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Structural optimization of flexible components under dynamic loading within a multibody system approach:

A comparative evaluation of optimization methods based on a 2-dof robot application.

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LTAS

Introduction – Optimization of a connecting rod

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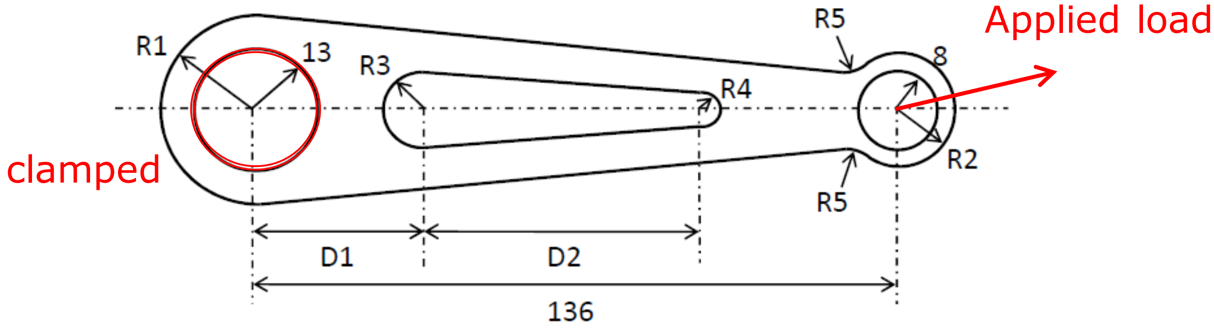
- A component based approach



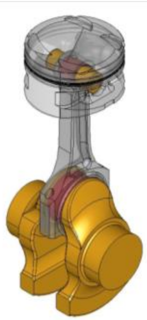
- Experience - Empirical load case
- Dynamic factor amplification for safety

