Implications of finite one-loop corrections for see-saw neutrino masses

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Abstract.   In the standard seesaw model, finite corrections to the neutrino mass matrix arise from one-loop self-energy diagrams mediated by a heavy neutrino. We discuss the impact that these corrections may have on the different low-energy neutrino observables paying special attention to their dependence with the seesaw model parameters. It is shown that sizable deviations from the tri-bimaximal mixing pattern can be obtained when these corrections are taken into account.

1. Introduction

One of the simplest realizations of the dimension five effective operator $\mathcal{O}_5 \sim LLHH$ is certainly given by the *standard* seesaw model [1–5]. By extending the standard model Lagrangian with fermionic electroweak singlets (right-handed (RH) neutrinos for brevity) light neutrino masses are generated by the exchange of the new states via a tree-level realization of $\mathcal{O}_5 \sim LLHH$. An appealing feature of this approach is that the smallness of neutrinos masses is due to the suppression induced by the new heavy states.

The seesaw Lagrangian spans an 18-dimensional parameter space determined by 6 CP phases and 12 real parameters. This parameter space is partially constrained by low-energy neutrino data [6]: these measurements—in principle—yield 9 constraints given by 3 light neutrino masses, 1 Dirac and 2 Majorana CP phases and 3 mixing angles. The remaining 9-dimensional parameter space is not constrained at all implying a region in parameter space leading to the correct low-energy observables can not be uniquely established.

In order to find those "spots" that are consistent with data typically scans of the seesaw parameter space are done. These scans rely on parametrizations of the seesaw parameters (Yukawa couplings) in such a way that instead of using low-energy data as an output criteria it is used as an input [1]. In this procedure light neutrino masses are taken to be entirely determined by the tree-level mass matrix with the Yukawa couplings restricted to be perturbative [8]. This is an accurate approximation as far as the one-loop contributions are not too large. However, depending on the parameter choice, regions in parameter space exist in which the one-loop contributions, and in particular the finite ones, are sizable [9; 10]. Thus, a reliable scan of the seesaw parameter space should include the finite one-loop corrections.

Here we will discuss the importance of the finite one-loop corrections for seesaw neutrino masses. We will present the general formulas for the self-energy diagrams that determine them,

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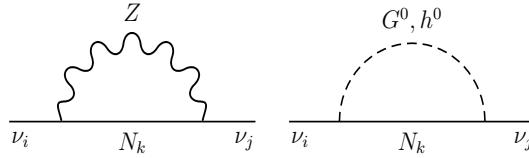[1] For a discussion of parametrizations of the seesaw see [7].

Figure 1. Self-energy diagrams accounting for $\boldsymbol{m_\nu}^{\text{(1-loop)}}$.

showing how they can affect the different entries of the light neutrino mass matrix. As an implication of these corrections we will analyze the impact they may have on the leptonic mixing angles by assuming that the tri-bimaximal mixing (TBM) is realized at the tree-level. We will closely follow reference [9].

2. One-loop finite corrections

The new states being $SU(2) \times U(1)$ singlets do not couple to standard model gauge bosons. Thus, in the basis in which the Majorana RH neutrino mass matrix is diagonal the interactions induced by the presence of these states, that for concreteness we assume to be three, are described by the following Lagrangian:

$$-\mathcal{L} = -i\bar{N}_{R_i}\,\gamma_\mu \partial^\mu N_{R_i} + \tilde{\phi}^\dagger \bar{N}_{R_i}\lambda_{ij}\ell_{Lj} + \frac{1}{2}\bar{N}_{R_i}CM_{R_i}\bar{N}_R^T + \text{h.c.} \tag{1}$$

Here ℓ_L are the lepton electroweak doublets, $\phi^T = (\phi^+ \phi^0)$ is the Higgs $SU(2)$ doublet, $M_{R_i} \equiv M_i$ are the RH neutrino massses, C is the charge conjugation operator and $\boldsymbol{\lambda}$ is a 3×3 Yukawa matrix in flavor space. Assuming $M_R \gg v$ (with $v \simeq 174$ GeV) the effective neutrino mass matrix is obtained once the full 6×6 neutral fermion mass matrix is diagonalized:

$$\boldsymbol{m_\nu}^{\text{(tree)}} = -v^2 \boldsymbol{\lambda}^T \hat{\boldsymbol{M_R}}^{-1} \boldsymbol{\lambda}\,. \tag{2}$$

