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Short Communication

Multi-frequency nonlinear energy transfer from linear oscillators to mdof essentially nonlinear attachments

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Abstract

We report on multi-frequency energy transfer from a two-mode, initially excited linear system to a multi-degree-of-freedom (mdof) essentially nonlinear attachment. This occurs through simultaneous resonance interactions of both linear modes with a set of nonlinear normal modes (NNMs) of the attachment, and is studied utilizing numerical wavelet transforms. The multi-frequency nonlinear energy transfer discussed herein differs from multi-frequency energy transfer caused by resonance capture cascading where sequential energy transfer from a set of linear modes to single-dof nonlinear attachments takes place.

1. Introduction

In previous works (for example, Ref. [1]) *passive nonlinear and irreversible energy transfer* of broadband vibration energy from a main (linear) damped system to a damped,

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single-degree-of-freedom (sdof) essentially nonlinear attachment has been studied. It was shown that this irreversible energy transfer is caused by 1:1 *resonance capture* [2,3] of a mode of the linear system and the nonlinear attachment. An interesting feature of linear systems with essentially nonlinear sdof attachments is the possibility of *resonance capture cascades*, that is, of a sequence of resonance captures involving the sdof nonlinear attachment and isolated modes of the linear system. This results in a series of energy pumping events occurring at different frequencies, with sudden transitions to lower frequencies between sequential events. Such resonance capture cascades result in multi-frequency energy transfer from the linear system to the sdof nonlinear attachment, but the energy extraction from the linear modes takes place sequentially and not simultaneously. This note considers an alternative design based on multi-degree-of-freedom (mdof) essentially nonlinear attachments that enable simultaneous energy absorption from multiple modes of the linear system.

2. A new way for multi-frequency nonlinear energy transfer

It was shown previously by Vakakis et al. [1] that sdof essentially nonlinear attachments are capable of passively absorbing energy from multiple linear modes through *resonance capture cascading*. The resulting multi-frequency energy pumping takes place sequentially, as the nonlinear attachment engages in resonance capture with a linear mode in the neighborhood of its natural frequency, before engaging the next mode at a different frequency range. It was shown that resonance capture cascading leads to multi-frequency irreversible energy transfer from the modes involved to the nonlinear attachment.

An example of a resonance capture cascade is shown in Fig. 1, where the wavelet transform (WT) of the transient response $v(t)$ of an sdof essentially nonlinear attachment weakly attached to a 2 dof impulsively forced linear oscillator is depicted:

$$\begin{aligned}
&\ddot{u}_1 + u_1(\omega_0^2 + 2\alpha) - \alpha u_2 = 0\\
&\ddot{u}_2 + u_2(\omega_0^2 + \alpha + \varepsilon) - \alpha u_1 - \varepsilon v = 0\\
&\ddot{v} + Cv^3 + \varepsilon\beta\dot{v} + \varepsilon(v - u_2) = 0
\end{aligned} \tag{1}$$

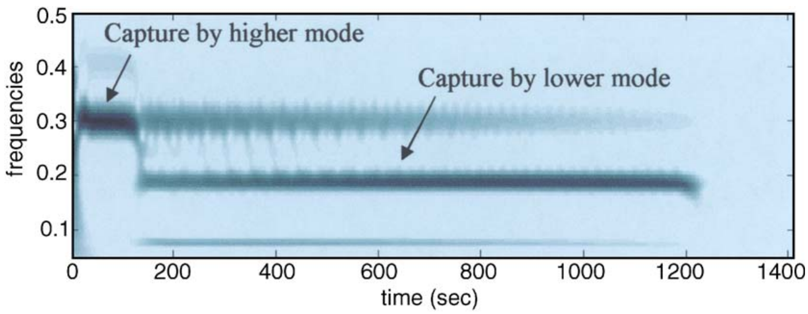

Fig. 1. Wavelet analysis of the transient response of an sdof nonlinear attachment engaged in a resonance capture cascade (frequencies in Hz).

The parameters used in this particular simulation were $\alpha = 1$, $\omega_0 = 1$, $\beta = 2$, $C = 3$ and $\varepsilon = 0.1$, with initial conditions $\dot{u}_1(0) = 25$ and all other initial conditions zero. Before we discuss the dynamics of this system we provide a brief summary of the applied WT algorithm.