➔ Not optimal

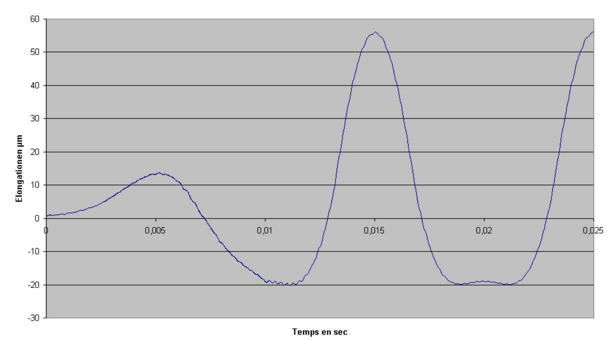
- Multibody system based approach

Loop

Optimization process



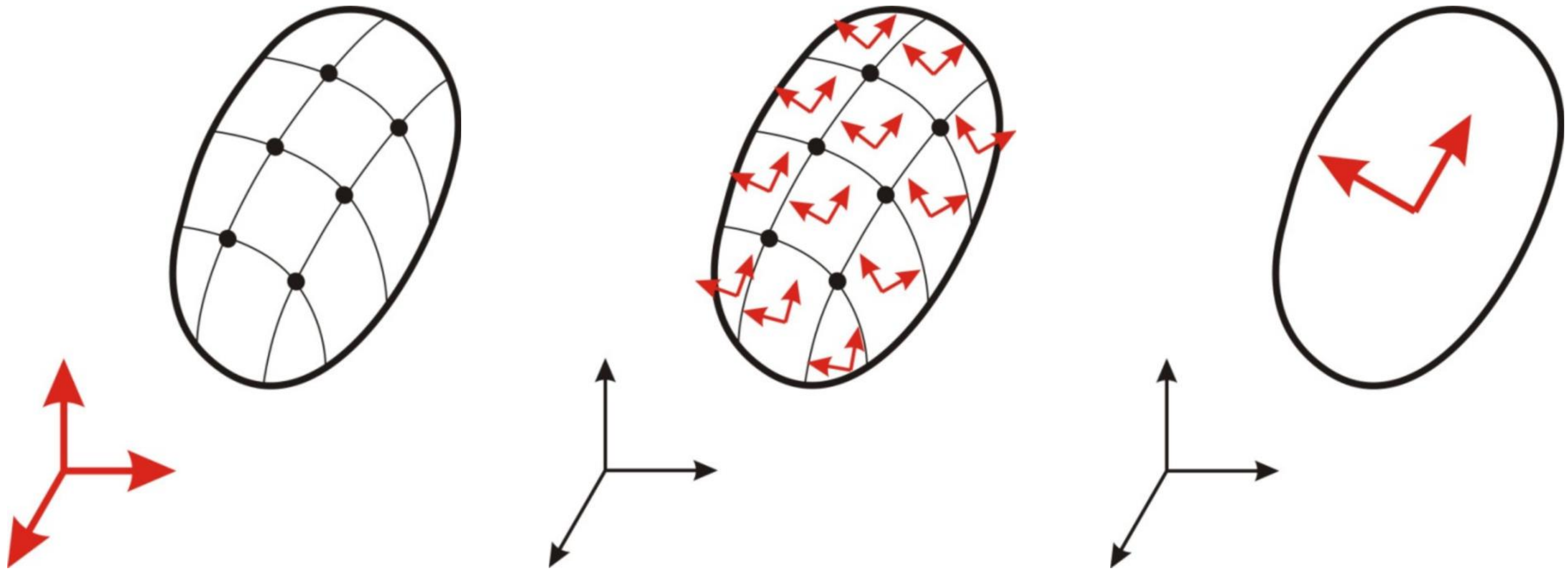
Geometrical modeling



Multibody system dynamics

MBS: Several parameterizations

Inertial Frame | Corotational Frame | Floating Frame



No distinction | Rigid motion + small deformation

Absolute coordinates (FE) | Rigid + Elast. Coord.

Inertia forces are easily computed in an inertial reference frame.
Internal forces are easily computed in a body-attached frame.

Equation of FEM-MBS dynamics

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- Motion of the flexible body (FEM) is represented by absolute nodal coordinates **q** (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{ext} - \mathbf{g}^{int} - \mathbf{g}^{gyr}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}$$

- The solution is based on a Lagrange multiplier method

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T(\mathbf{q}, t)\boldsymbol{\lambda} &= \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ \mathbf{\Phi}(\mathbf{q}, t) &= \mathbf{0}, \end{aligned}$$

with the initial conditions

$$\mathbf{q}(0) = \mathbf{q}_0 \text{ and } \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0.$$

Time Integration

- The set of nonlinear DAE is solved using the generalized-α method (Chung and Hulbert, 1993)
- Definition of a pseudo acceleration vector **a**:

$$(1-\alpha_m)\,\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\,\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n,$$

- Newmark integration formulae

$$\begin{aligned}\mathbf{q}_{n+1} &= \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2\,(1/2-\beta)\,\mathbf{a}_n + h^2\beta\mathbf{a}_{n+1}\\ \dot{\mathbf{q}}_{n+1} &= \dot{\mathbf{q}}_n + h\,(1-\gamma)\,\mathbf{a}_n + h\gamma\mathbf{a}_{n+1},\end{aligned}$$

- Solve iteratively the linearized dynamic equation system (Newton-Raphson scheme)

$$\begin{aligned}\mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{\Phi}_{\mathbf{q}}^T\Delta\boldsymbol{\lambda} &= \Delta\mathbf{r}\\ \mathbf{\Phi}_{\mathbf{q}}\Delta\mathbf{q} &= \Delta\mathbf{\Phi}\end{aligned}$$

$$\text{where } \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda} - \mathbf{g}$$

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The Equivalent Static Load method

The EQSL Method

- Difficulties of dealing with dynamic constraints and loadings
- Definition of the Equivalent Static Load:

  When a dynamic load is applied to a structure, the equivalent static load is defined as the static load that produces the same displacement field as the one created by the dynamic load at an arbitrary time. (Kang, Park & Arora, 2005)

- Introduction of the concept on a linear structure

  Equilibrium equation:
  $$\mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t) + \mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t)$$
  $$\Leftrightarrow \mathbf{K}(\mathbf{x})\mathbf{y}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t)$$

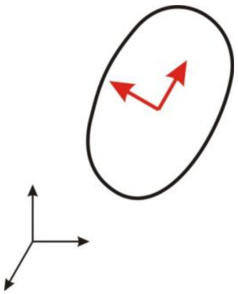
  The EQSL:
  $$\mathbf{f}_{eq}(t) = \mathbf{s}(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{y}}(t)$$

  - In a discrete time domain, it exists one EQSL for each integration time step.
  - The dynamic response optimization problem is transformed in a static response optimization problem with multiple load cases.

The EQSL Method for MBS optimization

- Equations of motion for body *i*

Floating Frame



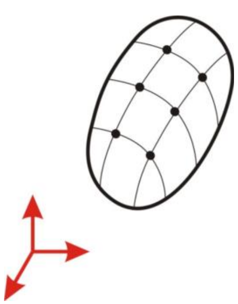
$$\begin{bmatrix} \mathbf{m}^i_{RR} & \mathbf{m}^i_{R\theta} & \mathbf{m}^i_{Rf} \\ & \mathbf{m}^i_{\theta\theta} & \mathbf{m}^i_{\theta f} \\ \text{sym.} & & \mathbf{m}^i_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{R}}^i \\ \ddot{\boldsymbol{\theta}}^i \\ \ddot{\mathbf{q}}^i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{K}^i_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{R}^i \\ \boldsymbol{\theta}^i \\ \mathbf{q}^i_f \end{bmatrix} = - \begin{bmatrix} \mathbf{C}^T_{R^i} \\ \mathbf{C}^T_{\theta^i} \\ \mathbf{C}^T_{q^i_f} \end{bmatrix} \boldsymbol{\lambda} + \begin{bmatrix} \mathbf{g}^i_R \\ \mathbf{g}^i_\theta \\ \mathbf{g}^i_f \end{bmatrix}$$