As pointed out in references [11; 12] finite corrections to the above matrix arise from the one-loop self-energy Feynman diagrams depicted in figure 1. These corrections remove the 3×3 null matrix in the left-upper block of the full 6×6 fermion mass matrix. Thus, their finiteness can be understood as due to the fact that there are no counterterms that could be used to absorb possible divergences arising from them. Full details of the calculation can be found in the appendix of reference [9], the result reads

$$\boldsymbol{m_\nu}^{\text{(1-loop)}} = v^2 \boldsymbol{\lambda}^T \hat{\boldsymbol{M_R}}^{-1} \left\{ \frac{g^2}{64\pi^2 M_W^2} \left[m_h^2 \ln\left(\frac{\hat{\boldsymbol{M_R}}^2}{m_h^2}\right) + 3M_Z^2 \ln\left(\frac{\hat{\boldsymbol{M_R}}^2}{M_Z^2}\right) \right] \right\} \boldsymbol{\lambda}\,. \tag{3}$$

Some words are in order regarding this result. The correction is not suppressed with respect to the tree-level result by additional factors of $v\,\boldsymbol{\lambda}\,\boldsymbol{M_R}^{-1}$. Thus, it is expected to be smaller than the tree-level mass term solely by a factor of order $(16\pi^2)^{-1}\ln(M_R/M_Z)$, implying it might have sizable effects. The logarithms involving the RH neutrino masses imply the correction is not proportional to the tree-level mass matrix in (2), unless the RH neutrino mass spectrum is degenerate. Accordingly, barring this case the correction can modify the tree-level result in a sizable way.

By using the Casas-Ibarra parametrization [13], namely

$$\boldsymbol{\lambda} = \frac{\sqrt{\hat{\boldsymbol{M_R}}}\,\boldsymbol{R}\,\sqrt{\hat{\boldsymbol{m_\nu}}}\,\boldsymbol{U}^\dagger}{v}\,, \tag{4}$$
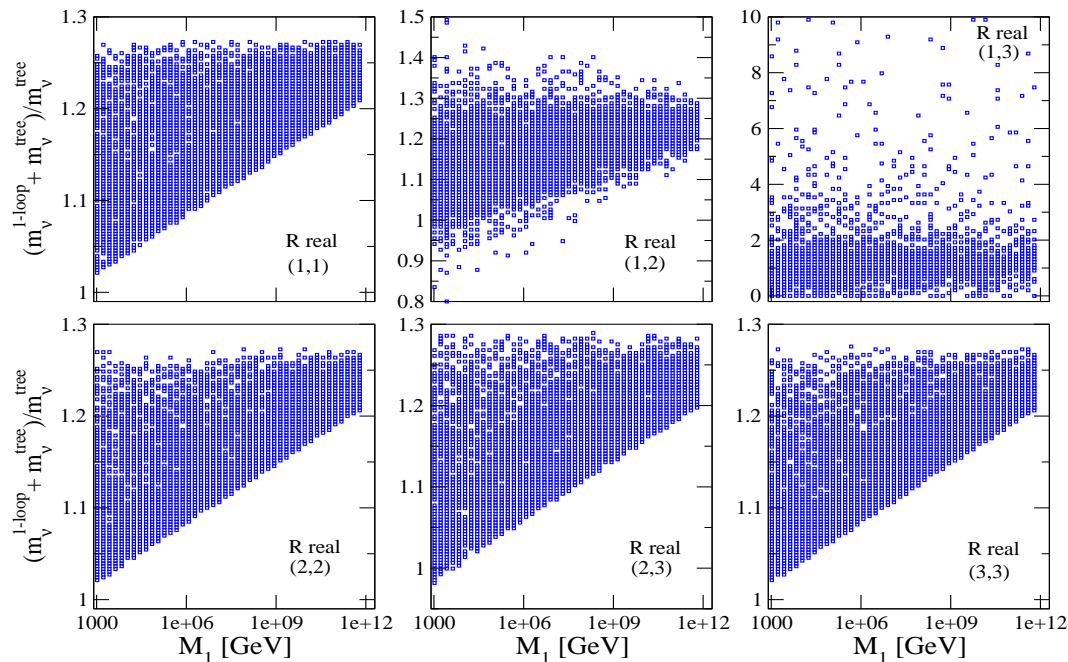
Figure 2. Ratio between the 1-loop and tree level elements of the light neutrino mass matrix as a function of the lightest RH neutrino mass M_1. A normal light neutrino mass spectrum has been assumed.

where $\boldsymbol{R}$ is a general complex orthogonal matrix, we scan the parameter space assuming a normal hierarchical light neutrino spectrum and a real $\boldsymbol{R}$. The results for the elements of the mass matrix are displayed in figure 2. It can be seen that even in this conservative case [2] the corrections can be of $30-50\%$, the exception being the 13 entry for which the contribution is far larger due to the vanishing of this entry at tree-level at around $\theta_{13} \sim 5°$.

