An incremental-secant mean-field homogenization scheme for elasto-plastic and damage-enhanced elasto-plastic composite materials

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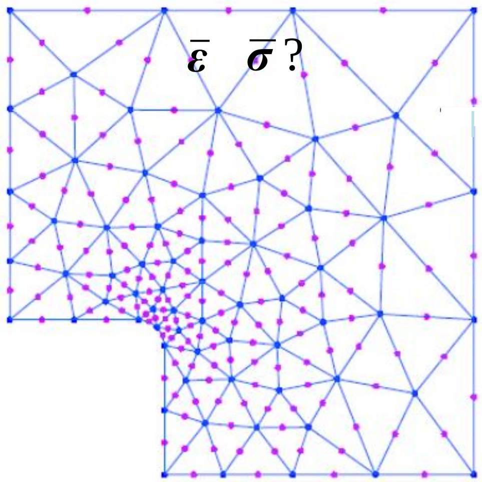
- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point $\bar{\boldsymbol{\varepsilon}}$ is know, $\bar{\boldsymbol{\sigma}}$ is sought



- Multiscale methods
  - Macro-scale
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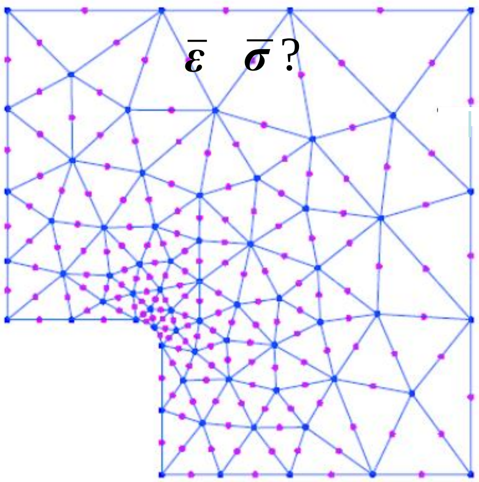
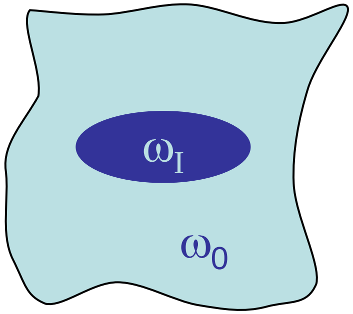
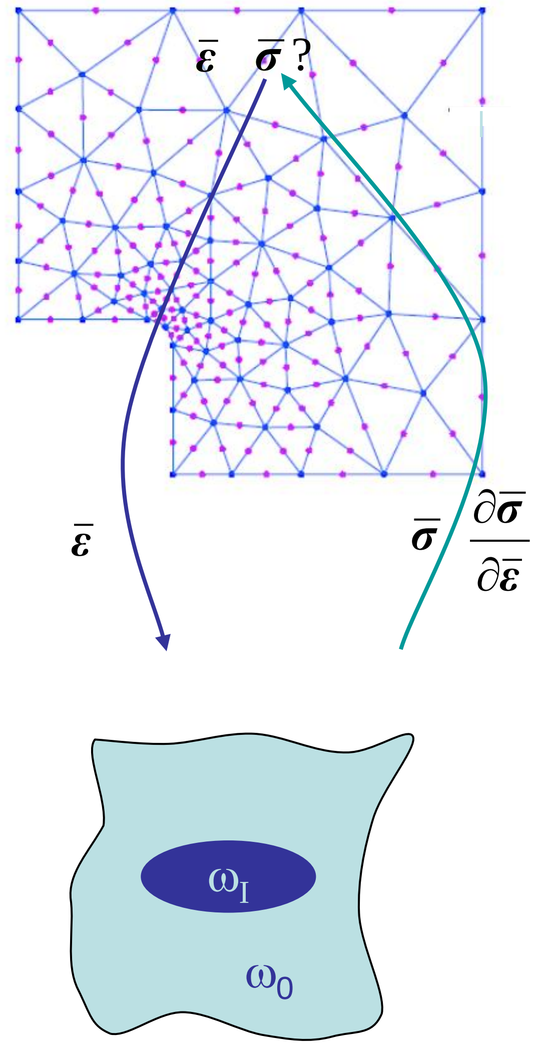


  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



Introduction

- Multiscale methods
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is know, $\bar{\sigma}$ is sought
  - Transition
    - Downscaling: $\bar{\varepsilon}$ is used as input of the MFH model
    - Upscaling: $\bar{\sigma}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



Assumptions:

$L_{macro} >> L_{RVE} >> L_{micro}$

- Semi analytical Mean-Field Homogenization
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

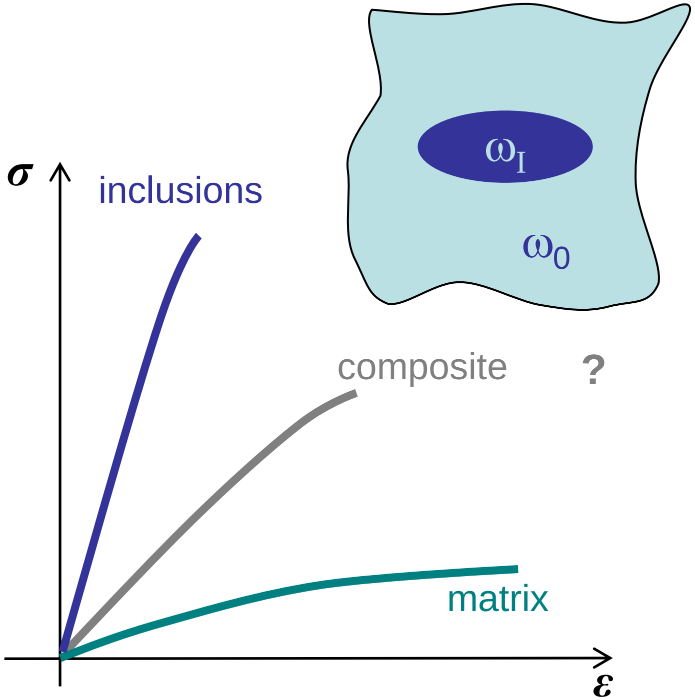
$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$

    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon}\ ?$$



- Mean-Field Homogenization for different materials
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{0} = \overline{\boldsymbol{C}}_{0} : \boldsymbol{\varepsilon}_{0} \end{cases}$$

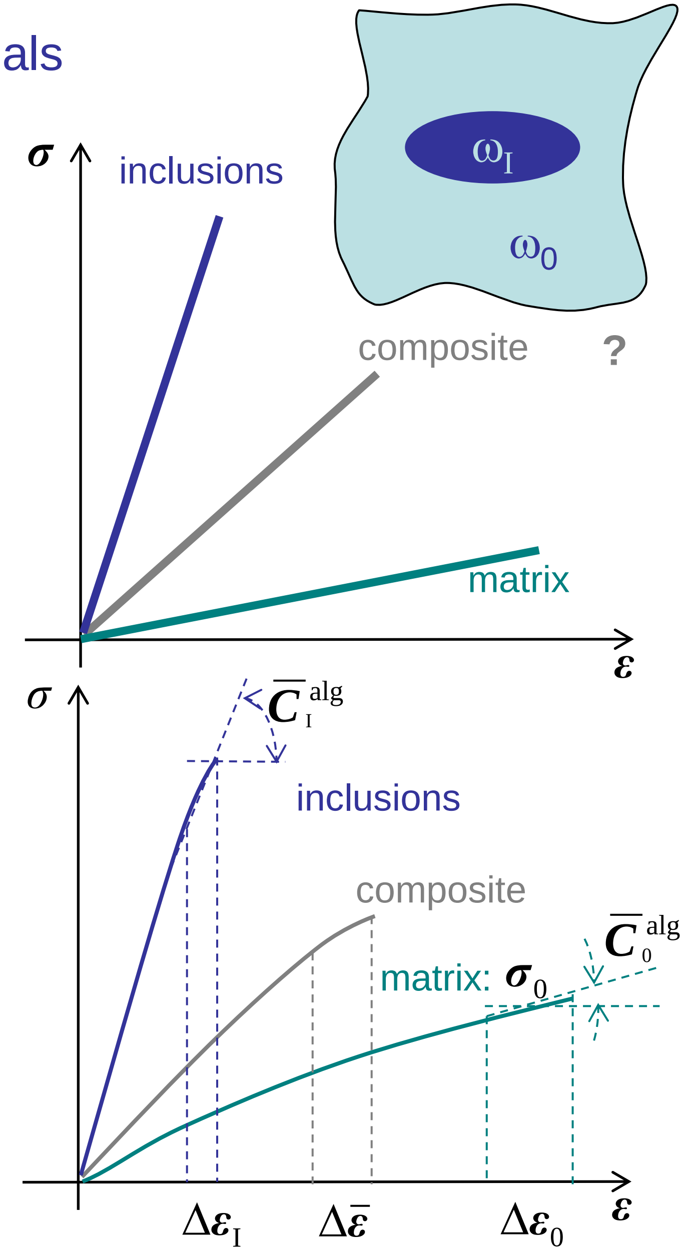
    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby result