$$\mathbf{K}^i_{ff}\mathbf{q}^i_f = \underbrace{-\mathbf{m}^i_{fR}\ddot{\mathbf{R}}^i - \mathbf{m}^i_{f\theta}\ddot{\boldsymbol{\theta}}^i - \mathbf{m}^i_{ff}\ddot{\mathbf{q}}^i_f - \mathbf{C}^T_{q^i_f}\boldsymbol{\lambda} + \mathbf{Q}^i_f}_{\mathbf{g}^i_{eq}(t)}$$

EQSL for body *i* at time *t*
(Kang, Park & Arora, 2005)

➔ *The EQSL method is tailored to a floating frame formalism.*
➔ *Each body is optimized independtly.*

Inertial Frame



- Linearized equations of the equations of motion

$$\mathbf{M}(t_i)\Delta\ddot{\mathbf{q}}(t_i) + \mathbf{C}_t(t_i)\Delta\dot{\mathbf{q}}(t_i) + \mathbf{K}_t(t_i)\Delta\mathbf{q}(t_i) + \boldsymbol{\Phi}^T_{\mathbf{q}}(t_i)\Delta\boldsymbol{\lambda}(t_i) = 0$$

$$\mathbf{K}_t(t_i)\Delta\mathbf{q}(t_i) = -\mathbf{M}(t_i)\Delta\ddot{\mathbf{q}}(t_i) - \mathbf{C}_t(t_i)\Delta\dot{\mathbf{q}}(t_i) - \boldsymbol{\Phi}^T_{\mathbf{q}}(t_i)\Delta\boldsymbol{\lambda}(t_i)$$

While the structure of the equations seems similar to the equilibrium equation of a static linear structure, the optimization process can not be directly based on this equation.

Differences between the two MBS approaches

Floating Frame

- Decoupling between the component flexibility
  - One stiffness matrix $\mathbf{K}^i$ is defined per component.
- The matrix K^i is constant with respect to the system configuration in the body attached frame.
- Decoupling between rigid body motions and deformations

Inertial Frame

- No decoupling between the component flexibility
  - $\mathbf{K}_t$ is related to the whole system.
- The matrix $\mathbf{K}_t$ evolves with respect to system configuration.
- No decoupling between rigid body motions and deformations in the displacement vector **q.**

In general

Floating Frame

- Originally developed for <u>rigid MBS</u>
  - Flexibility introduced later
- Unable to represent geometric stiffening

Inertial Frame

- Developed to obtain an <u>integrated approach of the flexibility in MBS</u>
- For instance, stress analysis is straightforward

A <u>post-processing step</u> to define the EQSL with an ***inertial*** frame approach

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1. For each component, it is possible to extract its tangent stiffness matrix by selecting suitable generalized coordinates

2. To avoid storing $\mathbf{K}_t$ at each time step, a reference state is considered (t_{ref})➔ Need of suitable transformations

3. Key point: introduction of a <u>corotational frame</u> in a post processing step for each component
   - Enables to define the deformation in the attached-body frame
   - Enables to define the appropriate transformations to go back to the reference state

Using the cororational frame

$$\mathbf{K}_t(t_i)\Delta\mathbf{q}(t_i) = -\mathbf{M}(t_i)\Delta\ddot{\mathbf{q}}(t_i) - \mathbf{C}_t(t_i)\Delta\dot{\mathbf{q}}(t_i) - \mathbf{\Phi}_{\mathbf{q}}^T(t_i)\Delta\boldsymbol{\lambda}(t_i)$$

