Derivation of optimal scroll compressor wrap for minimization of leakage losses

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ABSTRACT

The scroll wraps of a scroll compressor play a very important role in the compressor's efficiency due to their impact on leakage and leakage irreversibilities. In this work, a short summary of the scroll compressor geometry is presented. Using the geometry of the scroll wraps, an optimal scroll wrap geometry is derived for a given set of constraints on compressor displacement, scroll wrap thickness and volume ratio. This optimal scroll wrap geometry can be used as a starting point for further optimization in a detailed compressor model that includes the effects of mechanical losses.

1. INTRODUCTION

The concept of a scroll compressor has been around for more than one hundred years, having been patented by Leon Creux in 1905. Since then, many academic researchers have studied the scroll compressor and its geometry. In addition, a great deal of scroll compressor optimization has been conducted by industry, some of which has made its way into the public domain.

Prior work has focused on the derivation of volumes for the chambers, including work of Morishita (1984), Yangisawa (1990), Bush and Beagle (1992), Gravesen (2001), Wang (2005), and Blunier (2009). None of these prior works have accurately modeled the suction chamber geometry, though Bell (2011) provides analytic solutions for the scroll chamber geometry.

2. DEFINITION OF GEOMETRIC PARAMETERS

2.1 Scroll Wrap Definitions

Fundamentally, the scroll compressor geometry is formed of involutes of a circle. The involute of a circle is the line that would be formed if you were to "unwrap" a circle with radius r_b. Figure 1 shows what an involute of a circle looks like. The coordinates of a point on the involute can be given by

$$x = r_b \left(\cos\phi + (\phi - \phi_0)\sin\phi \right)$$
$$y = r_b \left(\sin\phi - (\phi - \phi_0)\cos\phi \right)$$

(1)

if the origin is taken to be the center of the base circle. While the mathematical curve for the involute is defined between involute angles of ϕ_0 (on the base circle) and ϕ_e (at the end of the scroll wrap), for the actual scroll wrap, the relevant part is for ϕ between ϕ_s and ϕ_e. The part of the involute between ϕ_0 and ϕ_s does not exist because in this region the geometry of the scroll wrap is governed by the discharge geometry.

The scroll compressor is formed of two scroll wraps, each of which is formed of two scroll involutes. Typically one of the scrolls is fixed while the other orbits around it, creating the compression pockets. For the fixed scroll wrap, the two involutes are unwrapped from the same base circle, but they have different initial angles. The subscripts beginning with *i* refer to the "inner" involute and those beginning with *o* refer to the "outer" involute. The thickness of the scroll wrap can be given by
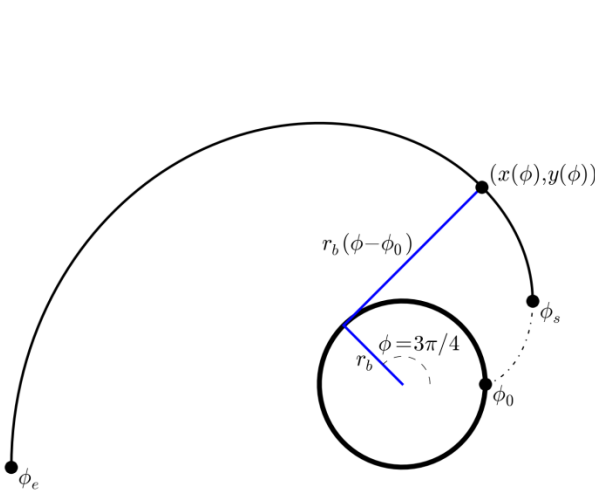
$$t_s = r_b(\phi_{i0} - \phi_{o0}) \tag{2}$$
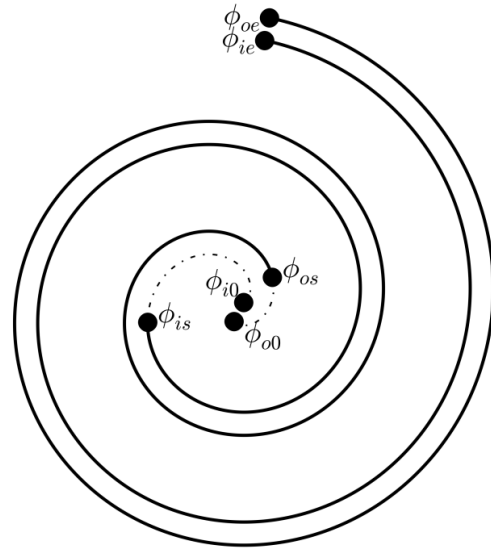