$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

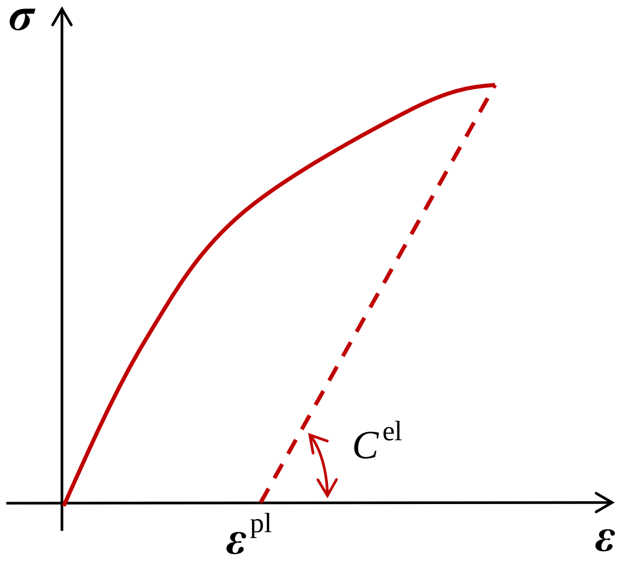
with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

  - Non-linear materials
    - Define a Linear Comparison Composite
    - Common approach: incremental tangent

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_{0}^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$

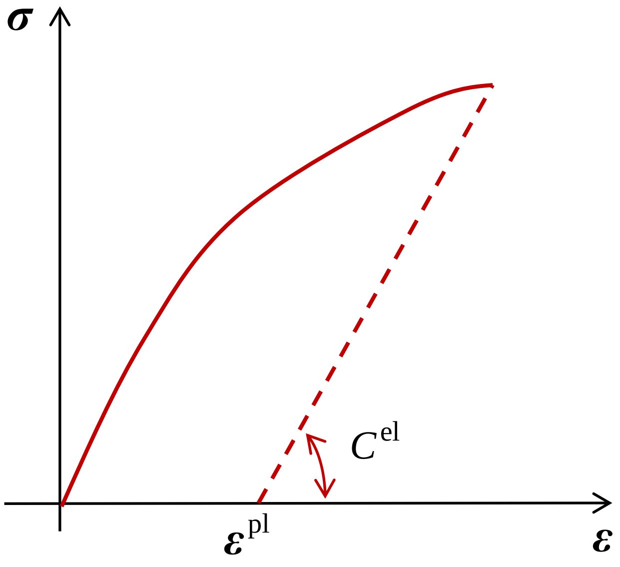


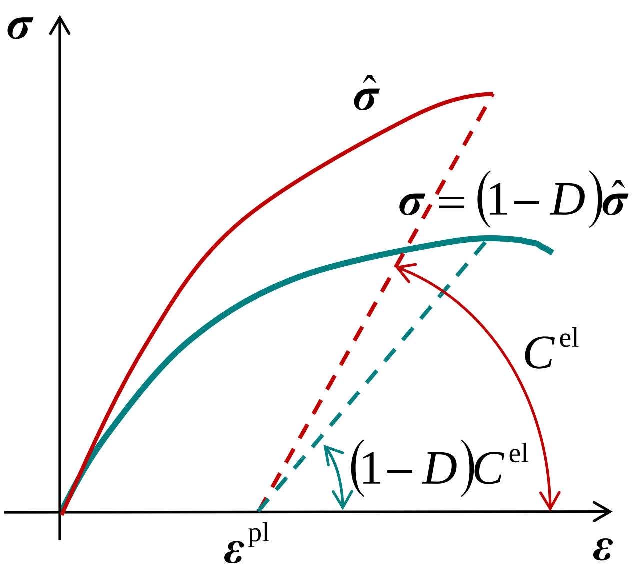
- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$



Damage-enhanced mean-field-homogenization

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \frac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$

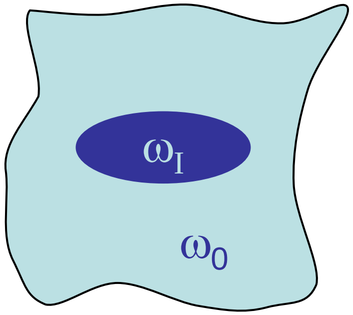




Damage-enhanced mean-field-homogenization

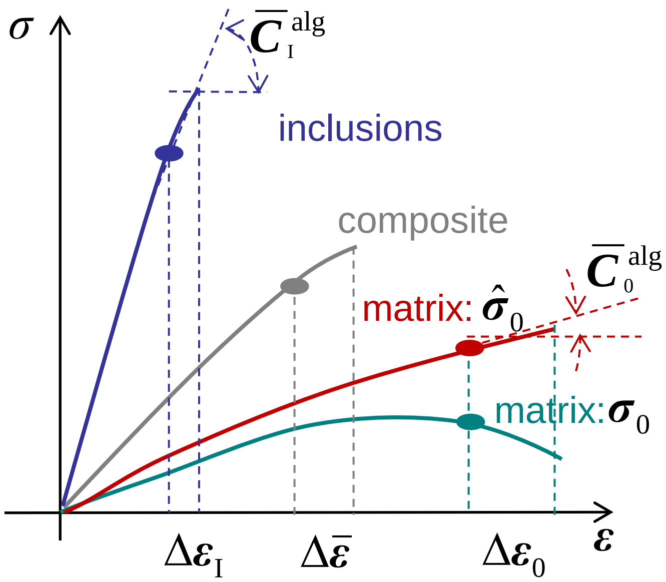
- Incremental-tangent model with damage in the matrix
  - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$



  - Non-linear phases behaviours
    - Elasto-plastic inclusions

$$\delta \boldsymbol{\sigma}_\mathrm{I} = \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_\mathrm{I}$$



    - Non-local damaged matrix

$$\delta \boldsymbol{\sigma}_0 = \left[(1 - D) \bar{\boldsymbol{C}}^{\mathrm{alg}} - \hat{\boldsymbol{\sigma}}_0 \otimes \frac{\partial \bar{F}_D}{\partial \boldsymbol{\varepsilon}_0} \right] : \delta \boldsymbol{\varepsilon}_0 - \hat{\boldsymbol{\sigma}}_0 \frac{\partial \bar{F}_D}{\partial p} \delta p$$

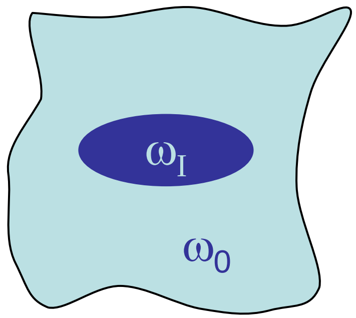
    - Composite

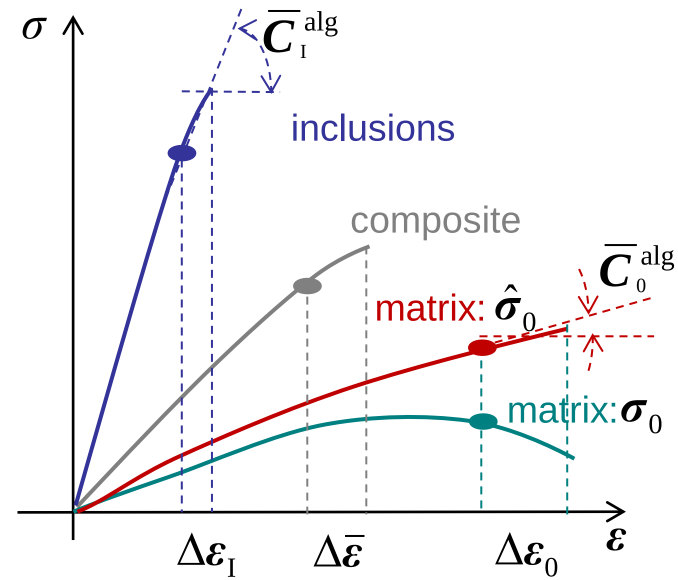
$$\delta \bar{\boldsymbol{\sigma}} = v_\mathrm{I} \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_\mathrm{I} + v_0 (1 - D) \bar{\boldsymbol{C}}_0^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_0 - v_0 \hat{\boldsymbol{\sigma}}_0 \otimes \frac{\partial \bar{F}_D}{\partial \boldsymbol{\varepsilon}_0} : \delta \boldsymbol{\varepsilon}_0 - v_0 \hat{\boldsymbol{\sigma}}_0 \frac{\partial \bar{F}_D}{\partial p} \delta p$$

Mori-Tanaka on one loading interval: $\Delta \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} \left(\mathrm{I}, (1 - D) \bar{\boldsymbol{C}}_0^{\mathrm{alg}}, \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{alg}} \right) : \Delta \boldsymbol{\varepsilon}_0$

- Incremental-tangent model with damage in the matrix
  - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$