$$\mathbf{K}_t^b(t_{ref})\mathbf{u}^b(t) = \mathbf{g}_{eq}^b(t)$$

Flowchart of the optimization process using the EQSL method

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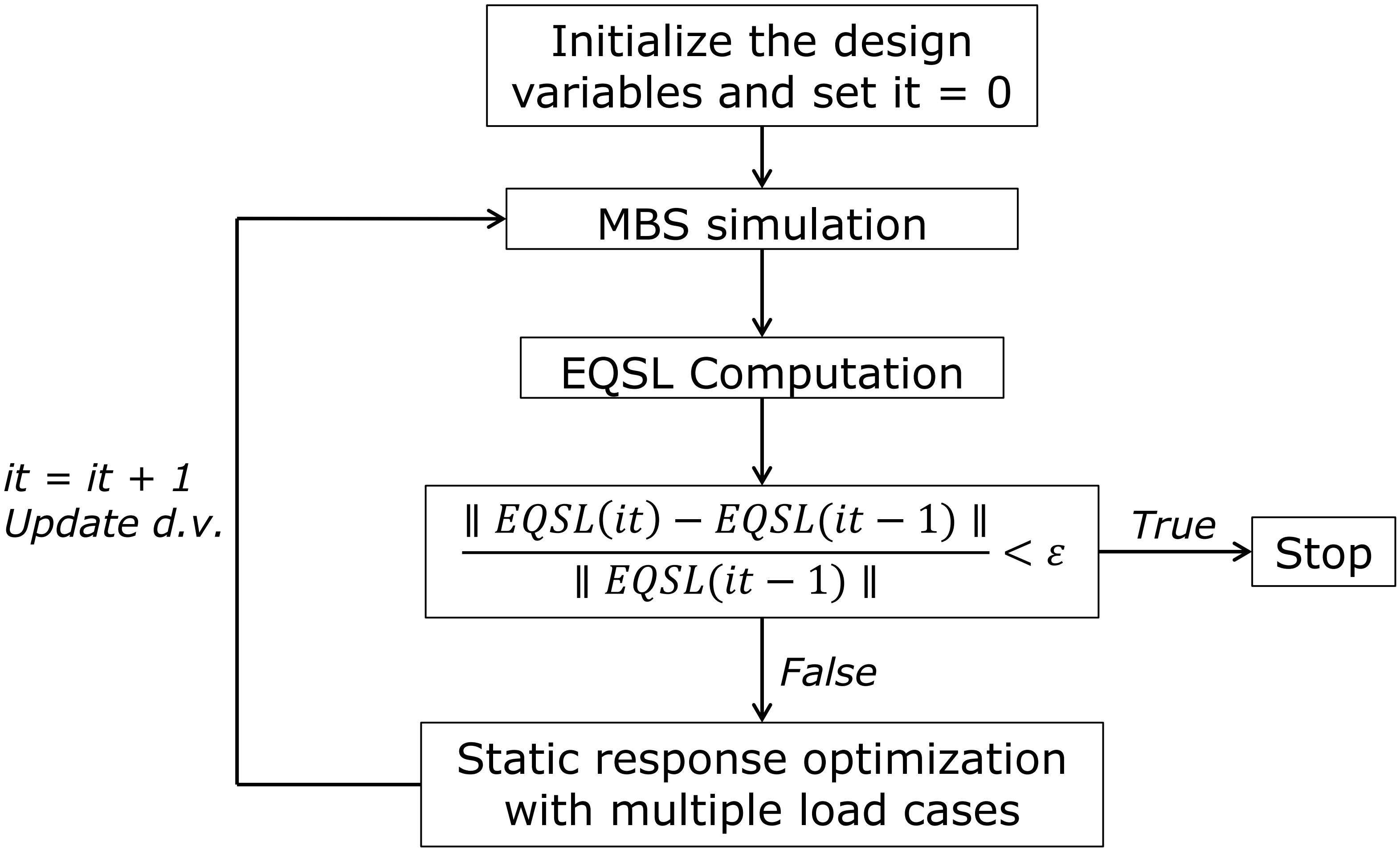
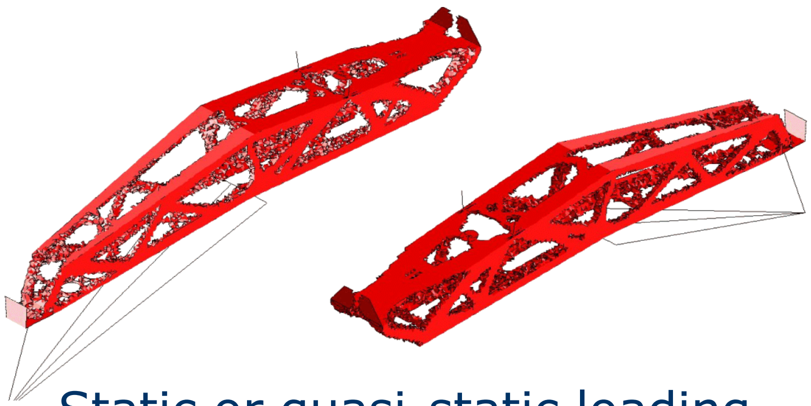