Figure 1: One Involute

Figure 2: Two involutes forming the fixed scroll

The fixed scroll is then mated with the orbiting scroll, which has the same shape, but is mirrored through the origin and shifted by the orbiting radius. When the two scroll wraps are put together, a set of pockets are formed between the scroll wraps, as shown in Figure 3.
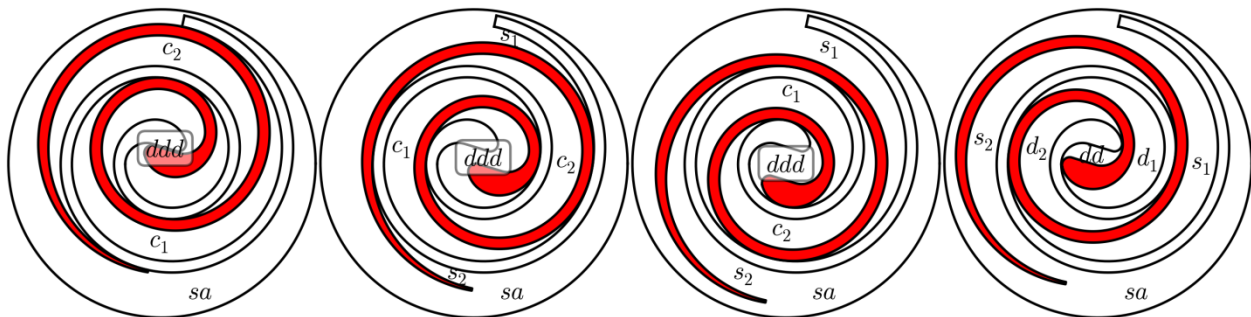

Figure 3: Pockets over the course of one rotation ($\theta=0$, $\pi/2$, π, $3\pi/2$)

The calculation of the volumes of the chambers over the course of a rotation are quite complicated. Details of these calculations are beyond the scope of this paper, but the full derivations are presented in Bell (2011), including the most comprehensive treatment of forces, moments and centroids to date.
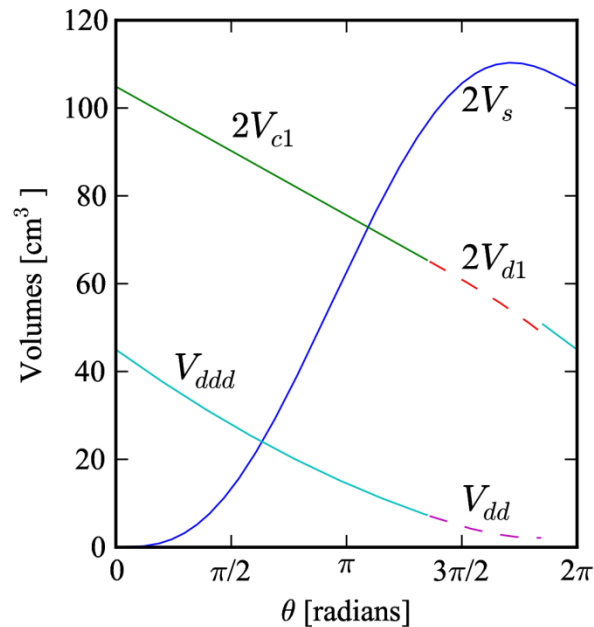
Figure 4: Volumes over one revolution