The WT can be viewed as a basis for functional representation but is at the same time a relevant technique for time–frequency analysis. In contrast to the fast Fourier transform (FFT) which assumes signal stationarity, the WT involves a windowing technique with variable-sized regions. Small time intervals are considered for high-frequency components whereas the size of the interval is increased for lower-frequency components, thereby giving better time and frequency resolutions than the FFT. The Matlab program used for the WT computations reported in this note was developed at the University of Liège by Dr. V. Lenaerts in collaboration with Dr. P. Argoul from the ‘‘Ecole Nationale des Ponts et Chaussées’’ (Paris, France). Two types of mother wavelets $\psi_M(t)$ are considered: (a) The Morlet wavelet which is a Gaussian-windowed complex sinusoid of frequency ω_0, $\psi_M(t) = \mathrm{e}^{-t^2/2}\,\mathrm{e}^{\mathrm{j}\omega_0 t}$; and (b) the Cauchy wavelet of order n, $\psi_M(t) = \left[\mathrm{j}/(t+\mathrm{j})\right]^{n+1}$, where $\mathrm{j} = (-1)^{1/2}$. The frequency ω_0 for the Morlet WT and the order n for the Cauchy WT are user-specified parameters which allow one to tune the frequency and time resolutions of the results. It should be noted that these two mother wavelets provide similar results when applied to the signals considered in the present study. The plots shown represent the amplitude of the WT as a function of frequency (y-axis) and time (x-axis). Heavy shaded areas correspond to regions where the amplitude of the WT is high whereas lightly shaded regions correspond to low amplitudes. Such plots enable one to deduce the temporal evolutions of the dominant frequency components of the signals analyzed. In recent works by Argoul and co-workers [4–6], the Continuous Cauchy WT was applied to system identification of linear dynamical systems.

Returning to the dynamics depicted in the plot of Fig. 1, we note that a resonance capture cascade occurs, whereby, in the initial phase of the motion, the nonlinear attachment resonates with the higher linear mode, resulting in irreversible energy transfer from this mode to the nonlinear attachment at the neighborhood of the mode's natural frequency [1,7]. As energy decreases due to damping dissipation escape from this initial resonance capture the attachment engages in resonance with the lower linear mode resulting in energy transfer to the nonlinear attachment at a lower frequency range. In the final phase of the motion the dynamics escapes from this second resonance capture and decays to zero due to damping dissipation. The overall result of this resonance capture cascade is multi-frequency energy transfer from both modes of the linear subsystem to the nonlinear attachment; however, this energy transfer takes place in a sequential manner, since the sdof attachment is incapable of simultaneous resonance with *both* linear modes.

This leads us naturally to the following question which is the focal point of this note: is it possible by using mdof nonlinear attachments to extract simultaneously energy from multiple linear modes, through simultaneous dynamic (resonance) interactions of multiple nonlinear normal modes (NNMs) of the attachments with multiple modes of the linear system? In an attempt to answer this question, we modify the attachment, linear system configuration (1), by considering the system of Fig. 2, composed of a 2dof linear system, weakly connected to a 3dof essentially nonlinear attachment. Assuming that the two modes of the uncoupled linear system (for $\varepsilon = 0$) possess natural frequencies ω_1 and ω_2, the equations of motion of the

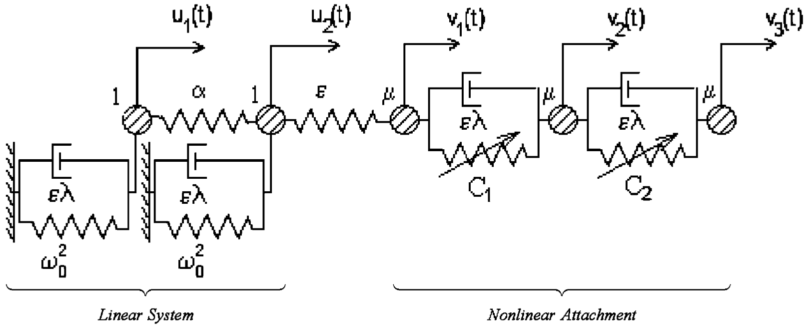

Fig. 2. The mdof system under consideration.