The "fully integrated" method

Evolution of virtual prototyping

- Structural optimization

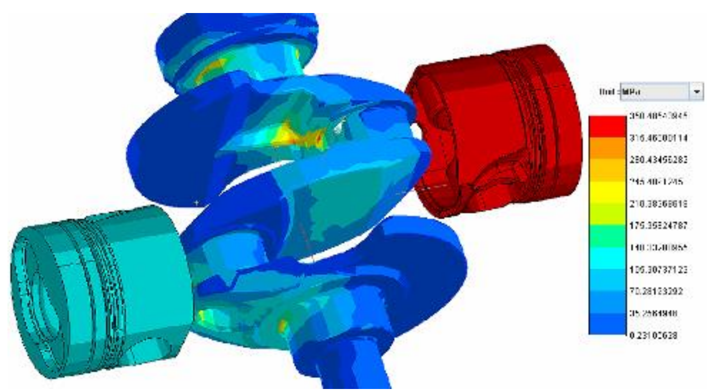


Static or quasi-static loading

- Flexible multibody systems

Dynamic loading

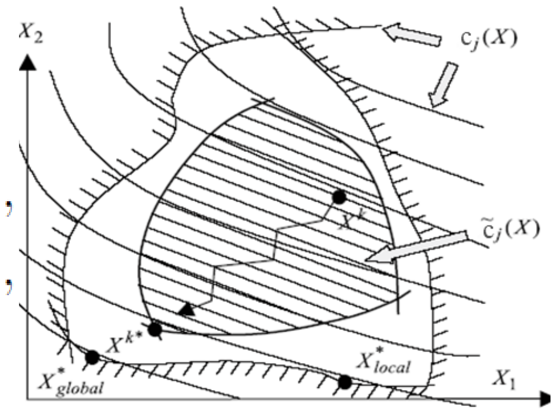
- Integrated optimization of flexible components in multibody systems



General form of the optimization problem

- Design problem casted in a mathematical programming problem

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \varphi(\mathbf{x}) \\
\text{subject to} \quad & \text{Equilibrium equation} \\
& c_j(\mathbf{x}) \leq \overline{c}_j, \qquad j = 1, \ldots, n_c, \\
& \underline{x}_v \leq x_v \leq \overline{x}_v, \qquad v = 1, \ldots, n_v,
\end{aligned}
$$



  - Provides a general and robust framework to the solution procedure
  - Various efficient solvers can be used.

- Integrated method - Formulation:

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \varphi(\mathbf{x}) \\
\text{subject to} \quad & \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{\Phi}_q^T(\mathbf{q}, t)\boldsymbol{\lambda} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t), \\
& \mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0}, \\
& c_j(\mathbf{x}, t) \leq \overline{c}_j, \qquad j = 1, \ldots, n_c, \\
& \underline{x}_v \leq x_v \leq \overline{x}_v, \qquad v = 1, \ldots, n_v.
\end{aligned}
$$

Sensitivity analysis

- Finite difference scheme can be CPU-time consuming.

- A semi-analytical method has been developed by O. Brüls and P. Eberhard (2008) which can be integrated in the generalized-α scheme.

$$\mathbf{M}\frac{d\ddot{\mathbf{q}}}{dp_u} + \mathbf{C}_t\frac{d\dot{\mathbf{q}}}{dp_u} + \mathbf{K}_t\frac{d\mathbf{q}}{dp_u} + \mathbf{\Phi}_{\mathbf{q}}^T\frac{d\boldsymbol{\lambda}}{dp_u} = -\frac{\partial \mathbf{r}}{\partial p_u}$$

$$\mathbf{\Phi}_{\mathbf{q}}\frac{d\mathbf{q}}{dp_u} = -\frac{\partial}{\partial p_u}\mathbf{\Phi}$$

  - Sensitivity equations are linear with respect to $\frac{d\mathbf{q}}{dp_u}$ and $\frac{d\boldsymbol{\lambda}}{dp_u}$.

  - Same structure as the linearized equations of motion
    - Same integration procedure except for the residuals
    - Tangent iteration matrix is the same as the one of the original problem
    - No need to apply a Newton-Raphson procedure