The displacement of the compressor is given by the size of the pocket that is pinched off after one rotation (c_1 plus c_2 from frame 1 of Figure 3). The displacement of the compressor is given by

$$V_{disp} = -2\pi h_s r_b r_o \left(3\pi - 2\phi_{ie} + \phi_{i0} + \phi_{o0} \right) \tag{3}$$

and the volume ratio of the compressor is given by:

$$V_{ratio} = \frac{V_{disp}}{2V_{c,d}} = \frac{3\pi - 2\phi_{ie} + \phi_{i0} + \phi_{o0}}{-2\phi_{os} - 3\pi + \phi_{i0} + \phi_{o0}} \tag{4}$$

which is the ratio of the displacement ratio of the compressor to the volume of the compression chamber right at the point where the compression pocket opens up to the discharge region. Typically the volume ratio is selected as the result of an optimization that combines material costs and compressor performance, but for the purposes here, we can assume that the volume ratio is known.

2.2 Derived Terms
If the volume ratio and displacement of the compressor are known, it is then possible to determine the scroll compressor geometry in order to match the desired volume ratio and displacement. A number of different sets of constraints on the scroll geometry are possible, but the constraints employed here are that the volume ratio, displacement, and the thickness of the scroll wrap are fixed.

The orbiting radius of the orbiting scroll is given by

$$r_o = r_b \pi - t_s \tag{5}$$

and if the displacement V_{disp}, volume ratio V_{ratio} and thickness of the scroll wraps t_s are imposed, then there are three equations (2), (3), and (4) and 6 unknowns (ϕ_{ie}, ϕ_{i0}, ϕ_{o0}, ϕ_{os}, h_s, and r_b).

Two further constraints are needed on the scroll geometry in order to fix the rest of the scroll geometry. Either ϕ_{o0} or ϕ_{i0} is a free variable, the other being fixed by the scroll wrap thickness for a given base circle radius. Increasing the value of ϕ_{i0} just rotates the scroll wrap, so for simplicity, ϕ_{i0} is set to zero. The value of ϕ_{os} is set to 0.3 radians.

Therefore, with the additional constraints imposed here, there remains just one free variable, which can either be taken to be the scroll wrap height h_s or the base circle radius r_b, and here the base circle radius was taken as the free variable with the height adjusted to meet the displacement constraint. A method is presented in the next section to optimize the selection of the base circle radius.

With these constraints, it is possible to obtain an analytic solution for the relevant scroll wrap parameters. The outer involute initial angle is then given by

$$\phi_{o0} = -t_s / r_b \tag{6}$$

and after some algebra and simplification, the height of the scroll wrap is given by

$$h_s = \frac{V_{disp}}{2\pi r_b^2 V_{ratio} (\pi + \phi_{o0})(2\phi_{os} + 3\pi - \phi_{o0})} \tag{7}$$

and the ending angle of the scroll is given by

$$\phi_{ie} = \frac{V_{disp}}{4\pi h_s r_b^2 (\pi + \phi_{o0})} + \frac{3\pi + \phi_{o0}}{2} \tag{8}$$

where both the fixed and orbiting scrolls have the same ending angle. If another set of constraints is desired, it is possible to use a non-linear solver to obtain the scroll wrap geometry.

