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# Nonlinear guitar loudspeaker simulation

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# **ABSTRACT**

In this study, we simulated in real time the sound of a guitar amplifier loudspeaker, including its non-linear behavior. The simulation method is based on a non-linear convolution of the signal emitted by the instrument with the Volterra kernels, which were measured in anechoic conditions with a sine-sweep technique. The model has been implemented in a "VST" (Virtual Studio Technology) audio plug-in.

The loudspeaker simulation can be performed in real time with the Volterra kernels up to the third order and offers a good accuracy. Informal tests revealed that simulated and real sounds were very close, although approximately 50 percents of the tested musicians were still able to hear a small difference.

### 1. INTRODUCTION

Musicians use loudspeakers with behaviors different from Hi-Fi loudspeaker's [1]. Where the maximal linearity is desired for Hi-Fi loudspeakers, the nonlinear behavior is often preferred for guitar loudspeakers. These nonlinearities are welcome since they significantly define the final "sound character" of a guitarist.

The purpose of this article is to show how to take into account these nonlinearities in order to simulate a guitar amplifier loudspeaker in his box (called "cab").

These simulations allow musicians to:

- Have many sounds with different simulated loudspeakers.
- Play with their tube-amplifier and benefit of their strong nonlinearities at high level, with a controlled output sound pressure level.
- Have a greater portability of their equipment which is often very heavy and cumbersome.



Figure 1: Replacement of the Cab by its simulation

# 2. NONLINEAR MODELING

# 2.1. Volterra series

The modeling method developed here is based on a black-box approach by means of the Volterra series theory. It takes into account the nonlinear behavior of the loudspeaker.

The following series expansions express the output y[n] of a nonlinear system truncated to the k-th order [2]:

$$y[n] = \sum_{\tau_{1} = -\infty}^{\infty} h_{1}[\tau_{1}] x[n - \tau_{1}]$$

$$+ \sum_{\tau_{1} = -\infty}^{\infty} \sum_{\tau_{2} = -\infty}^{\infty} h_{2}[\tau_{1}, \tau_{2}] x[n - \tau_{1}] x[n - \tau_{2}]$$

$$+ \dots$$

$$+ \sum_{\tau_{k} = -\infty}^{\infty} \dots \sum_{\tau_{k} = -\infty}^{\infty} h_{k}[\tau_{1}, \dots, \tau_{k}] x[n - \tau_{1}] \dots x[n - \tau_{k}]$$

(1)

Where  $h_k[\tau_1,...,\tau_k]$  is the discrete k-th order Volterra kernel and x[n] is the input signal.

Assuming that the loudspeaker is a nonlinear system without memory [2], we can rewrite system (1) as:

$$y[n] = h_1[n] \otimes x[n] + h_2[n] \otimes x^2[n] + \dots + h_k[n] \otimes x^k[n]$$
(2)

The memory of the system could be modeled, but with more complexity [4]. This is not taken into account in this research.

# 2.2. Impulse Response (IR) measurement

The goal is to determine the Volterra kernels. We have used a logarithmic sine sweep technique [2] which provides a way to separate the different orders of distortion from the linear part.

Choosing a signal x[n] and its inverse  $x^{-1}[n]$  such that:

$$x[n] = A\sin(\phi[n])$$
with  $\phi[n] = B\left[\left(\frac{w_2}{w_1}\right)^{\frac{n}{N-1}} - 1\right]$ 
and  $B = \frac{w_1(N-1)/fs}{\ln\left(\frac{w_2}{w_1}\right)}$ 

$$x^{-1}[n] = x[N-1-n] \left(\frac{w_2}{w_1}\right)^{\frac{-n}{N-1}}$$
 (4)

Where T, N,  $w_1$ ,  $w_2$  are the sweep duration, number of samples of the sweep, initial and final frequency respectively and with  $n \in [0, N-1]$ .

If  $w_1$  and  $w_2$  are taken sufficiently large, the convolution of the sweep and its inverse leads to a scaled and time-shifted impulsion [3]:

$$\sum_{k=0}^{N-1} x[k] x^{-1}[n-k] = C \delta[n-N-1]$$
 (5)

So that, if the system is linear:

$$y[n] \otimes x^{-1}[n] = y_1[n] \otimes x^{-1}[n] = C h_1[n-N-1]$$
(6)

#### 2.3. Volterra kernels reconstruction

The reconstruction is complicated by the fact that  $x^k[n]$  has several frequency contributions. For example,  $x^3[n]$  has a contribution in  $\phi[n]$  and  $3\phi[n]$  since:

$$x^{3}[n] = \frac{A^{3}}{4} [3\sin(\phi[n]) - \sin(3\phi[n])]$$
 (7)

We can express  $x^k[n]$  as a sum of  $x[n+\Delta_k]$  if we take into account that:

$$k\phi[n] = kB \left[ \left( \frac{w_2}{w_1} \right)^{\frac{n}{N-1}} - 1 \right] = \phi[n + \Delta_k] - B(M-1)$$

$$\Delta_k = (N-1) \frac{\ln(k)}{\ln(w_2/w_1)}$$
(8)

Proceeding the deconvolution:

$$z[n] = y[n] \otimes x^{-1}[n] \tag{9}$$

We obtain a series of impulse responses  $z_k[n]$  which are delayed from  $\Delta_k$  with respect to the first order impulse  $z_1[n]$  as can be seen in Figure (2).