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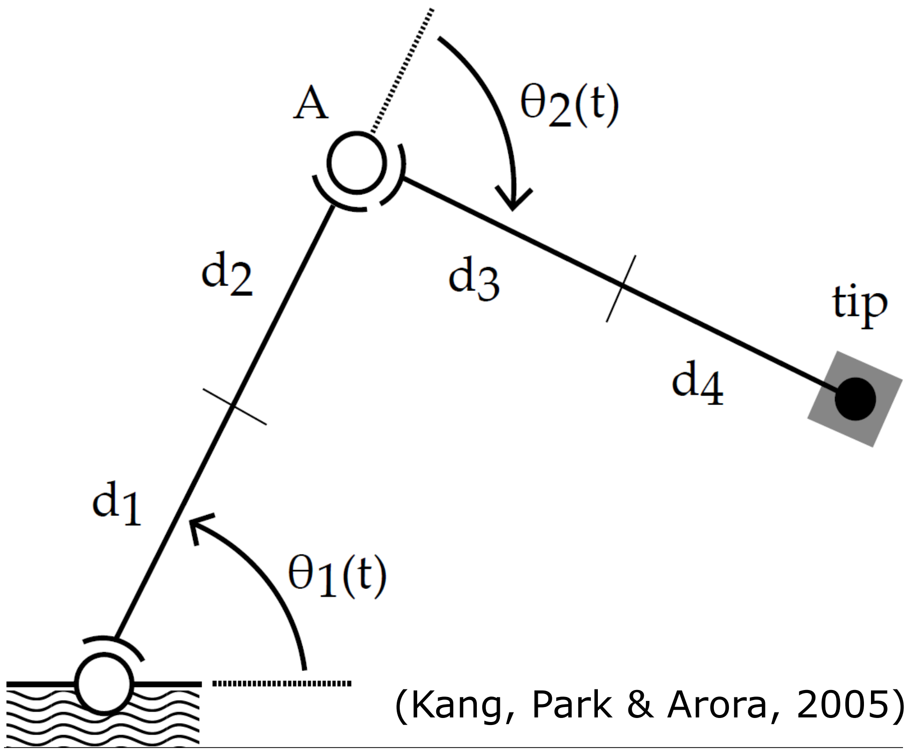
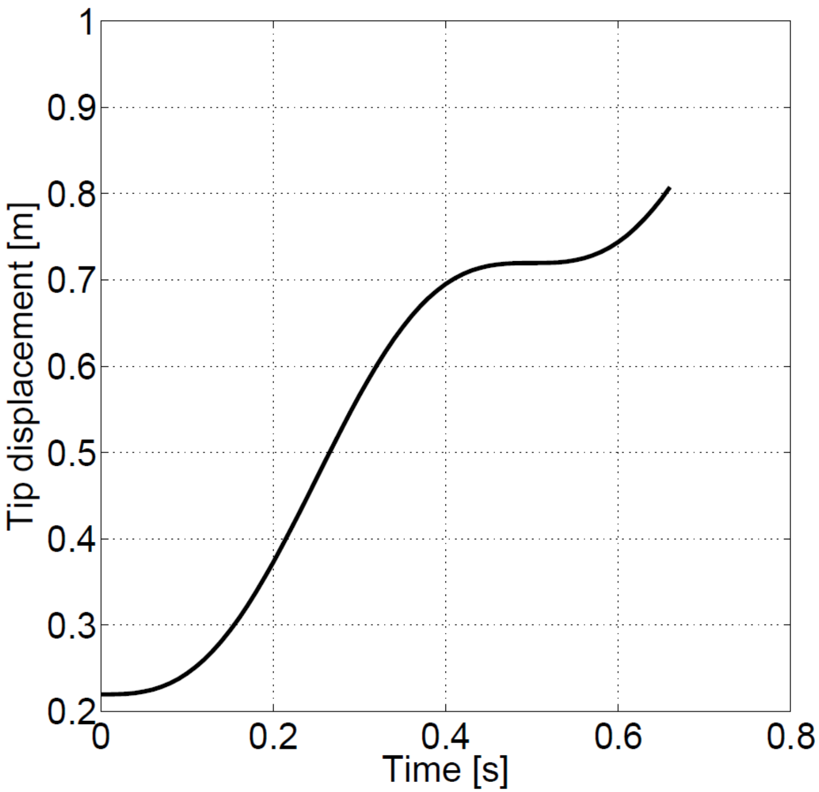
Numerical Applications

A 2-dof robot subject to tracking trajectory constaints



(Kang, Park & Arora, 2005)



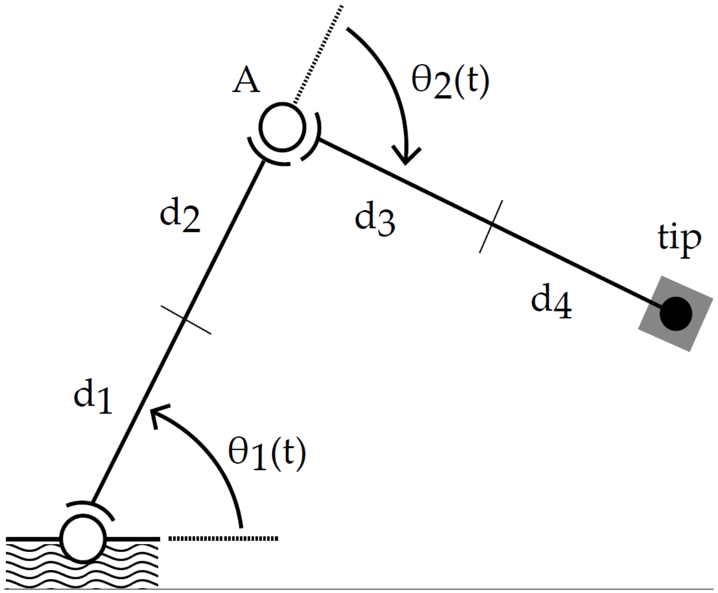
- Minimize the mass
- 4 beam elements
- Design variables: diameters
- Imposed rotations at hinges
- Time step: 0,0005 [s]

$$\Delta x_{tip}(t) = \Delta y_{tip}(t) = \frac{0.5}{T}\left(t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right)$$