For the same volume ratio, scroll wrap thickness and displacement, the larger r_b is, the smaller h_s must be to maintain the same displacement, volume ratio and scroll wrap thickness. This yields a family of solutions from a very narrow cylinder to a "pancake" scroll design. Selected members of this family are shown in Figure 5. All scroll wraps are plotted at the same scale.
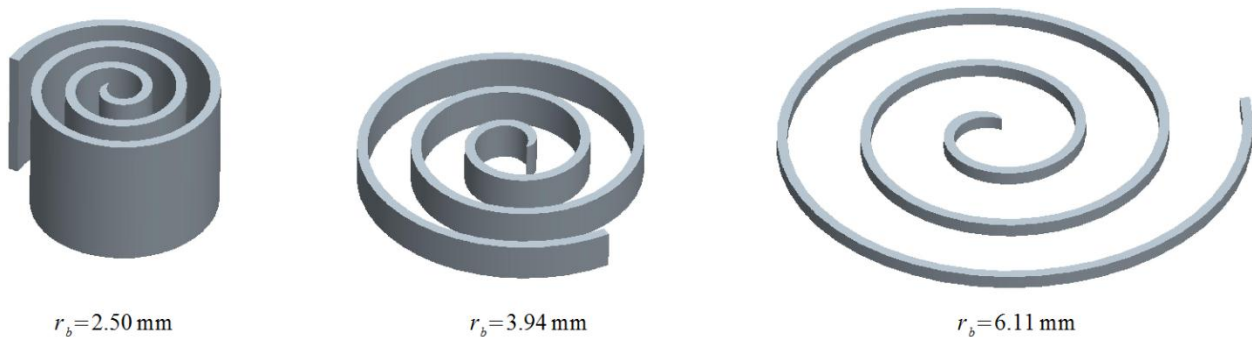
$r_b = 2.50\,\text{mm}$ $\qquad\qquad r_b = 3.94\,\text{mm}$ $\qquad\qquad r_b = 6.11\,\text{mm}$

Figure 5: Family of scroll wraps for a volume ratio of 2.7, displacement of 104.8 cm³, and wrap thickness of 4.66 mm

3. DERIVATION OF OPTIMAL BASE CIRCLE RADIUS

As shown in the previous section, for a given volume ratio, displacement, and scroll wrap thickness, a family of different scroll wraps can be obtained. The range of scroll wraps, from a narrow cylinder to a pancake scroll, offer different performance due to the variation in the leakage rates. It is therefore useful to develop a simple model for the leakage terms in order get a first guess for the optimal scroll wrap geometry from a leakage standpoint. In the

above analysis, the base circle radius r_b was a free variable, but the model presented here can predict the optimal base circle radius with reasonable accuracy.

To begin the analysis, it is first assumed that some portion of the scroll wrap does not contribute to radial leakage. This can be understood by considering the suction chamber. Over the course of the first rotation, the outermost conjugate point moves 2π radians towards the center of the compressor. Radial area between the suction chamber and the suction area does not contribute to leakage since there is effectively no pressure difference to drive the flow. Therefore, an effective ending angle of the scroll wrap is defined by

$$\phi_{ie}^* = \phi_{ie} - \pi \tag{9}$$

which removes the contribution of half of the suction chamber since over the course of one rotation, the mean conjugate angle that divides the suction chamber and the compression chamber is the inner ending angle minus a half rotation or π radians. The same argument is employed for the inner starting angle in the discharge region. Once the discharge region has equalized in pressure the radial leakage area no longer contributes to leakage. Therefore in the discharge region, another π radians are removed from the scroll involute, yielding an effective inner involute starting angle of

$$\phi_{is}^* = \phi_{is} + \pi \tag{10}$$

which removes the contribution from the portion of the rotation where the discharge region is equalized in pressure. Thus the total radial leakage area based on the inner involutes of the fixed and orbiting scrolls can be given by

$$A_{radial}^* = 2\delta_{radial} \int_{\phi_{is}^*}^{\phi_{ie}^*} r_b(\phi - \phi_{i0}) d\phi \tag{11}$$

which yields

$$A_{radial}^* = 2r_b\delta_{radial} \left(\frac{(\phi_{ie}^*)^2}{2} - \frac{(\phi_{is}^*)^2}{2} \right) \tag{12}$$

because the inner initial angle ϕ_{i0} was fixed at 0 in order to derive the involute parameters.