  - Non-linear phases behaviours
    - Elasto-plastic inclusions

$$\delta \boldsymbol{\sigma}_\mathrm{I} = \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_\mathrm{I}$$

    - Non-local damaged matrix

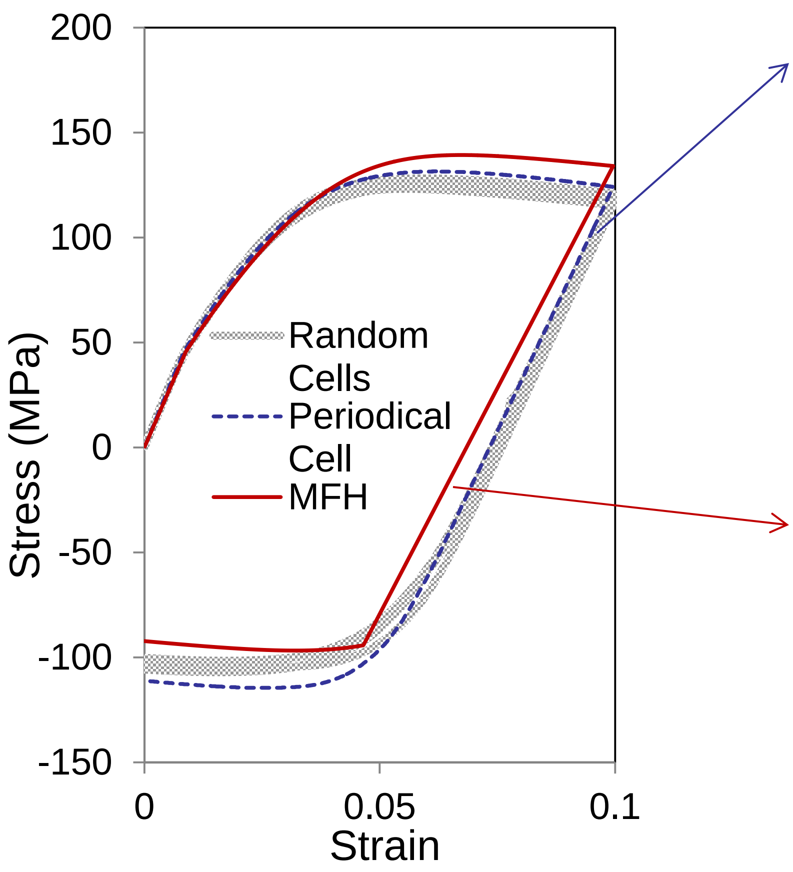
$$\delta \boldsymbol{\sigma}_0 = \left[(1-D) \bar{\boldsymbol{C}}^{\mathrm{alg}} - \hat{\boldsymbol{\sigma}}_0 \otimes \frac{\partial \bar{F}_D}{\partial \boldsymbol{\varepsilon}_0} \right] : \delta \boldsymbol{\varepsilon}_0 - \hat{\boldsymbol{\sigma}}_0 \frac{\partial \bar{F}_D}{\partial p} \delta p$$

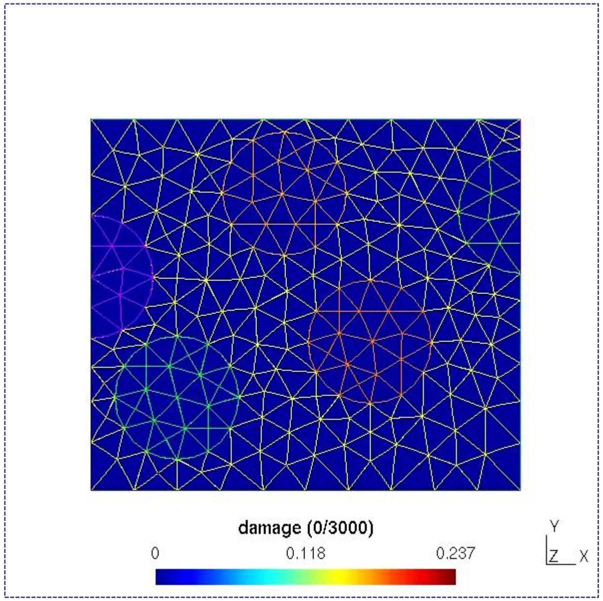
    - Composite

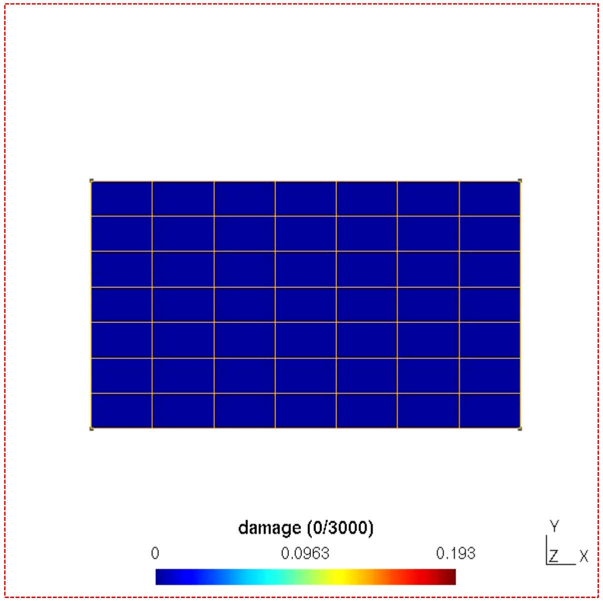
$$\delta \bar{\boldsymbol{\sigma}} = v_\mathrm{I} \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_\mathrm{I} + v_0 (1-D) \bar{\boldsymbol{C}}_0^{\mathrm{alg}} : \delta \boldsymbol{\varepsilon}_0 - v_0 \hat{\boldsymbol{\sigma}}_0 \otimes \frac{\partial \bar{F}_D}{\partial \boldsymbol{\varepsilon}_0} : \delta \boldsymbol{\varepsilon}_0 - v_0 \hat{\boldsymbol{\sigma}}_0 \frac{\partial \bar{F}_D}{\partial p} \delta p$$