First numerical application

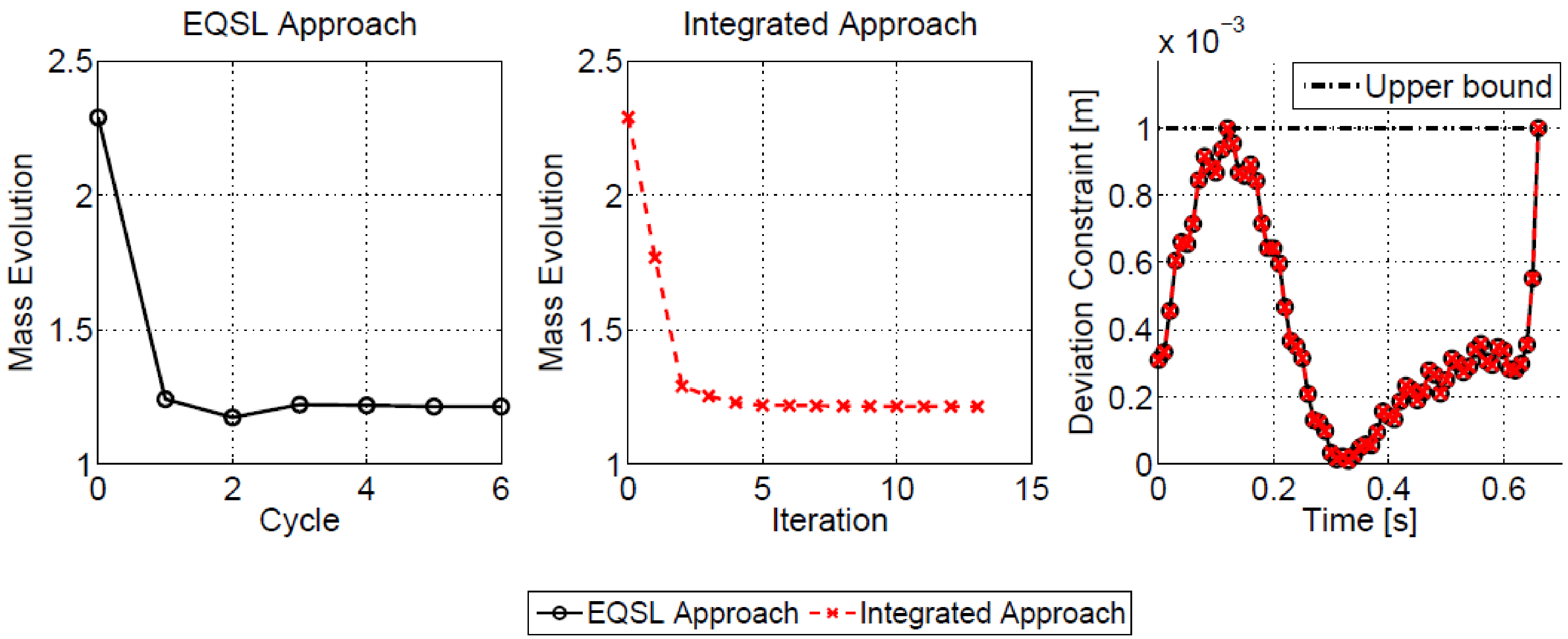
$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & m(\mathbf{x}) \\
\text{subject to} \quad & \sqrt{\delta y_a^2(t_n) + \delta y_{tip}^2(t_n)} \leq 0.001 \ [\text{m}], \quad n = 1, \ldots 67, \\
& 0.02 \ [\text{m}] \leq x_v \leq 0.06 \ [\text{m}], \quad v = 1, \ldots, 4.
\end{aligned}
$$

where $\delta y_a(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the vertical deflections in the inertial frame of the first link at the hinge A and of the second link at the tip.



First numerical application - Results

	Mass [kg]	Iterations	Inner iterations	d_1 [mm]	d_2 [mm]	d_3 [mm]	d_4 [mm]
EQSL Method	1.213	6	61	45.40	32.76	37.99	26.83
Integrated Method	1.214	13	/	45.44	32.69	38.08	26.78



