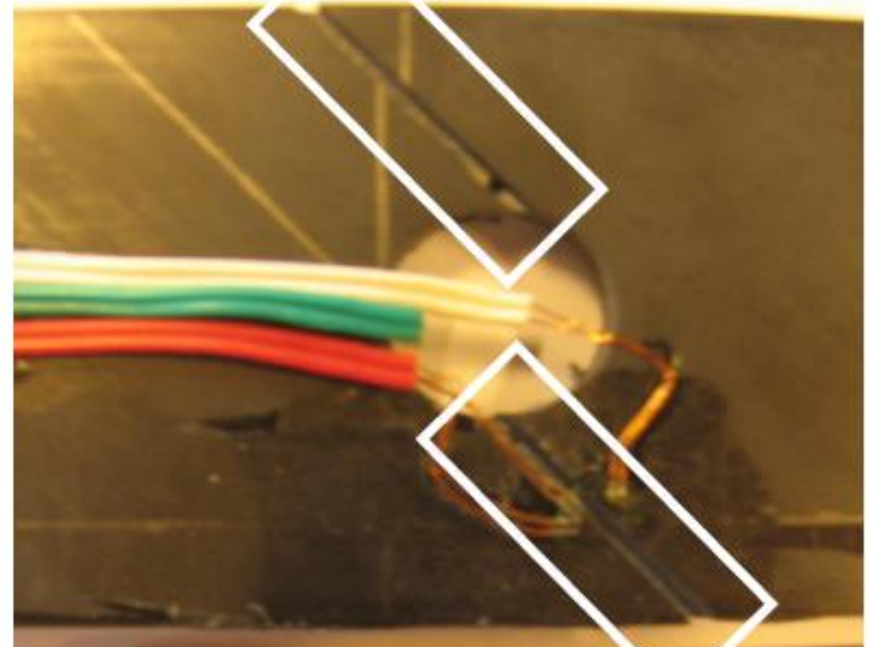
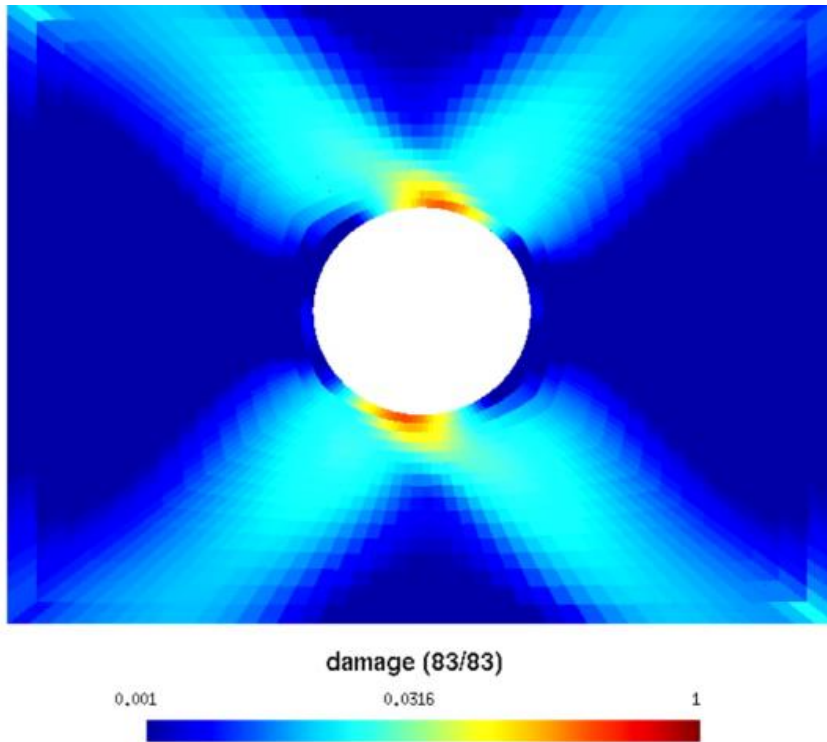
- Laminate: calibration
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$  stacking sequence