system are expressed as

$$
\begin{aligned}
&\ddot{x}_1 + \varepsilon\lambda\dot{x}_1 + \left(\omega_1^2 + \frac{\varepsilon}{2}\right)x_1 - \varepsilon\left(\frac{x_2}{2} + v_1\right) = 0\\
&\ddot{x}_2 + \varepsilon\lambda\dot{x}_2 + \left(\omega_2^2 + \frac{\varepsilon}{2}\right)x_2 - \varepsilon\left(\frac{x_1}{2} - v_1\right) = 0\\
&\mu\ddot{v}_1 + \varepsilon\lambda(\dot{v}_1 - \dot{v}_2) + \varepsilon\left(v_1 + \frac{x_2 - x_1}{2}\right) + C(v_1 - v_2)^3 = 0\\
&\mu\ddot{v}_2 + \varepsilon\lambda(2\dot{v}_2 - \dot{v}_1 - \dot{v}_3) + C(v_2 - v_1)^3 + C(v_2 - v_3)^3 = 0\\
&\mu\ddot{v}_3 + \varepsilon\lambda(\dot{v}_3 - \dot{v}_2) + C(v_3 - v_2)^3 = 0
\end{aligned}
\tag{2}
$$

where the variables x_1 and x_2 are the linear modal co-ordinates, and v_i are the displacements of the particles of the nonlinear attachment. In the following numerical simulations we consider initial excitation of the linear part of the system, with the nonlinear attachment being initially at rest. The aim is to study how energy gets transferred irreversibly (gets pumped) from the directly excited linear system to the nonlinear attachment, and the frequency content of this energy interaction.

Before considering energy interactions in the coupled system, it is instructive to discuss the dynamics of the two degenerate systems resulting in the limit of zero coupling, e.g., as $\varepsilon \to 0$. The degenerate nonlinear attachment possesses three NNMs; these are synchronous free periodic motions where all coordinates vibrate in-unison in similarity to the modes of classical linear vibration theory [8]. The first mode possesses zero frequency and corresponds to a rigid-body mode of the decoupled nonlinear attachment. In addition, there exist an in-phase NNM, $[v_2(t) - v_3(t)] = [v_1(t) - v_2(t)]$, and an out-of-phase NNM, $[v_2(t) - v_3(t)] = -[v_1(t) - v_2(t)]$. Based on these observations, we introduce at this point the nonlinear modal coordinates $z_1(t)$, $z_2(t)$ and $z_3(t)$, defined as

$$z_3(t) = [v_2(t) - v_3(t)] + [v_1(t) - v_2(t)], \quad z_2(t) = [v_2(t) - v_3(t)] - [v_1(t) - v_2(t)]$$

$$z_1(t) = v_1(t) + v_2(t) + v_3(t)$$

which denote the responses of the three NNMs of the decoupled nonlinear attachment. We note at this point that the nonlinear modal coordinates $z_i(t)$ cannot be used to decouple the equations of motion of the degenerate (decoupled) nonlinear attachment, since the principle of superposition does not hold in the nonlinear case. These coordinates are utilized only as a means to monitor the responses of the NNMs of the nonlinear attachment, in an effort to study the frequency content of the energy extraction from the linear system. The corresponding backbone curves (e.g., the frequency-energy dependences) of the modes of the two degenerate systems are depicted in Fig. 3. At crossing points between different backbone curves (such as points A, B and C in Fig. 3) internal resonances may occur, since at these points the frequency of one of the NNMs becomes identical to the natural frequency of one of the linear modes.

When coupling is introduced ($\varepsilon > 0$), the combined system is expected to possess NNMs that result as perturbations of the aforementioned modes of the two degenerate linear and nonlinear components. The resulting dynamics are expected to exhibit complicated behavior due to the dynamic interaction of the linear and essentially nonlinear components; close to points of internal resonances bifurcations of NNMs [1] occur that further complicate the dynamics. In Fig. 4 we depict the transient responses of the system of Fig. 2 with parameters,

$$\varepsilon = 0.25, \quad C = 0.15, \quad \lambda = 0.1, \quad \mu = 0.33, \quad \omega_1 = 1, \quad \omega_2 = \sqrt{3}$$

and initial conditions, $\dot{x}_1(0) = 5$, $\dot{x}_2(0) = -5$ with all other initial conditions zero. This corresponds to initial excitation of the anti-phase mode of the linear system. Comparing the responses of the linear and nonlinear components, we observe that the nonlinear attachment passively absorbs vibration energy from the directly excited linear system; moreover, this energy is absorbed in multiple frequencies, an indication of the complex dynamic interaction between the linear and nonlinear components. In Figs. 4c and d we depict the FFTs of the nonlinear modal responses $z_2(t)$ and $z_3(t)$, from which we deduce the presence of strong frequency components close to the natural frequencies of both linear modes (e.g., the eigenfrequencies of the modes of the degenerate linear subsystem).