Cannot be used in M-T as not (always) definite positive

- DNS vs. FE/MFH
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage

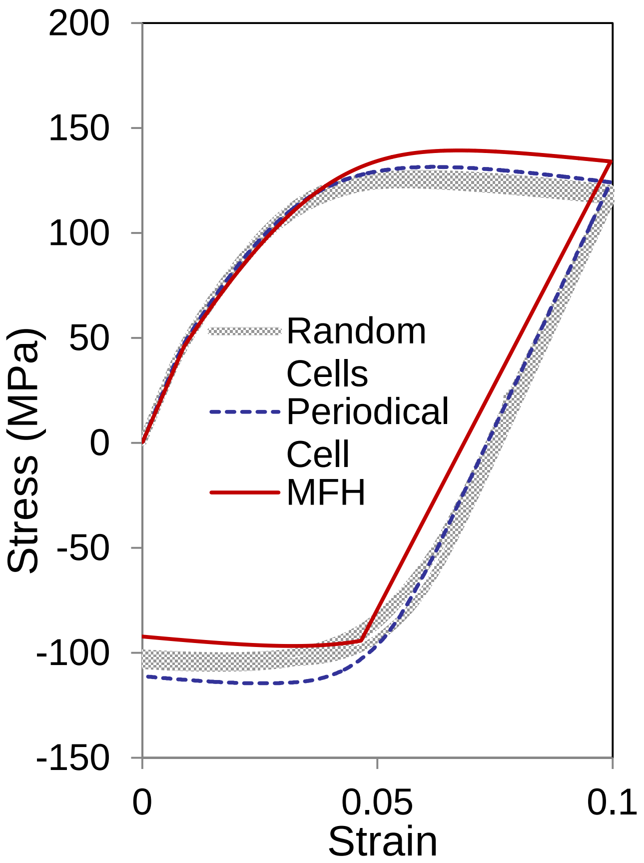


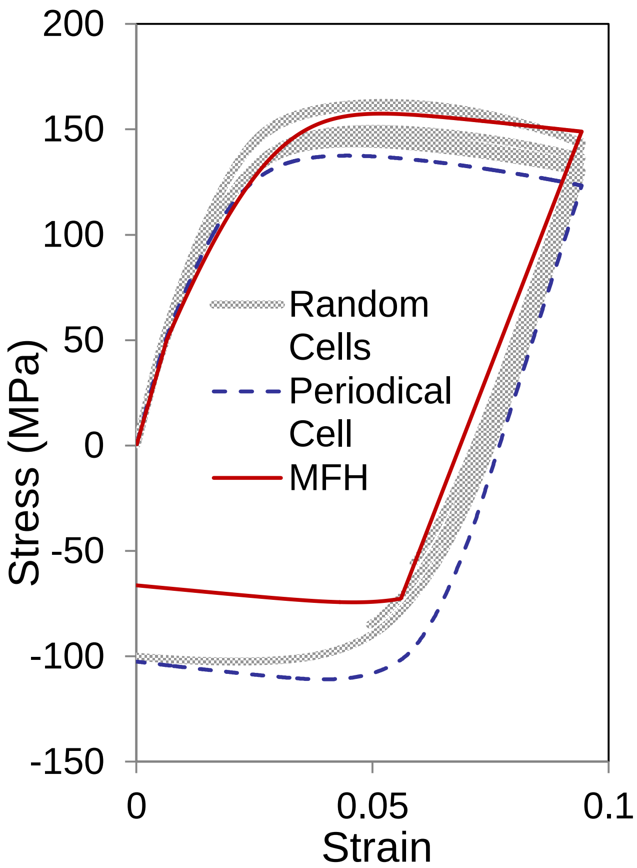




Non-local damage mean-field-homogenization

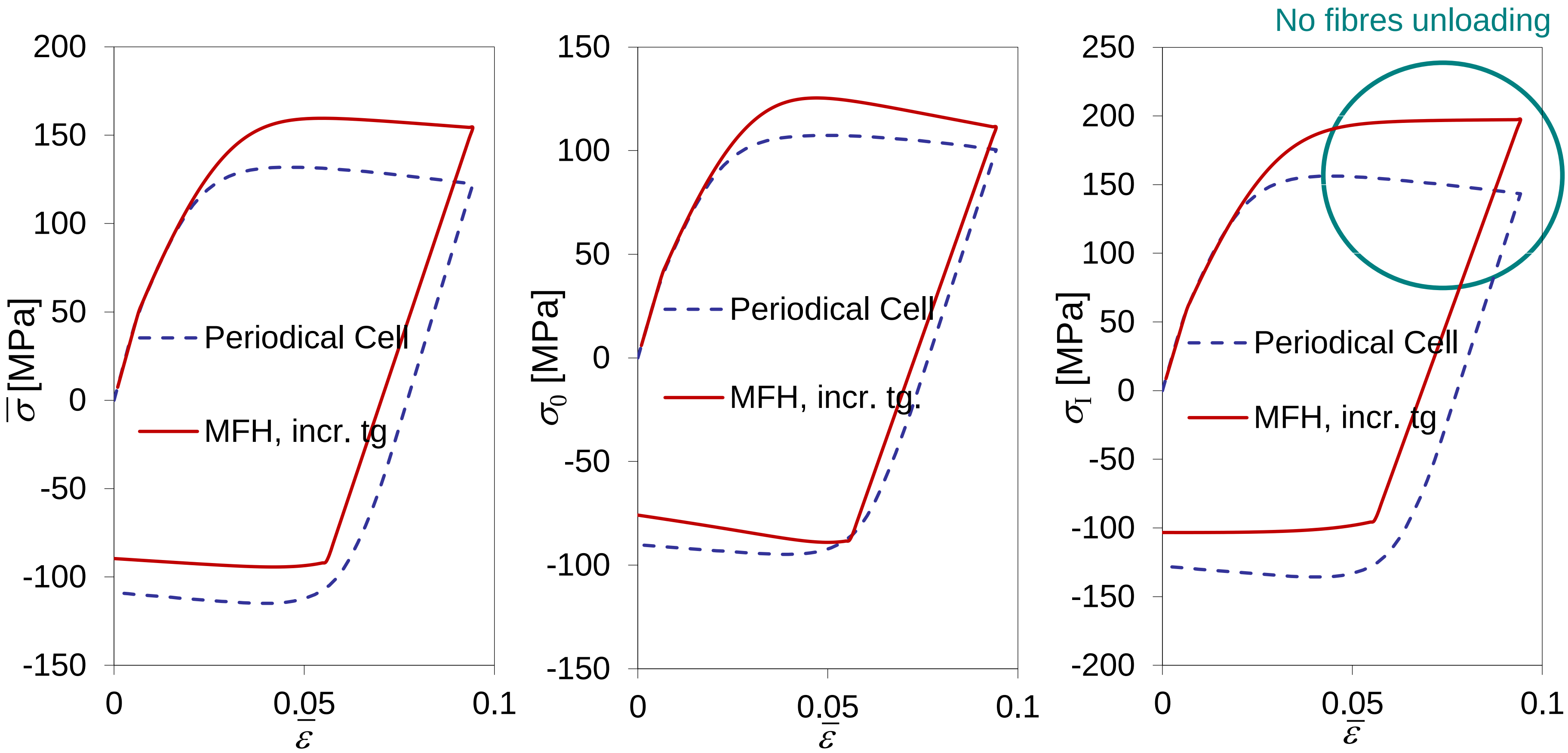
- Limitation of the method
  - Fictitious composite
    - 30%-UD fibres
    - 50%-UD fibres





  - Less accurate during softening for high fibres-volume-ratios

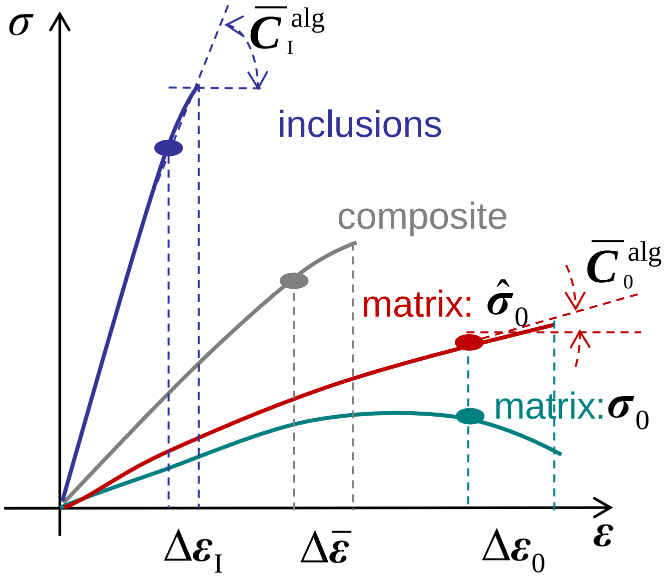
- Limitation of the method (2)
  - Fictitious composite
    - 50%-UD fibres
    - Analyse phases behaviours
  - Due to the incremental formalism, stress in fibres cannot decreases during loading



- Problem
  - We want the fibres to get unloaded during the matrix damaging process
    - For the incremental-tangent approach

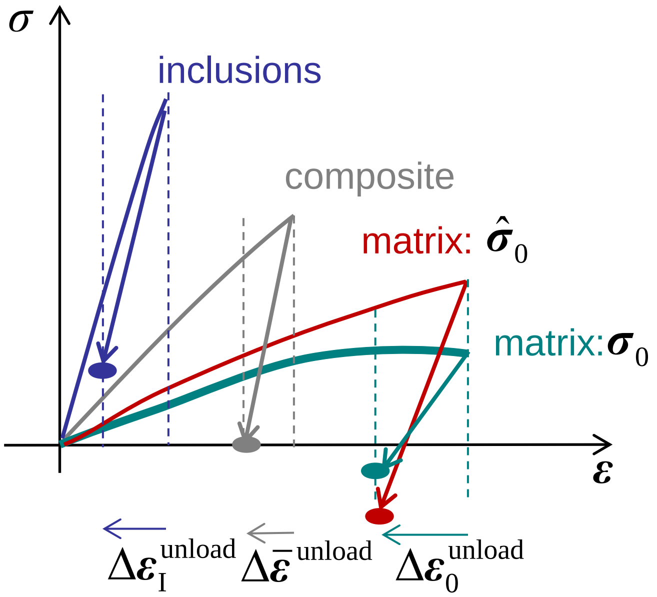
$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\overline{\boldsymbol{C}}_{0}^{\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_{0}$$

    - To unload the fibres ($\boldsymbol{\varepsilon}_{\mathrm{I}} < 0$) with such approach would require $\overline{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{alg}} < 0$
    - We cannot use the incremental tangent MFH
  - We need to define the LCC from another stress state



New incremental-secant mean-field-homogenization

- Idea
  - New incremental-secant approach
    - Perform a virtual elastic unloading from previous solution
      - Composite material unloaded to reach the stress-free state
      - Residual stress in components



New incremental-secant mean-field-homogenization

- Idea
  - New incremental-secant approach
    - Perform a virtual elastic unloading from previous solution
      - Composite material unloaded to reach the stress-free state
      - Residual stress in components
    - Apply MFH from unloaded state
      - New strain increments (>0)