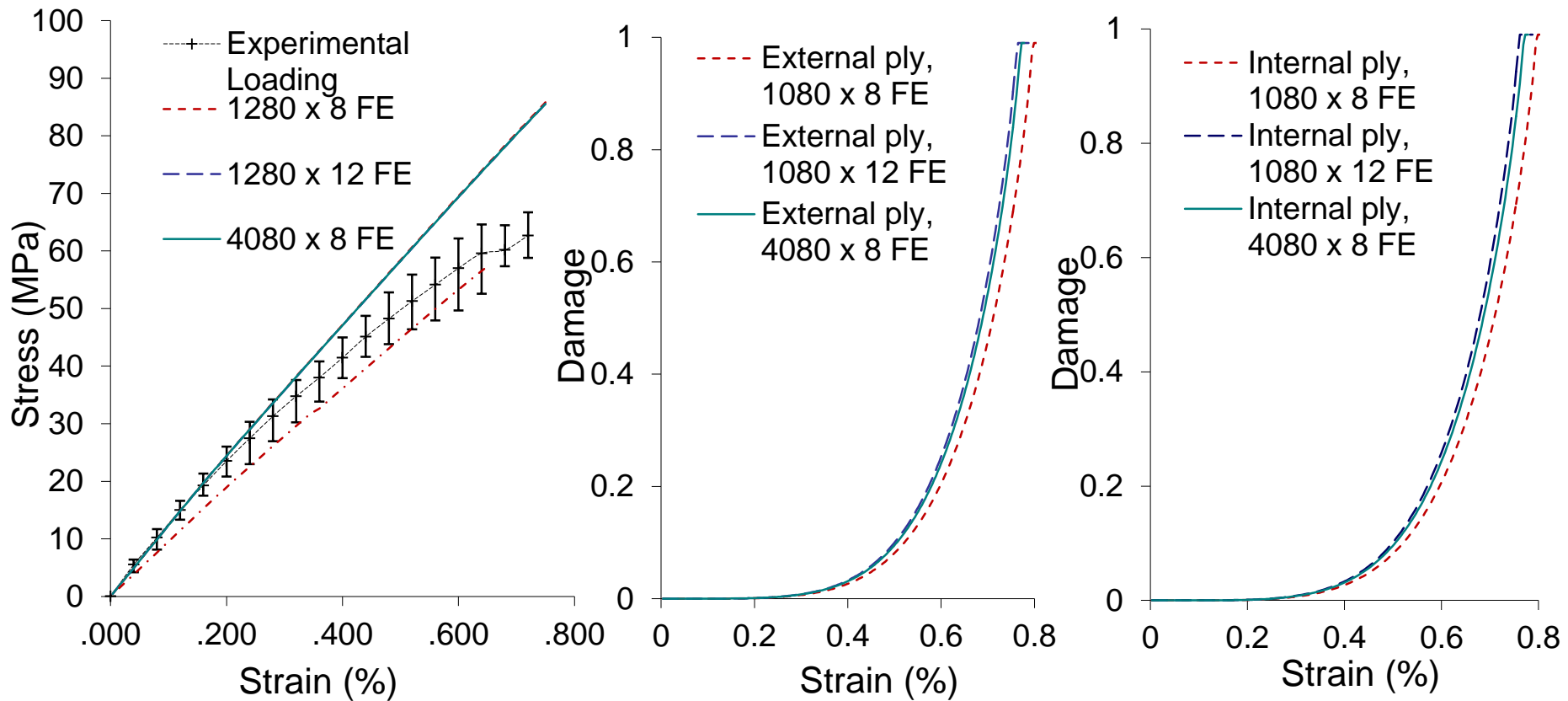
- Laminate plate with hole
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$  stacking sequence



- Laminate plate with hole (2)
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$  stacking sequence



- Laminate plate with hole (3)
  - Carbon-fibres reinforced epoxy
    - 60%-UD fibres
    - Elasto-plastic matrix with damage
  - $[-45_2/45_2]_S$  stacking sequence





# Conclusions

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- **New damage-enhanced incremental secant MFH approach**
  - Efficient computationally
  - Allows fibres unloading during matrix softening
- **Non-local damage-enhanced MFH**
  - Good description of the meso-scale response
  - Can be used to study coupons problems
- **Perspective**
  - From damage to crack