Traffic-induced Vibration TD-1

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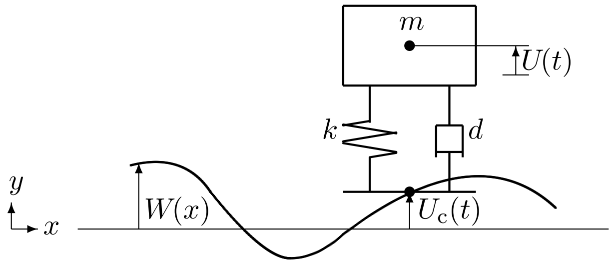

Figure 1: Schematic representation of the problem.

- We consider a highly simplified model of a vehicle that moves over an uneven support at a constant horizontal velocity of v.
- Let the vehicle body be represented by a mass, and let the vehicle suspension be represented by a spring and dashpot that link the vehicle body to the contact point with the support.
- Let the support unevenness be modeled by a mean-square stationary zero-mean second-order stochastic process $\{W(x),\ x \in \mathbb{R}\}$ with power spectral density function s_W.
- Let $\{U_c(t),\ t \in \mathbb{R}\}$ be the vertical displacement of the contact point caused by the support unevenness.
- Let $\{U(t),\ t \in \mathbb{R}\}$ be the resulting vertical displacement of the vehicle body with respect to the static equilibrium configuration on a flat support.

1. Using the fact that the vehicle rolls at a horizontal velocity of v, write the equation that relates the vertical displacement of the contact point $\{U_c(t),\ t \in \mathbb{R}\}$ to the support unevenness $\{W(x),\ x \in \mathbb{R}\}$.
2. Write the equations that relate the mean function, the autocorrelation function, and the power spectral density function of $\{U_c(t),\ t \in \mathbb{R}\}$ to those of $\{W(x),\ x \in \mathbb{R}\}$.
3. Express dynamical equilibrium and deduce the stochastic differential equation that governs the displacement $\{U(t),\ t \in \mathbb{R}\}$ of the vehicle body.
4. Consider the stochastic differential equation that you obtained under 3 as a convolution transformation whose input is the stochastic process $\{U_c(t),\ t \in \mathbb{R}\}$ and whose output is the stochastic process $\{U(t),\ t \in \mathbb{R}\}$. Write the expression for the FRF $\hat{h}$ associated with this convolution filter. Interpret your result: indicate frequency regions wherein the vehicle body follows the motion of the contact point (vibration transmission) and wherein the vehicle body is isolated from the motion of the contact point (vibration isolation).
5. Write the equations that relate the mean function and the power spectral density function of $\{U(t),\ t \in \mathbb{R}\}$ to those of $\{W(x),\ x \in \mathbb{R}\}$.
6. Consider $s_W(\xi) = s_0/\big(1 + \frac{|\xi|}{\xi_0}\big)^\alpha$ and the numerical values of $s_0 = 5 \times 10^{-5}\,\mathrm{m}^3$, $\alpha = 2$, $\xi_0 = 0.5\,\mathrm{m}^{-1}$, $m = 470\,\mathrm{kg}$, $k = 36 \times 10^{+3}\,\mathrm{N/m}$, and $d = 0.10 \times 2 \times \sqrt{mk}$. Plot s_W and $|\hat{h}|$, first on a linear scale and then on a loglog scale. Subsequently, plot s_U for $v = 20\,\mathrm{m/s}$ and $v = 40\,\mathrm{m/s}$, first on a linear scale and then on a loglog scale. Interpret your results.