The effective flank area is determined by the number of flank contact points in existence over the course of a rotation. The mean total number of flank contact points is given by

$$N_{flank} = 2\frac{\phi_{ie} - \phi_{is}}{2\pi} \tag{13}$$

and the flank leakage flow area for each contact point can be given by $h_s\delta_{flank}$, thus the total flank leakage area is given by

$$A_{flank}^* = F\delta_{flank}h_s N_{flank} \tag{14}$$

where F is a flow adjustment parameter. For a given flow area and pressure difference, more flow will go through the flank leakage. This can be understood by considering the hydraulic diameters of the leakage paths. In the radial leakage the hydraulic diameter is always twice the gap width, while for the flank leakage the conformal contact results in a hydraulic diameter that increases sharply away from the throat of the leakage path. The ratio of flank to radial frictional leakage mass fluxes is approximated from the mass flow correction terms, and is given by a value for F of 3. This value was slightly tuned in order to better fit the results from the optimization carried out on the Liquid-Flooded Ericsson Cycle compressor (Bell, 2011) for a volume ratio of 2.7. In practice, the value of this ratio is dependent on the thickness of the scroll wrap and the system operating parameters, but since the purpose of this paper is to derive a guess value for detailed optimization, this value is sufficiently accurate. Thus the total effective leakage is given by

$$A_{total}^* = A_{radial}^* + A_{flank}^* \tag{15}$$

and the results for the effective leakage areas as a function of base circle radius for a volume ratio of 2.7 and displacement of 104.8 cm^3 are shown in Figure 6. The effective radial leakage increases quasi-linearly with the base circle radius, while the effective flank leakage decreases with the base circle radius. Under these constraints the sum of the two terms yields a minimum effective leakage area at a base circle radius of 3.91 mm.
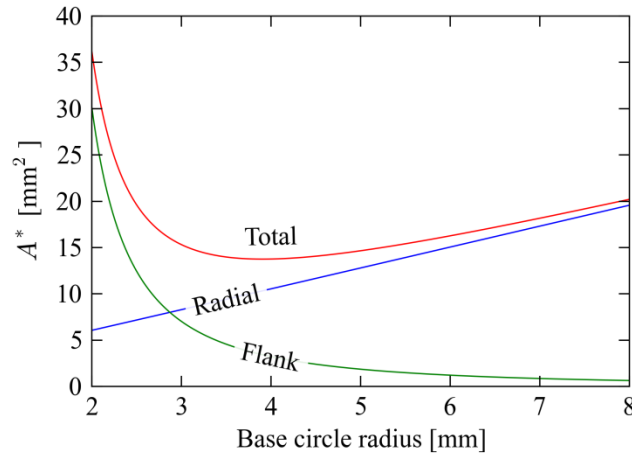
$$A^*_{total} = A^*_{radial} + A^*_{flank} \tag{16}$$

Figure 6: Effective flank and radial leakage areas for compressor with volume ratio of 2.7, displacement of 104.8 cm^3, scroll thickness of 4.66 mm

A numerical minimization routine can be employed to determine the optimal base circle radius that minimizes A^*_{total} over a range of displacement and volume ratios for a fixed scroll wrap width of 4.66 mm. Both flank and radial leakage gap widths are set to 12 µm. The results of this analysis are shown in Figure 7. The optimal base circle radii obtained from the detailed compressor modeling for the Liquid-Flooded Ericsson Cycle and the Liquid-Flooded CO$_2$ analyses (Bell, 2011) are also overlaid in order to demonstrate the effectiveness of this method for calculating an approximate optimal base circle radius. While both of these sets of optimal geometry are based on a liquid-flooded compressor, it is interesting to note that their optimal base circle radii follow closely with the model presented here. The Python code required to carry out the optimization is listed as the appendix.