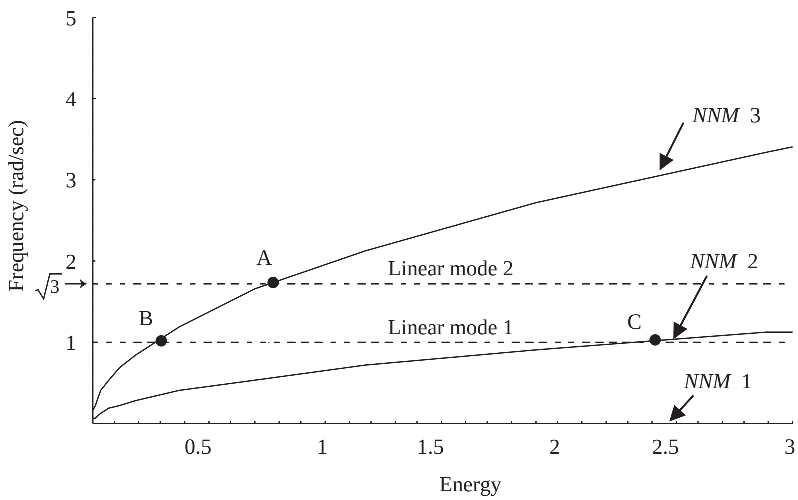

Fig. 3. Normal modes of the two uncoupled subsystems ($\varepsilon = 0$): - - - - -, linear——, nonlinear modes.

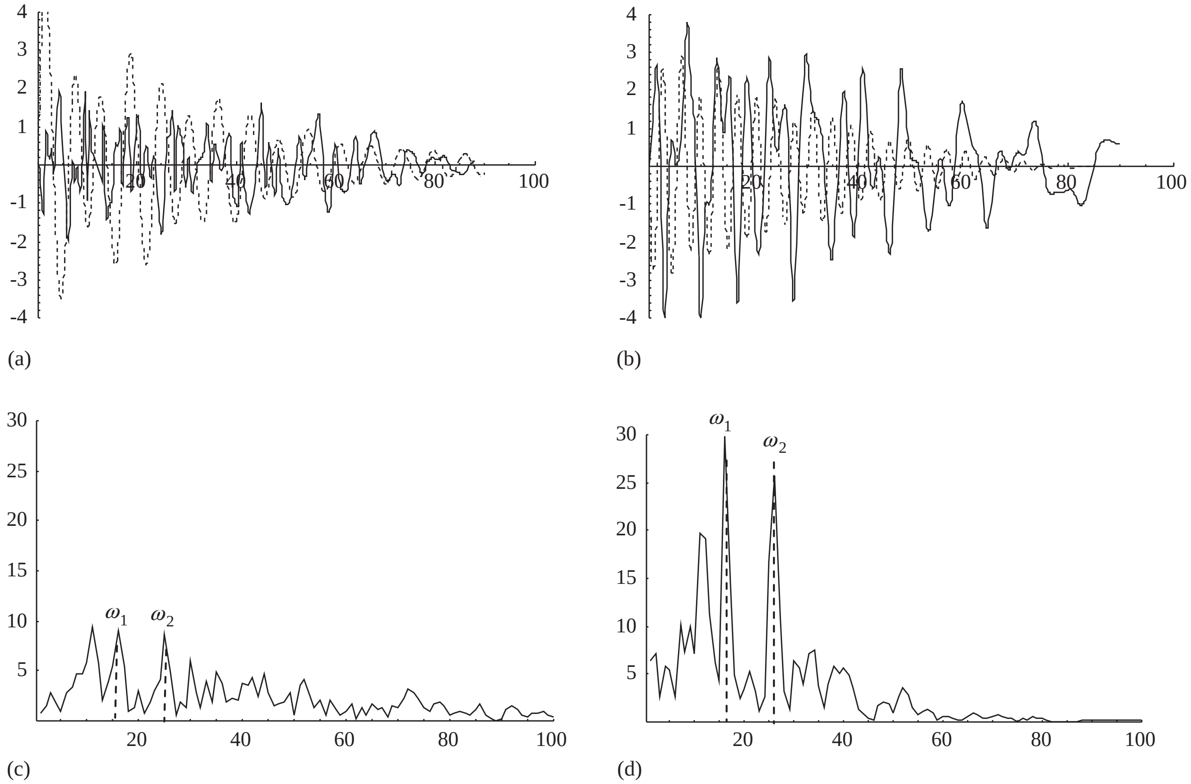

Fig. 4. Transient responses of the coupled system: (a) $x_1(t)$, - - - - -; $z_2(t)$, ——; (b) $x_2(t)$, - - - - -; $z_3(t)$, ——; (c) FFT of $z_2(t)$, (d) FFT of $z_3(t)$.