Second numerical application

$$\underset{\mathbf{x}}{\text{minimize}} \quad m(\mathbf{x})$$

$$\text{subject to} \quad \sqrt{\delta x_{tip}^2(t_n) + \delta y_{tip}^2(t_n)} \leq 0.001 \text{ [m]}, \quad n = 1, \ldots 51,$$

$$0.02 \text{ [m]} \leq x_v \leq 0.06 \text{ [m]}, \qquad i = 1, \ldots, 4.$$

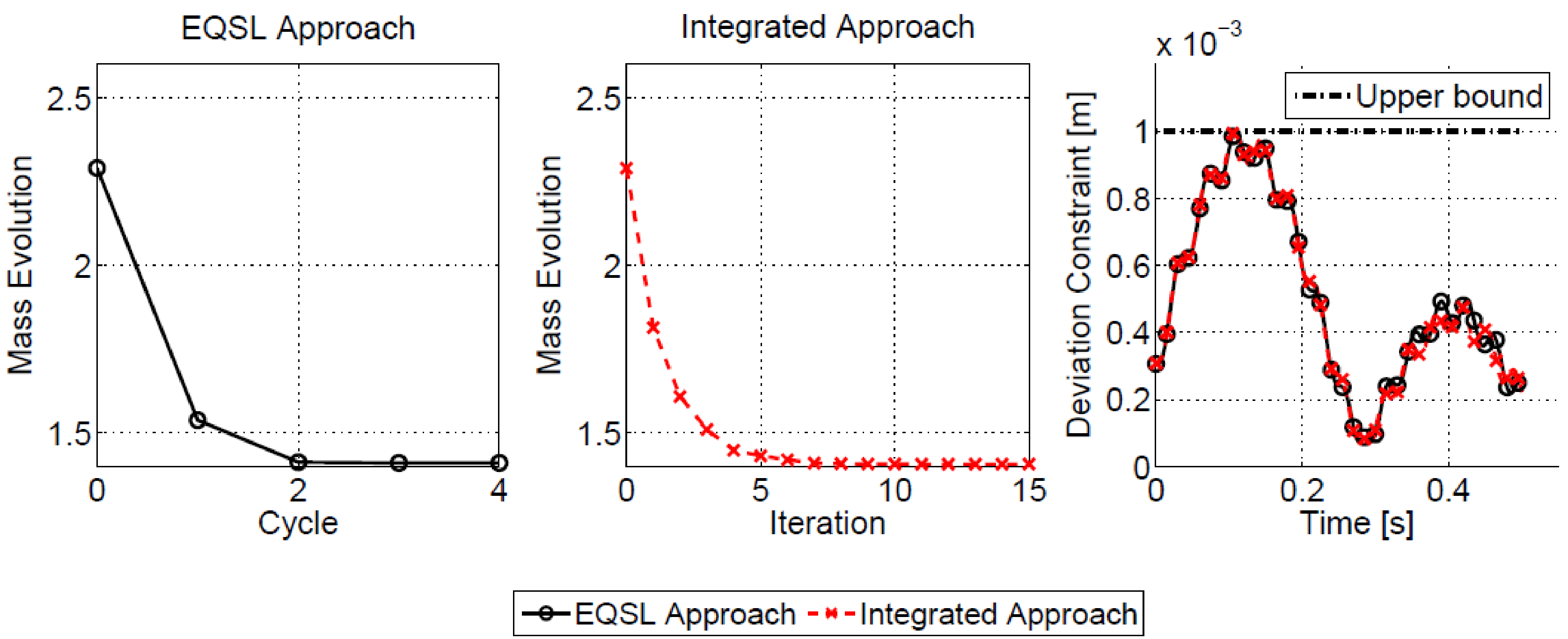
where $\delta x_{tip}(t_n)$ and $\delta y_{tip}(t_n)$ are respectively the horizontal and vertical deflections of the robot tip in the inertial frame.

- Only the extremity of the second robot link is concerned by the optimization constraint.
- It is a constraint on the global system behavior.
- With the EQSL method, the components are optimized independently.

  The first link does not appear in the constraint formulation while it is obvious that its flexibility has a contribution to the tip displacement.
- The problem can be overcome by using a sum over the deflection of all the components.
- This problem does not appear with the fully integrated method as the system is also treated as a whole during the optimization process.

Second numerical application - Results

	Mass [kg]	Iterations	Inner iterations	d_1 [mm]	d_2 [mm]	d_3 [mm]	d_4 [mm]
EQSL Method	1.411	4	38	47.88	34.51	42.11	30.08
Integrated Method	1.408	15	/	48.59	34.82	41.60	29.02



Conclusions

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- We proposed a method to derive the EQSL adapted to the nonlinear finite element based MBS formalism.
- Both methods can converge towards the same optimum for the considered example.
- Fundamental difference:
  - Fully integrated method: 1 dynamic analysis per iteration
  - EQSL method: 1 dynamic analysis + a set of static analysis per cycle
- For slowly varying body loads, the EQSL method normally requires less dynamic simulations and one dynamic analysis is more time consuming than one static analysis.
- The formulation of global behavior constraint can become rather complex with the EQSL method as the components are decoupled (e.g. multiple loop system).

Perspectives

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- Ongoing work investigates systems with design dependent loading and more advanced cases as different behaviors are expected for the methods.

- A Lie group formulation enables to have a constant tangent stiffness matrix in the material frame and enables to have a measure of the deformation in the material frame.

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Thank You Very Much For Your Attention

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