It is straightforward to generate a similar plot for a different scroll wrap thickness. These results show that for a given displacement, as the volume ratio increases, the optimal base circle radius decreases. Furthermore, for a given volume ratio, as the displacement is increased, the optimal base circle radius increases. This chart can be generally employed in the design of scroll wraps, whether for flooded or dry compression applications. The inclusion of geometrically-dependent mechanical losses and scroll wrap manufacturing cost would result in different optimal scroll wrap geometry.
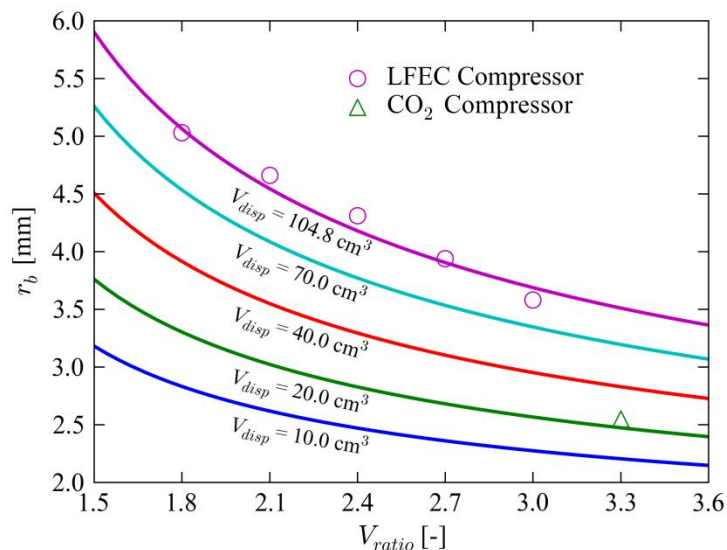
**Figure 7: Optimal base circle radius as a function of displacement
and volume ratio for a scroll with thickness 4.66 mm**

CONCLUSIONS

A method for the calculation of an optimal base circle radius has been presented that allows for a means of predicting with good accuracy the base circle radius of the scroll compressor that will minimize the leakage losses. This method can be used as a starting point for further optimization that includes the effects of mechanical losses which are also strongly dependent on the scroll wrap geometry.

ACKNOWLEDGEMENTS

NOMENCLATURE

Variables			Subscripts	
A	m^2	Area	0	Initial
F	-	Flow correction parameter	s	Starting
h_s	m	Scroll height	e	Ending
N	-	Number of	i0	Inner initial
r_b	m	Base circle radius	is	Inner starting
r_o	m	Orbiting Radius	ie	Inner ending
t_s	m	Scroll thickness	o0	Outer initial
x	m	x-Coordinate	os	Outer starting
y	m	y-Coordinate	oe	Outer ending
ϕ^*	rad	Effective involute angle		
ϕ	rad	Involute angle		
δ	m	Gap width		
V_{disp}	m^3	Displacement Volume		
V_{ratio}	-	Volume ratio		

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APPENDIX

This example is written in Python. Requires scipy and numpy packages. When this code is run, it should yield a base circle radius of 3.915 mm.

```python
from scipy.optimize import fmin #Multi-dimensional minimization routine
import numpy as np #Python matrix math package
from math import pi

def OptimalRb(Vratio,Vdisp,t,phi_os=0.3,F=3):
    def OBJECTIVE(rb):
        phi_i0=0.0
        phi_is=np.pi
        delta=12e-6
        phi_o0=phi_i0-t/rb
        hs=(Vdisp)/(rb**2*2*pi*Vratio*(pi+phi_o0)*(2*phi_os+3*pi-phi_o0))
        phi_ie=Vdisp/(4*pi*hs*rb**2*(pi+phi_o0))+(3*pi+phi_o0)/2.0
        A_radial=2*delta*rb*((phi_ie-np.pi)**2/2-(phi_is+np.pi)**2/2-phi_i0*(phi_ie-phi_is-2*np.pi))*1e6
        Nfl=(phi_ie-phi_is)/(2*pi)
        A_flank=2*Nfl*delta*hs*1e6*F
        return A_flank+A_radial
    return fmin(OBJECTIVE,0.003,ftol=1e-12)

if __name__=='__main__':
    rb=OptimalRb(1.8,40e-6,0.00466)
    print 'Optimal rb is: ' + str(float(rb)*1000) + ' mm'
```