Selection and Identification of Lumped Models of the Arterial Vasculature Using Multiple Regression and Backward Elimination in the Time Domain.

Thierry Pochet, Paul Gerard, Bernard Lambermont, Olivier Detry, Vincent D'Orio,
Jean-Olivier Defraigne, Anny Fossion, and Raymond Limet
Hemodynamics Research Centre (HemoLiège), University of Liège
6, quai Banning, B-4000 Liège, Belgium

Abstract: Although electrical networks are used with increasing insistence in both clinical and experimental investigations to describe hemodynamic properties of the arterial vasculature, the rationale for the selection of a particular model is, however, rarely discussed. In most cases the mathematical procedure is time consuming, highly specific and thereby, reproducible with great difficulties. The aim of the present paper is to provide a rapid method of analysis which supports the identification of parameters together with the choice of the best analog model. Five models with gradual complexity and described by a second order differential equation are considered. Selection and identification is performed through a multiple regression in the time domain.

INTRODUCTION

Simple electrical networks are becoming of increasing interest to provide insights into the hemodynamic behavior of the pulmonary [1] and the systemic [2] arterial vasculature as well as of organ networks [3]. Furthermore, they are also used to quantify the coupling between native or prosthetic ventricles and their respective vascular output.

However, to the extent that collected hemodynamic data may be analyzed owing to the use of lumped models, the capacity of interpretation of their parameters is related to the success to overcome two main problems. The first one is related to the choice of the best and most suitable analog model. The second one is the capability to compute the values that characterize the network components. Unfortunately the rationale that subtends the choice of a particular model remains often scarce.

Thereby, the main objective of the present study is to provide a consistent time domain method which enables to solve these two fundamental problems by means of a systematic and time sparing procedure which can be easily run using usual and widespread softwares. It is presented using hemodynamic data coming from the pulmonary circulation.

METHODS

A large class including the most usual models are described by a second order linear differential equation. Five such analog models of increasing complexity have been considered. They are composed of resistances R_i ($i=1,...,3$), a compliance C and an inertance L. Their electrical representation are given on figure 1.

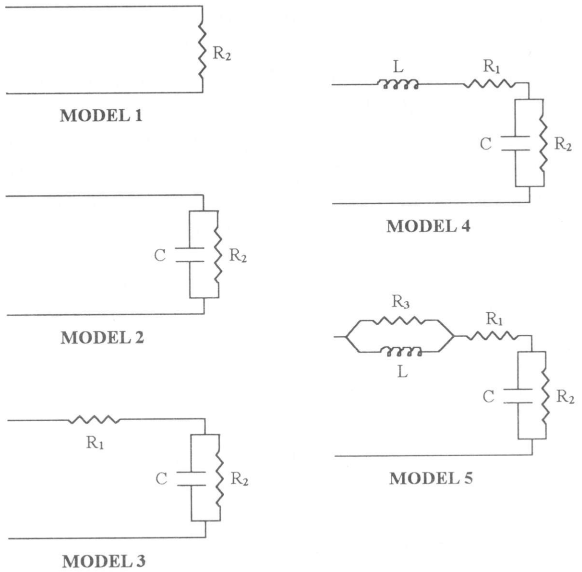

Figure 1. Electrical representation of the 5 analog models.

After integration of their equation, the following general equation is obtained :

$$IQ = C_1\, IP + C_2\, P_1 + C_3\, Q_1 + C_4 DQ_1 + C_5\, DP_1 \tag{1}$$

with :

$$IQ = \int_{t_0}^{t} Q.dt\ ;\ Q_1 = Q(t) - Q(t_0)\ ;\ DQ_1 = Q'(t) - Q'(t_0)$$

$$IP = \int_{t_0}^{t} P.dt\ ;\ P_1 = P(t) - P(t_0)\ ;\ DP_1 = P'(t) - P'(t_0) \tag{2}$$

where Q is the flow, P the driving pressure, Q' and P' their first derivatives with respect to time t. The coefficients C_i ($i=1,...,5$) are known functions of the model RCL parameters [4]. When successively simplifying the models from model 5 to model 1, the last term of equation (1) disappears at each step.