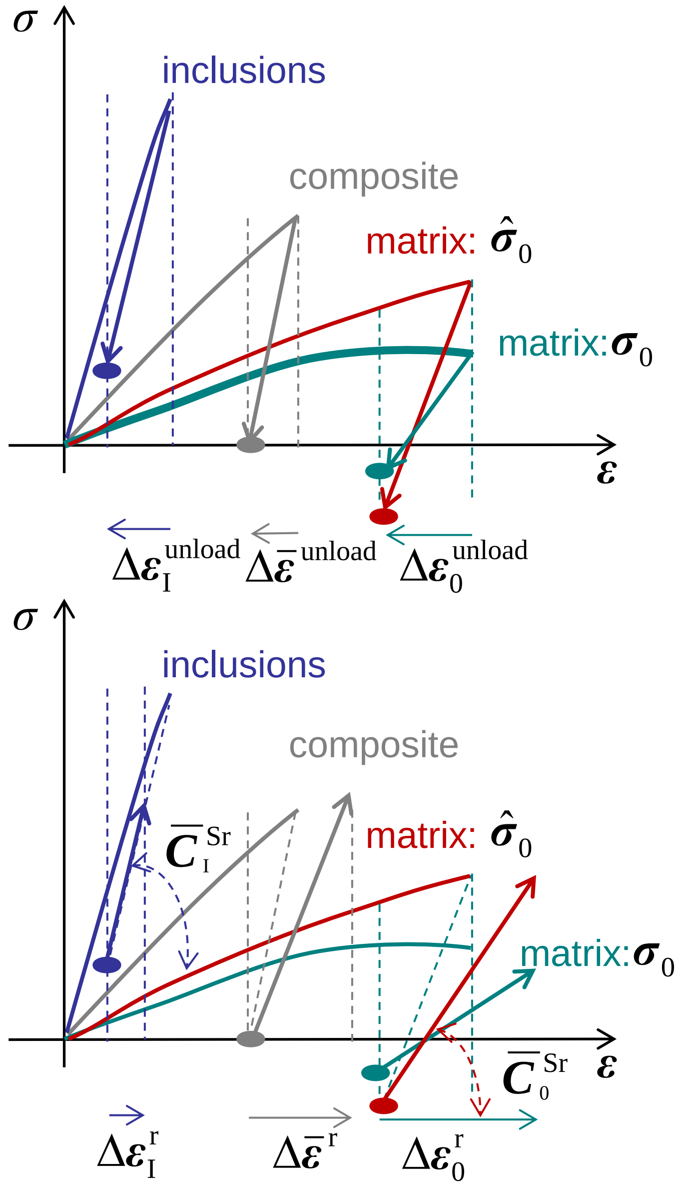
$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I/0}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I/0}} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I/0}}$$

      - Use of secant operators

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$

      - Possibility of have unloading

$$\begin{cases} \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} > 0 \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} < 0 \end{cases}$$



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- New incremental-secant approach
  - First step: virtual elastic unloading
    - Composite material unloaded to reach the stress-free state

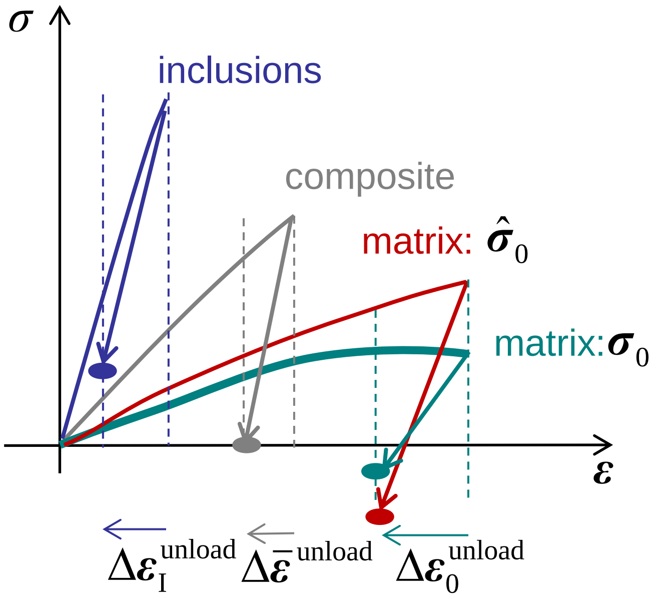
$$\Delta\bar{\boldsymbol{\varepsilon}}^{\text{unload}} = \left(\bar{\boldsymbol{C}}^{\text{elD}}\right)^{-1} : \bar{\boldsymbol{\sigma}}$$

    - Residual strain in components

$$\begin{cases} \boldsymbol{\varepsilon}_{\text{I}}^{\text{res}} = \Delta\boldsymbol{\varepsilon}_{\text{I}} - \Delta\boldsymbol{\varepsilon}_{\text{I}}^{\text{unload}} \\ \quad = \boldsymbol{\varepsilon}_{\text{I}} - \boldsymbol{B}^{\varepsilon} : \left[v_{\text{I}}\boldsymbol{B}^{\varepsilon} + v_0 I\right]^{-1} : \Delta\bar{\boldsymbol{\varepsilon}}^{\text{unload}} \\ \boldsymbol{\varepsilon}_0^{\text{res}} = \Delta\boldsymbol{\varepsilon}_0 - \Delta\boldsymbol{\varepsilon}_0^{\text{unload}} \\ \quad = \boldsymbol{\varepsilon}_0 - \left[v_{\text{I}}\boldsymbol{B}^{\varepsilon} + v_0 I\right]^{-1} : \Delta\bar{\boldsymbol{\varepsilon}}^{\text{unload}} \end{cases}$$

    - Residual stress in components

$$\begin{cases} \boldsymbol{\sigma}_{\text{I}}^{\text{res}} = \boldsymbol{\sigma}_{\text{I}} - \boldsymbol{C}_{\text{I}}^{\text{el}} : \Delta\boldsymbol{\varepsilon}_{\text{I}}^{\text{unload}} \\ \hat{\boldsymbol{\sigma}}_0^{\text{res}} = \hat{\boldsymbol{\sigma}}_0 - \boldsymbol{C}_0^{\text{el}} : \Delta\boldsymbol{\varepsilon}_0^{\text{unload}} \end{cases}$$

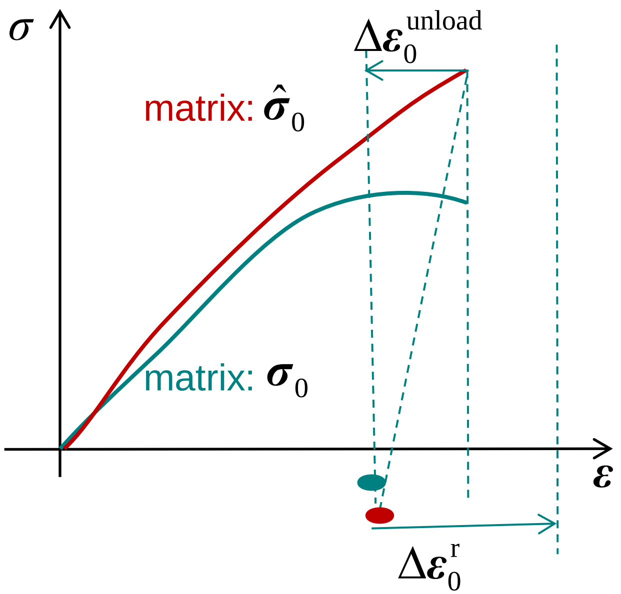


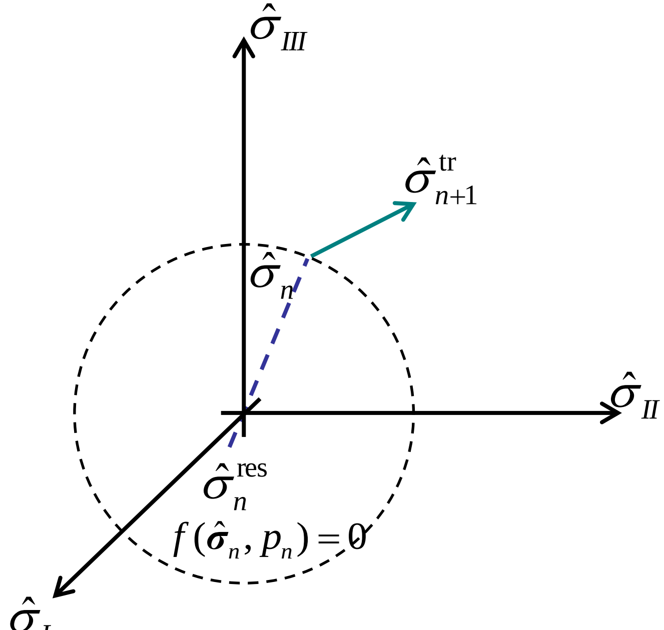
- New incremental-secant approach (2)
  - Second step: phase loading (here matrix)
    - Assume strain increment in each phase known

$$\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}}$$

    - Elastic trial

$$\widehat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} = \widehat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$





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$$\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}}$$

