Mémoires Société Royale des Sciences de Liège, 6e série, tome VIII, pp. 317-323 (1975)

33. — VIBRATIONAL STABILITY OF G+ MODES TOWARDS NON-RADIAL OSCILLATIONS FOR SOLAR MODELS

A. NOELS, M. GABRIEL, A. BOURY, R. SCUFLAIRE and P. LEDOUX
Institut d'Astrophysique, Université de Liège, Belgium

ABSTRACT

The vibrational stability of solar models towards g^+ modes has been studied from the ZAMS up to the age of 5 billion yr. The nuclear energy contribution to instability is calculated with the effective sensitivity of the reactions to change of temperature. The perturbation of convective terms is taken from an extension for non-radial oscillations of Unno's theory. The ZAMS model and the present sun are stable. However, for intermediate ages, models are unstable with respect to the lowest modes due to the effects of nuclear reactions. If, moreover, the mechanical effects of convection are taken into account, all models are unstable. However their theory is uncertain enough to require caution.

I. INTRODUCTION

In an attempt to solve the problem aroused by Davis' measurements (1972) of the solar neutrino flux, Dilke and Gough (1972) have suggested that, during its main sequence evolution, a 1 $M_\odot$ star encounters a phase of vibrational instability towards low order g^+ modes. They related this instability to the presence of a large positive He^3 gradient in the central regions. In the hypothesis that such an instability could induce a transient mixing of the nuclear burning core, the neutrino flux would indeed become smaller, as well as (although later on) the surface luminosity (Fowler 1972, Ezer and Cameron 1972, Rood 1972). Repetition of such phases of lower solar constant could also possibly match the geological ice ages, as thought by many a palaeoclimatologist. The present sun would correspond to such a phase of anomalously low neutrino flux. The aim of the work reported here, was to test the vibrational stability towards g^+ modes of models corresponding to the main sequence evolution of a 1 $M_\odot$ star of solar chemical composition. We find that an instability occurs at an age of about 2.4 10^8 yr. This instability is directly related to the value of the He^3 abundance, through its contribution in the destabilizing nuclear term. Dziembowski and Sienkiewicz (1973) had previously analysed similar models and found no instability. However, they apparently used static value of the temperature sensitivity of the nuclear energy generation rate instead of the effective value, which led them to underestimate the destabilizing term.

II. MODELS

The evolution of a 1 $M_\odot$ star of chemical composition : $X = 0.74$, $Z = 0.02$ was computed with a Henyey method from the phases preceding the onset of hydrogen burning, through the main sequence phases up to an age of 5 billion yr.

The ratio of the mixing length to the pressure scale height was chosen to be 1.55 in order to match the present solar luminosity and radius at an age of 4.5 billion yr.

The opacity coefficients were obtained by interpolation in Cox and Stewart's tables (1970). The rates of the nuclear reactions involved in the p-p chain and C-N-O cycle were those given by Fowler et al. (1967) except that for (p, p) (Bahcall and May, 1969), ($^{1}O^{6}$, p) (unpublished value from the same authors), and (He^3, He^3) (Dwarakanath and Winkler, 1971). The first two rows of table I give the central mass concentration and central hydrogen abundance for each model tested for vibrational stability. Model 1 corresponds to the ZAMS while the present sun is best represented by model 4.

TABLE I

Periods (in s) and e-folding times (in yr) obtained for the first $3g^+$ modes in the case $l = 1$, for 5 evolutionary models of 1 $M_{\odot}$ (X = 0.74, Z = 0.02).

The present sun is best represented by model 4. The corresponding mass concentration (ρ_c/ρ) and central hydrogen abundance (X_c) are given in the first 2 rows.

N°			g_1		g_2		g_3	
	ρ_c/ρ	X_c	P(s)	σ'^{-1}(yr)	P(s)	σ'^{-1}(yr)	P(s)	σ'^{-1}(yr)
1	43,68	0,7366	6,373(3)	4,452(7)	9,079(3)	2.190(7)	1,183(4)	7,367(6)
2	50,91	0,6806	5,983(3)	—2,494(7)	8,250(3)	—2,324(7)	1,047(4)	5,349(7)
3	68,81	0,5652	5,096(3)	—8,838(6)	6,761(3)	—2,739(7)	8,530(3)	1,275(7)
4 (☉)	110,4	0,3930	3,845(3)	4,613(5)	5,238(3)	1,656(7)	6,556(3)	7,001(6)
5	141,0	0,3181	3,701(3)	2,970(4)	4,669(3)	3,444(6)	5,872(3)	4,927(6)

III. NON-RADIAL ADIABATIC OSCILLATIONS

The theory of non-radial oscillations can be found for instance in Ledoux (1974). The fourth order system, taking into account the perturbation of the gravitational

TABLE II

Same as Table I in the case $l = 2$.