The fitting of the models is performed through a multiple regression of the dependent variable IQ on the five explanatory variables IP, P_1, Q_1, DQ_1 and DP_1. The standard stepwise procedures of backward elimination of the explanatory variables to be included in the regression equation is applied to decide which one of the models is to be chosen [4].

The choice of a model is always the result of a compromise between quality of the fit and simplicity of the model. This stepwize backward elimination procedure starts with the most complex model and sequentially remove useless variables. At each step, the variable for which the damaging effect on the fit is the least when it is removed, is withdrawed from the model. The process stops when any deletion leads to significant deterioration of the fit. Furthermore, any model with estimated regression coefficients leading to spurious (for example negative) values of the RCL elements must be rejected.

The procedure is highlighted in this paper on one example of data set coming from the pulmonary arterial vasculature of a pig. Pressure and flow were recorded by a pressure catheter (Sentron) located in the pulmonary artery and a perivascular flow probe (Transonic) encircled around the pulmonary artery. The output pressure was taken equal to the observed capillary pressure (7 mmHg). The pressure and flow signals recorded during one beat are given in figure 2. The sampling interval was 0.005 s.

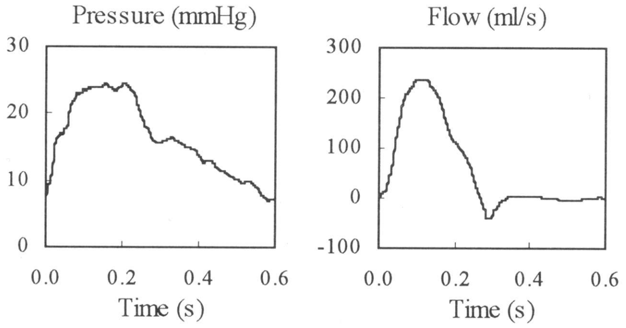

Figure 2. Recorded pressure and flow waves.

RESULTS

The results of the identification procedure are given in Table 1. The backward elimination process started with model 5 leading to a negative value for R_3. Thus, model 5 was rejected and the procedure continued with model 4. Fitting of model 3 led to a substantial deterioration of the fit measured by the relative increase of the residual sum of squares. Therefore, model 4 was selected as the most appropriate model among the five models.

Table 1. Results of the identification procedure.

Model	Model 5	Model 4	Model 3
R_1	0.032	0.030	0.041
R_2	0.136	0.136	0.123
C	1.237	1.216	1.486
L	0.00156	0.00154	
R_3	-0.597		
RSS	357.7	366.4	1586.2
RIRSS (%)		2.43	332

Resistances R_1, R_2 and R_3 are in mmHg s ml^{-1}, compliance C is in $ml.mmHg^{-1}$ and inertance L is in $mmHg.s^2.ml^{-1}$. RSS is the residual sum of squares, RIRSS is the relative increase of the residual sum of squares.

DISCUSSION

Grant et al. [1] had already discussed the problem of the choice among electrical networks in the spectral domain. We propose another strategy of fitting and selection of models. In order to provide a tool that allows the treatment of large amounts of data files in a standard way, our method satisfies the four following requirements: 1) it is rapid, non time consuming, using a non iterative method in the time domain; 2) it objectively allows the choice of a model; 3) it can be implemented on a spreadsheet software; 4) it is easily understandable allowing a clear discussion among members of a research team.

A systematic use of this process is useful and save, avoiding useless complexity and damaging simplification. For example, if the process stops at model 4, it probably means that the inertial properties cannot be neglected. But a process stopping at model 3 implies that the inertial effects, if present, are too weak to be revealed by the recorded data.

Equation (1) was used instead of the second order differential equation to avoid the often delicate numerical evaluation of the second order derivatives. The backward elimination procedure was used instead of the forward selection method because this process could stop too early.

CONCLUSION

We propose a method that allows simultaneously a rapid and systematic choice of a lumped model for the arterial networks and the identification of the parameters of the selected model. The method can easily run on popular spreadsheet or statistical softwares. Such a procedure may encourage to examine more frequently, on large samples and with repeated measurements, the evolution of the hemodynamic variables in various physiological situations.

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