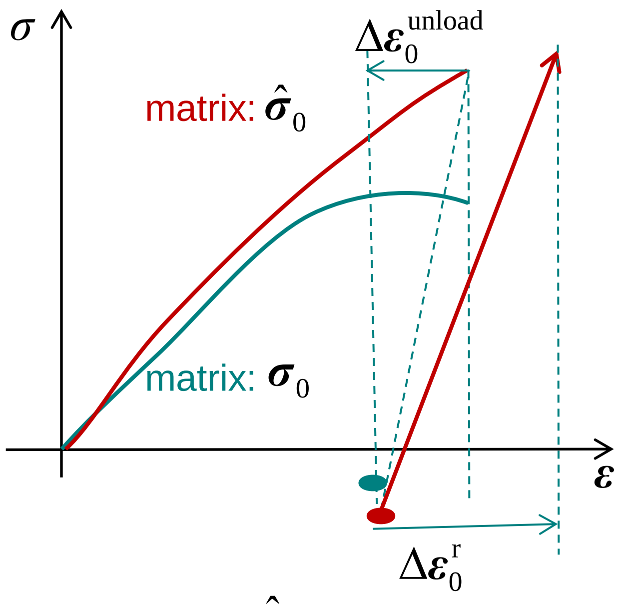
    - Elastic trial

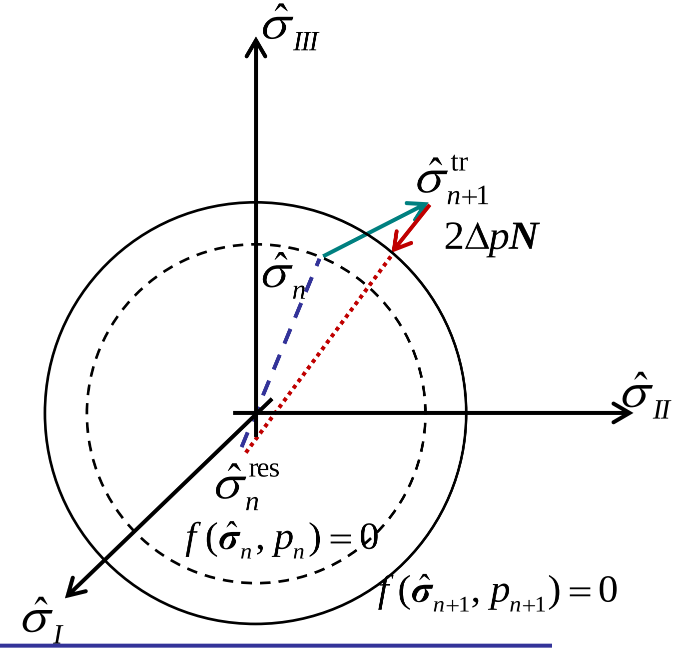
$$\hat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} = \hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$

    - Plastic corrector

$$\begin{cases} \hat{\boldsymbol{\sigma}}_0 = \hat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} - \Delta p \boldsymbol{N} \\ \boldsymbol{N} = \dfrac{3\left(\boldsymbol{C}^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}\right)^{\mathrm{dev}}}{2\left(\boldsymbol{C}^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}\right)^{\mathrm{eq}}} \end{cases}$$

      - Modification of the plastic flow (first order approximation)





New incremental-secant mean-field-homogenization

- New incremental-secant approach (2)
  - Second step: phase loading (here matrix)
    - Assume strain increment in each phase known

$$\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}}$$

    - Elastic trial

$$\widehat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} = \widehat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$

    - Plastic corrector

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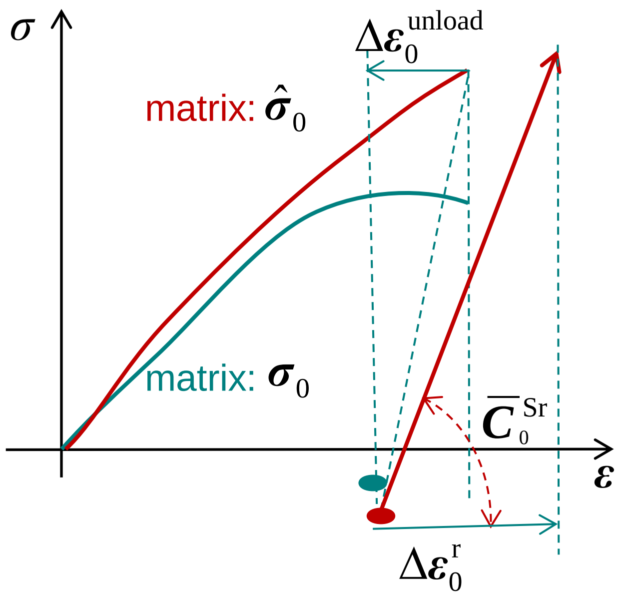
      - Modification of the plastic flow (first order approximation)
    - There exists an isotropic operator such that

$$\hat{\boldsymbol{\sigma}}_0 = \hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$



- New incremental-secant approach (2)
  - Second step: phase loading (here matrix)
    - Assume strain increment in each phase known

$$\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}}$$

    - Elastic trial

$$\hat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} = \hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$

    - Plastic corrector

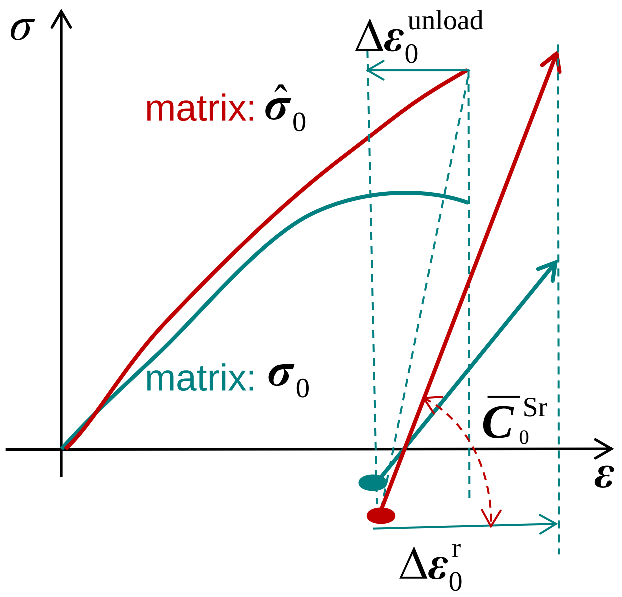
$$\begin{cases} \hat{\boldsymbol{\sigma}}_0 = \hat{\boldsymbol{\sigma}}_0^{\mathrm{tr}} - \Delta p \boldsymbol{N} \\ \boldsymbol{N} = \dfrac{3\left(\boldsymbol{C}^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}\right)^{\mathrm{dev}}}{2\left(\boldsymbol{C}^{\mathrm{el}} : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}\right)^{\mathrm{eq}}} \end{cases}$$

      - Modification of the plastic flow (first order approximation)
    - There exists an isotropic operator such that

$$\hat{\boldsymbol{\sigma}}_0 = \hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + \boldsymbol{C}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}$$

    - Coupled damage problem

$$\begin{cases} \Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p) \\ \boldsymbol{\sigma}_0 = (1-D)\hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + (1-D)\boldsymbol{C}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

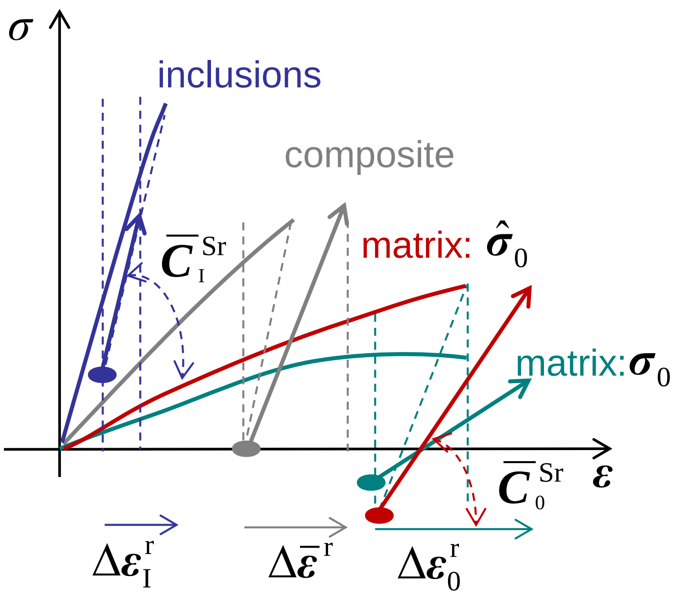


- New incremental-secant approach (3)
  - Third step: MFH
    - Inputs
      - Internal variable at last increment
      - Residual tensor after virtual unloading
      - $\Delta\bar{\boldsymbol{\varepsilon}}, \Delta p$ from FE resolution
    - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_\mathrm{I} \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_\mathrm{I} + \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{Sr}}, \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