The multi-frequency nature of the energy transfer to the nonlinear attachment becomes apparent by studying the frequency–time plots depicted in Fig. 5, depicting the WTs of the transient nonlinear responses $z_2(t)$ and $z_3(t)$ of Fig. 4. Considering these WTs we note the following:

(a) The out-of-phase NNM of the nonlinear attachment (corresponding to coordinate $z_3(t)$) absorbs energy at three basic frequencies, two of which are close to the natural frequencies of the linear in-phase and out-of-phase modes (of the linear subsystem), and one is lower. Hence, this NNM appears to resonate simultaneously with both linear modes, extracting energy simultaneously from both. The additional lower frequency indicates the presence of an essentially nonlinear mode that exists in the coupled system; as shown in Ref. [1] in systems of this type, composed of weakly coupled linear and nonlinear components, there can exist numerous branches of stable and unstable NNMs resulting from bifurcations near points of internal resonance.

(b) The in-phase NNM (corresponding to coordinate $z_2(t)$) exhibits similar behavior, though its interaction with the out-of-phase linear mode takes place after some time delay. However, this mode also absorbs energy in a multi-frequency fashion, resonating with both linear

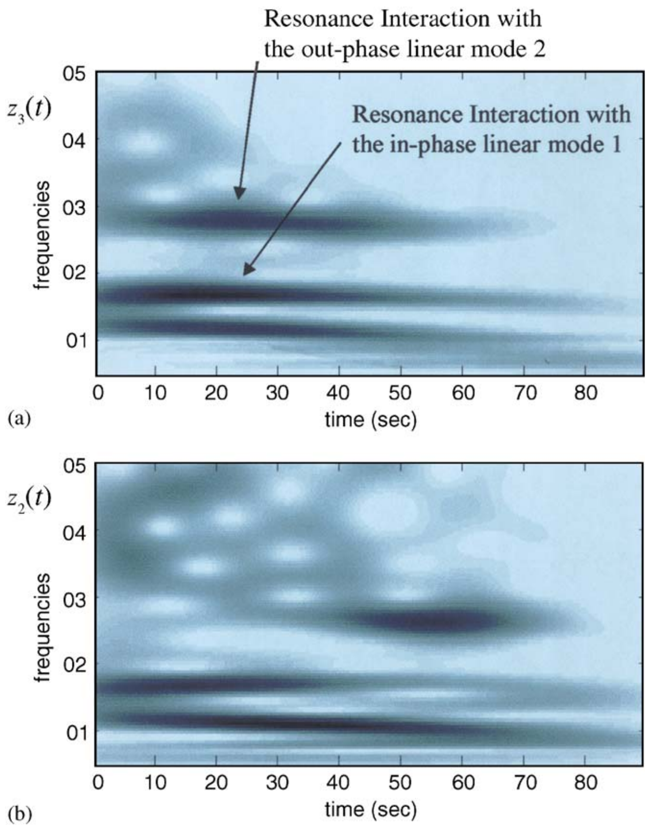

Fig. 5. Wavelet analysis of the transient responses of the nonlinear subsystem of the coupled system (frequencies in Hz): (a) $z_3(t)$, (b) $z_2(t)$.

modes; the presence of the lower NNM is again noted in this in-phase nonlinear modal response.

(c) The WTs provide important information not only on the frequency contents of the nonlinear modal responses (compare the WT results with the FFTs of the same transient signals presented in Fig. 4), but also on the temporal evolution of each individual frequency component as the interaction between the linear and nonlinear subsystems progresses in time. This underlines the usefulness of the WT in analyzing essentially nonlinear dynamical interactions of the type considered herein.

In order to fully understand the dynamical mechanism that governs the interaction between the linear and the essentially nonlinear components, one must perform an analytical study of the resonance interactions (and possibly, captures) between the different linear and nonlinear modes of the system in different frequency ranges. This analytical study could be performed using the analytical techniques developed by Panagopoulos et al. [7] and Vakakis et al. [1].

3. Concluding remarks

The reported results indicate that mdof essentially nonlinear attachments can extract energy from linear systems in a multi-frequency fashion, through simultaneous dynamic interactions of multiple modes of the nonlinear attachment with multiple modes of the linear system. This form of multi-frequency energy exchange is different from the resonance capture cascades encountered in previous works, where energy extraction to sdof nonlinear attachments occurs in a sequential manner.

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