The corrections in (3) can sizably modify the neutrino mass matrix, however not necessarily may have such an effect in low-energy observables. For example, if the modification of the neutrino mass matrix elements are correlated the effects on the neutrino mixing angles could be negligible as they are calculated from ratios of the mass matrix entries. In order to see whether this is the case we have assumed an input *ansatz* for the neutrino mixing angles that we have chosen to be the TBM pattern:

$$\theta_{23} = 45°, \quad \theta_{12} = 33.3°, \quad \theta_{13} = 0°. \tag{5}$$

Assuming $\boldsymbol{R}$ to be real, with these values, we calculate the corresponding Yukawa couplings from (4) for RH neutrino masses ranging from $[10^3, 10^{12}]$ GeV. With these Yukawa couplings we proceed to calculate the full effective mass matrix $\boldsymbol{m_\nu}^{\text{eff}} = \boldsymbol{m_\nu}^{(\text{tree})} + \boldsymbol{m_\nu}^{(\text{1-loop})}$ and the corresponding eigenvectors from which we extract the mixing angles. The results are displayed in figure 3 for the solar and reactor angle as well. It can be seen that sizable deviations from the TBM pattern are more pronounced for small values, $\mathcal{O}(\text{TeV})$, of M_1: in models with a light lightest RH neutrino there is more room for large hierarchies in the RH neutrino spectrum and accordingly the mismatch between the tree-level effective matrix and the finite one-loop correction (3), due the logarithmic functions, is larger. As in the previous case this example can be regarded as conservative and larger deviations could be expected in the general case of complex $\boldsymbol{R}$ (see ref. [9]).

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[2] Note that under this assumption three out of the six CP physical phases are taken to be zero.
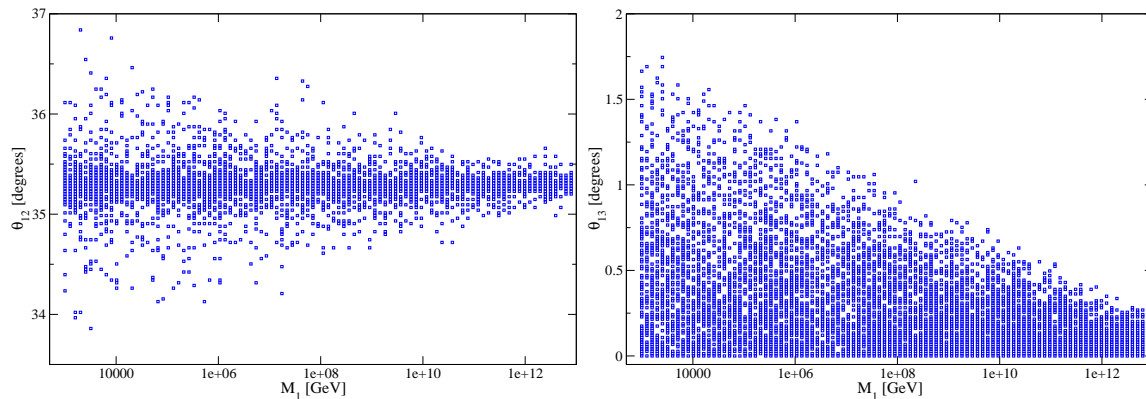
Figure 3. Deviations from the TBM pattern induced by the the one-loop finite corrections as a function of the lightest RH neutrino mass. The left panel (right panel) shows the results for the solar (reactor) neutrino mixing angle.

3. Conclusions

In the seesaw model finite one-loop corrections arise from self-energy diagrams mediated by a heavy RH neutrino. We have analyzed the impact that these corrections may have on the different entries of the effective light neutrino mass matrix, finding that the deviations from the tree-level result can be quite relevant. We also studied the influence of these corrections on low-energy observables by using the TBM mixing pattern as an input *ansatz*. As our results show sizable deviations from this pattern can be expected. In summary our findings prove one-loop finite corrections to seesaw neutrino masses are important and thus should be included in any analysis of the seesaw parameter space.

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