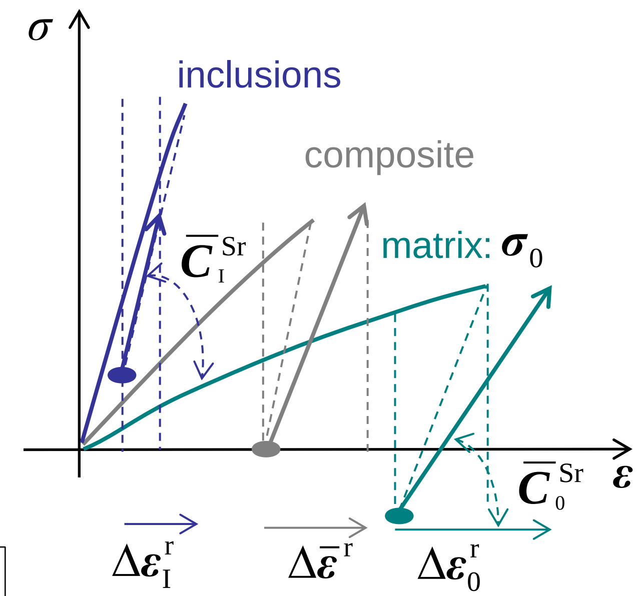
    - With the stress tensors

$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \boldsymbol{\sigma}_\mathrm{I} = \boldsymbol{\sigma}_\mathrm{I}^{\mathrm{res}} + \bar{\boldsymbol{C}}_\mathrm{I}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_\mathrm{I}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = (1-D)\hat{\boldsymbol{\sigma}}_0^{\mathrm{res}} + (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

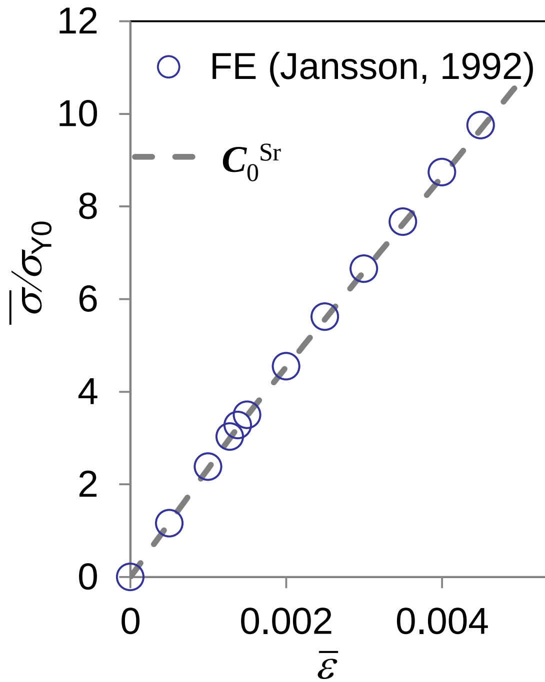


New incremental-secant mean-field-homogenization

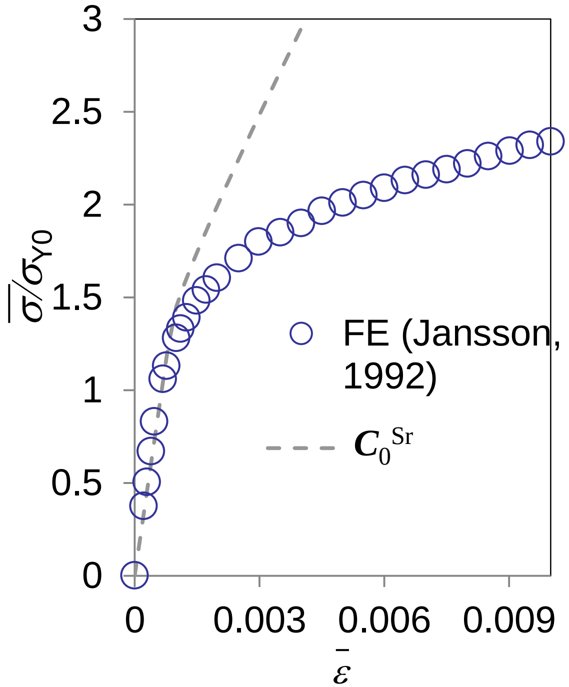
- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - For inclusions with high hardening (elastic)
    - Model is too stiff



Longitudinal tension



Transverse loading



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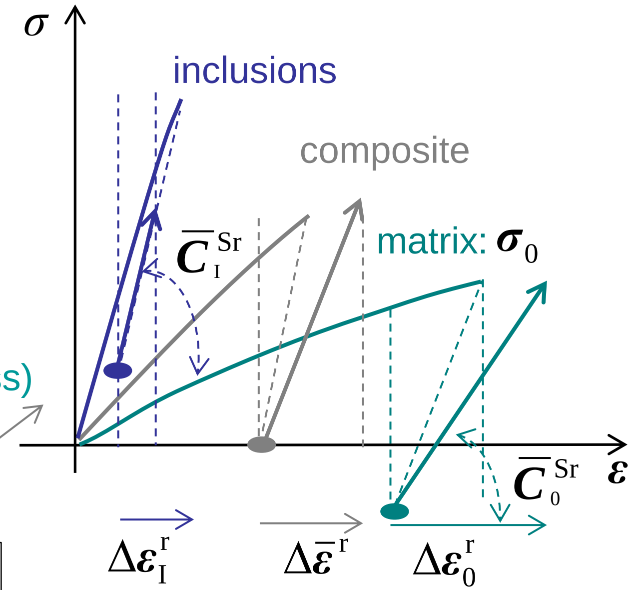
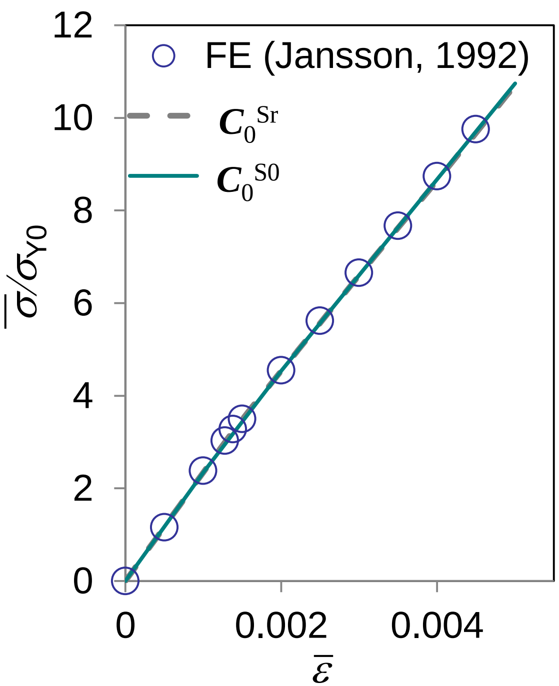
New incremental-secant mean-field-homogenization

- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if stiffer inclusions (negative residual stress)