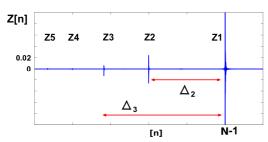


Figure 2: Loudspeaker impulse response obtained after deconvolution of the emitted signal, when it is excited by a logarithmic sine sweep.

Assembling terms that are time-shifted by the same  $\Delta_k$  and considering that a Cosine function can be transformed in a Sine by convolution with Hilbert Transform  $h_H[n]$  [4]. From (9), we obtain:

$$z_{1}[n] = C \left[ h_{1}[n] + \frac{3A^{2}}{4} h_{3}[n] + \frac{10A^{4}}{16} h_{5}[n] \right]$$

$$z_{2}[n] = \frac{AC}{2} \left[ \cos(B)h_{2}[n] \otimes h_{H}[n] \right]$$

$$+ \frac{A^{3}C}{8} \left[ 4h_{4}[n] \otimes h_{H}[n] \cos(B) \right]$$

$$\times -4h_{4}[n] \sin(B)$$

$$z_{3}[n] = -\frac{A^{2}C}{4} \left[ \frac{\sin(2B)h_{3}[n] \otimes h_{H}[n]}{\times -\cos(2B)h_{3}[n]} \right]$$

$$-\frac{A^{4}C}{16} \left[ \frac{5h_{5}[n] \otimes h_{H}[n] \sin(2B)}{\times -5h_{5}[n] \cos(2B)} \right]$$

$$z_{4}[n] = \frac{A^{3}C}{8} \left[ -h_{4}[n] \otimes h_{H}[n] \cos(3B) \right]$$

$$\times +4h_{4}[n] \sin(3B)$$

$$z_{5}[n] = \frac{A^{4}C}{16} \left[ h_{5}[n] \otimes h_{H}[n] \sin(4B) \right]$$

$$\times +h_{5}[n] \cos(4B)$$
(10)

As we can see in (10), impulse responses  $z_k[n]$  contain parts from several Volterra kernels.

We can obtain Volterra kernels  $h_k[n]$  from the measured impulse responses  $z_k[n]$  by expressing (10) in the frequency domain (for w>0):

$$H_{1}(w) = \frac{1}{C} \left\{ Z_{1}(w) + 3e^{j2B} Z_{3}(w) + 5e^{j4B} Z_{5}(w) \right\}$$

$$H_{2}(w) = \frac{2j}{AC} e^{jB} \left\{ Z_{2}(w) + 4e^{j2B} Z_{4}(w) \right\}$$

$$H_{3}(w) = -\frac{4}{CA^{2}} e^{j2B} \left\{ Z_{3}(w) + 5e^{j2B} Z_{5}(w) \right\}$$

$$H_{4}(w) = -\frac{8j}{CA^{3}} e^{j3B} Z_{4}(w)$$

$$H_{5}(w) = \frac{16}{CA^{4}} e^{j4B} Z_{5}(w)$$
(11)

Where  $H_k(w)$  and  $Z_k(w)$  are the Fourier Transform of Volterra kernels and measured impulses respectively.

Relations (11) are different from those described by Pr.Farina in [2].

We can find relations for w < 0 since [5]:

$$H_k(-w) = H_k^*(w)$$

$$\forall h_k[n] \in \Re$$
(11)

# 3. TESTING AND CONCLUSIONS

# 3.1. Listening test considerations

We have implemented, in real time, the loudspeaker simulator in a VST (Virtual Studio Technology) plug-in for audio sequencer. As can be seen in Figure (3), we have compared the sound that comes from the loudspeaker placed in an anechoic room with the one coming from our simulation.

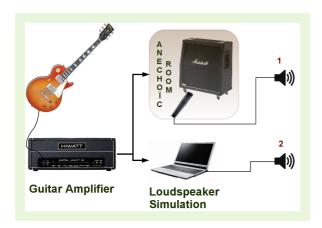


Figure 3: Comparison between real speaker and simulated speaker

# 3.2. Listening test results

To avoid subjective effects, we have made a blind test with 17 musicians. As we can see in Figure (4), we ask them to listen *sample 0* which is a record of a musician playing on the real loudspeaker and compare it with four other samples. In these four samples, three are the same as *sample 0* and one comes from our loudspeaker simulation (*sample 3*). Finally, approximately half of tested musicians still able to differentiate *simulated sound* from *real sound*.

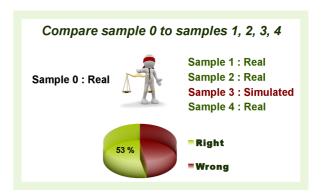


Figure 4: Blind tests between real sound loudspeaker and simulated sound loudspeaker

## 4. ACKNOWLEDGEMENTS

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