Longitudinal tension



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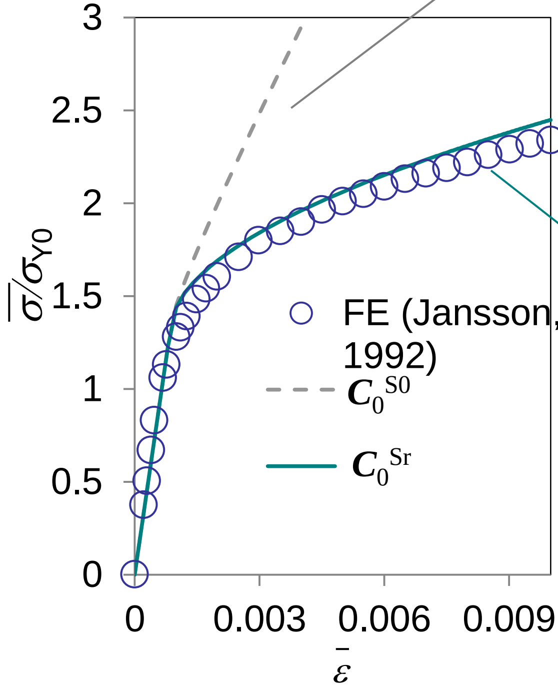
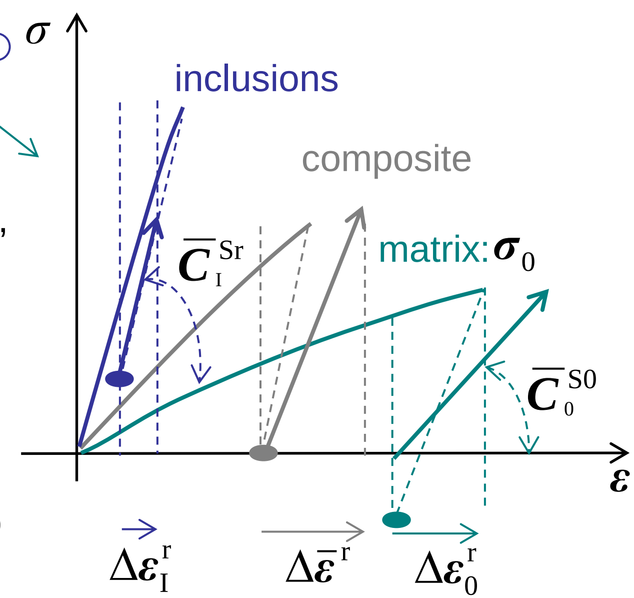
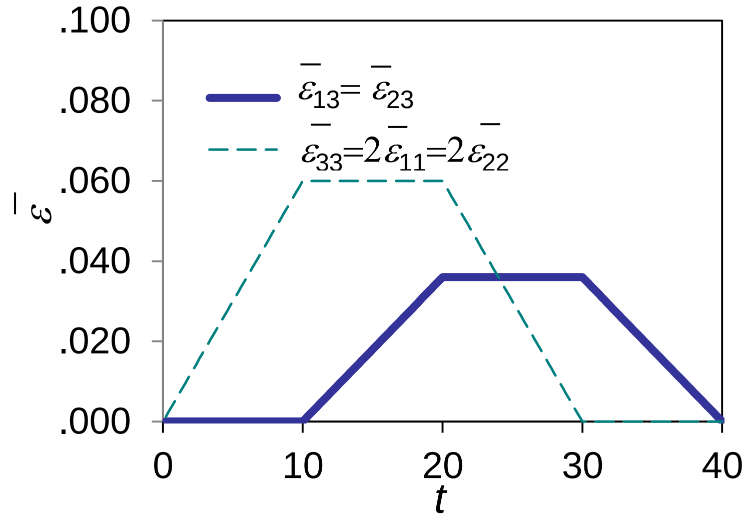
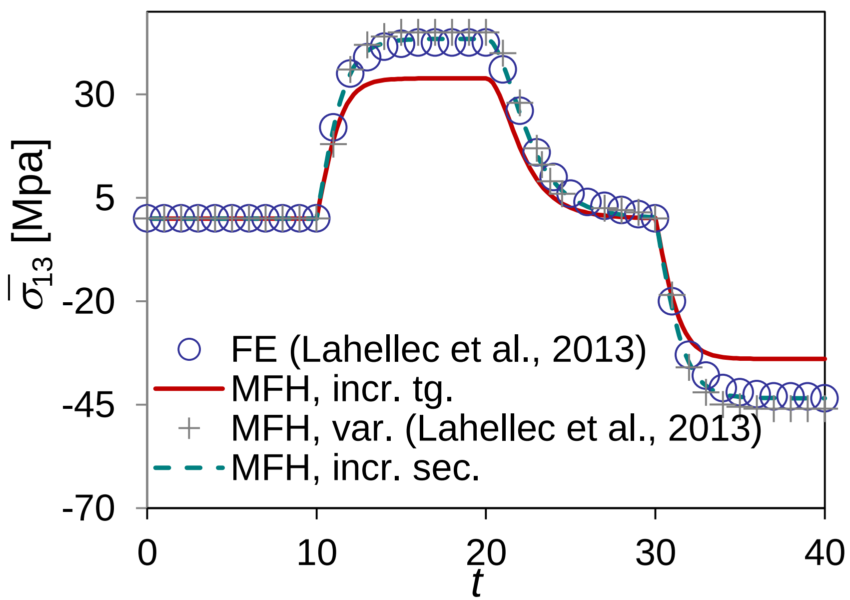
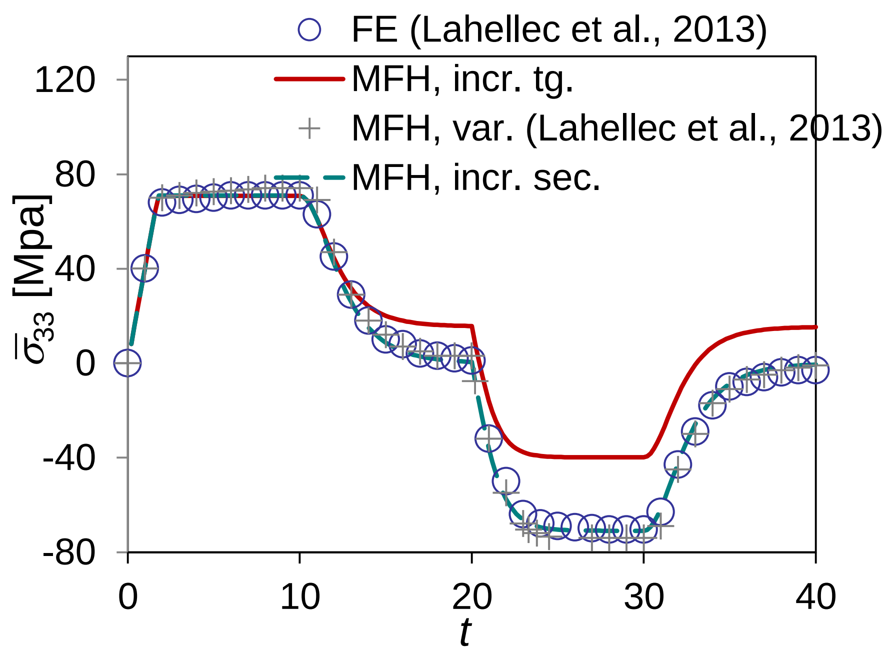




- Verification of the method
  - Spherical inclusions
    - 17 % volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-radial loading
  - Non-monotonic loading

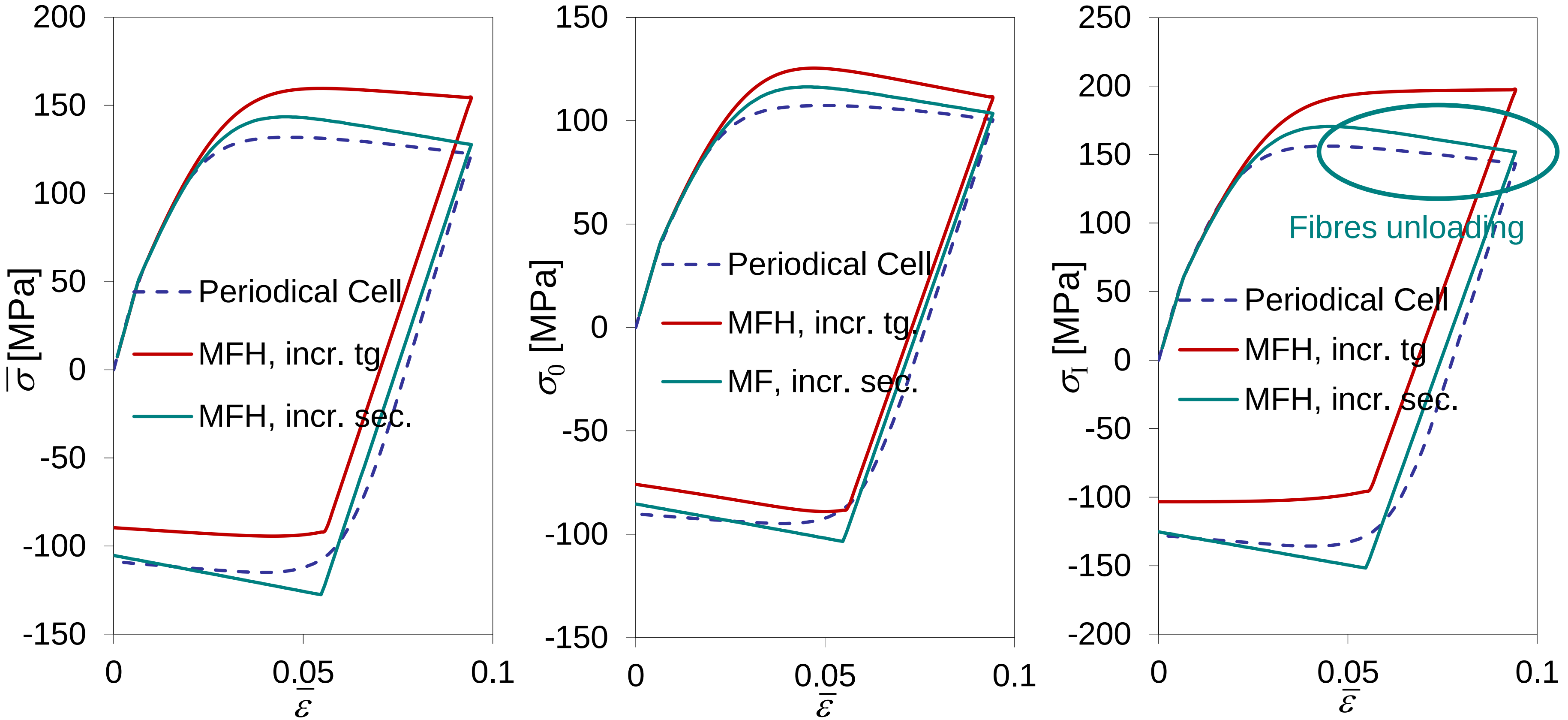






New incremental-secant mean-field-homogenization

- New results for damage
  - Fictitious composite
    - 50%-UD fibres
    - Analyse phases behaviours



Conclusions

- New incremental secant MFH approach

- For elasto-plastic materials
  - Accurate first-statistical-moments approach
  - Allows non-proportional non-monotonic loading
  - Efficient computationally

- For damage-enhanced materials
  - Allows to capture the fibres unloading during the matrix strain-softening

- Can also be used to predict meso-scale responses
  - Damage is reformulated in a non-local way
  - See talk of tomorrow