N°	g_1		g_2		g_3	
	P(s)	σ'^{-1}(yr)	P(s)	σ'^{-1}(yr)	P(s)	σ'^{-1}(yr)
1	4.370(3)	2.627(6)	5.821(3)	4.450(6)		
3	3.890(3)	6.023(5)	4.694(3)	3.096(6)	5.605(3)	4.675(6)
4	3.367(3)	8.290(4)	3.845(3)	1.955(5)	4.425(3)	6.602(5)

potential was integrated. The periods (in seconds) of oscillations corresponding to the first 3 g^+ modes are given for each model in the case $l = 1$ (table I) and $l = 2$ (table II). In figures 1 and 2, we have plotted $\frac{\delta p}{p}$ and $\frac{\delta r}{r}$ respectively, in the case $l = 1, g_1$ for all models, whose corresponding ages are indicated. Figure 3 represents $\frac{\delta p}{p}$ in the case $l = 1$, g_2.

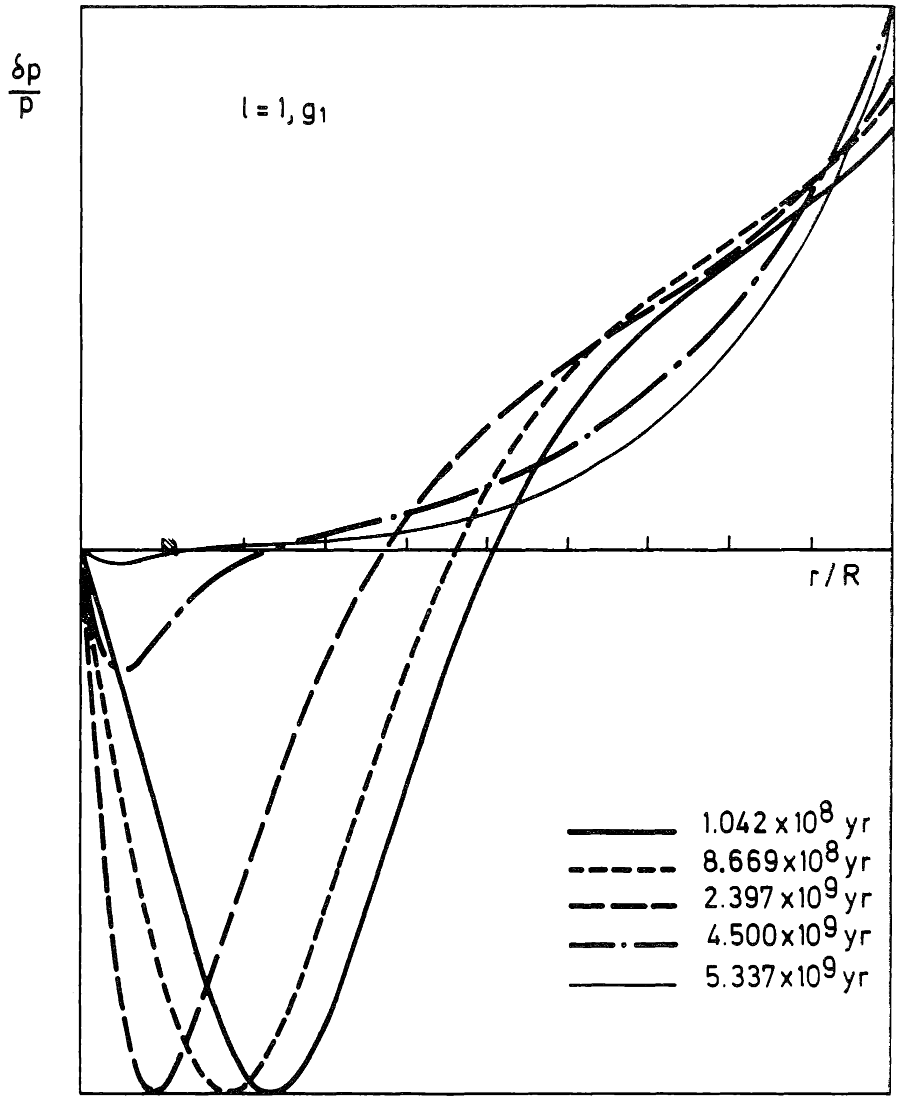

Fig. 1. — $\frac{\delta p}{p}$ as a function of $\frac{r}{R}$ in the case $l = 1$, g_1 for the models listed in table I; model 1 ——— (); model 2 (- - -), model 3 (— — —), model 4 (present sun) (— . —), model 5 (———).

IV. VIBRATIONAL STABILITY

Assuming a space and time dependence of the form $\delta r = \delta r\,(r)\,P_l^m(\cos\theta)\,e^{im\varphi}$ $e^{i\sigma_a t}\,e^{-\sigma' t}$, the coefficient of vibrational stability writes, in the Lagrangian formalism :

$$\sigma' = -\frac{1}{2\sigma_a^2}\,\frac{\displaystyle\int_0^{M_a}\frac{\delta T}{T}\,\delta\left[\varepsilon - \frac{1}{\rho}\vec{\nabla}.\vec{F}\right]dm + \int_0^{M_a}\left(\Gamma_3 - \frac{5}{3}\right)\frac{\delta\rho}{\rho}\,\delta\left[\varepsilon_2 + \frac{1}{\rho}\overline{\vec{\nabla}.\vec{\nabla}p}\right]dm}{\displaystyle\int_0^{M}\vec{\delta r}.\vec{\delta r}^*\,dm} \qquad (1)$$

$P_l^m(\cos\theta)$ is the associated Legendre polynomial of degree l and order m, σ_a is the

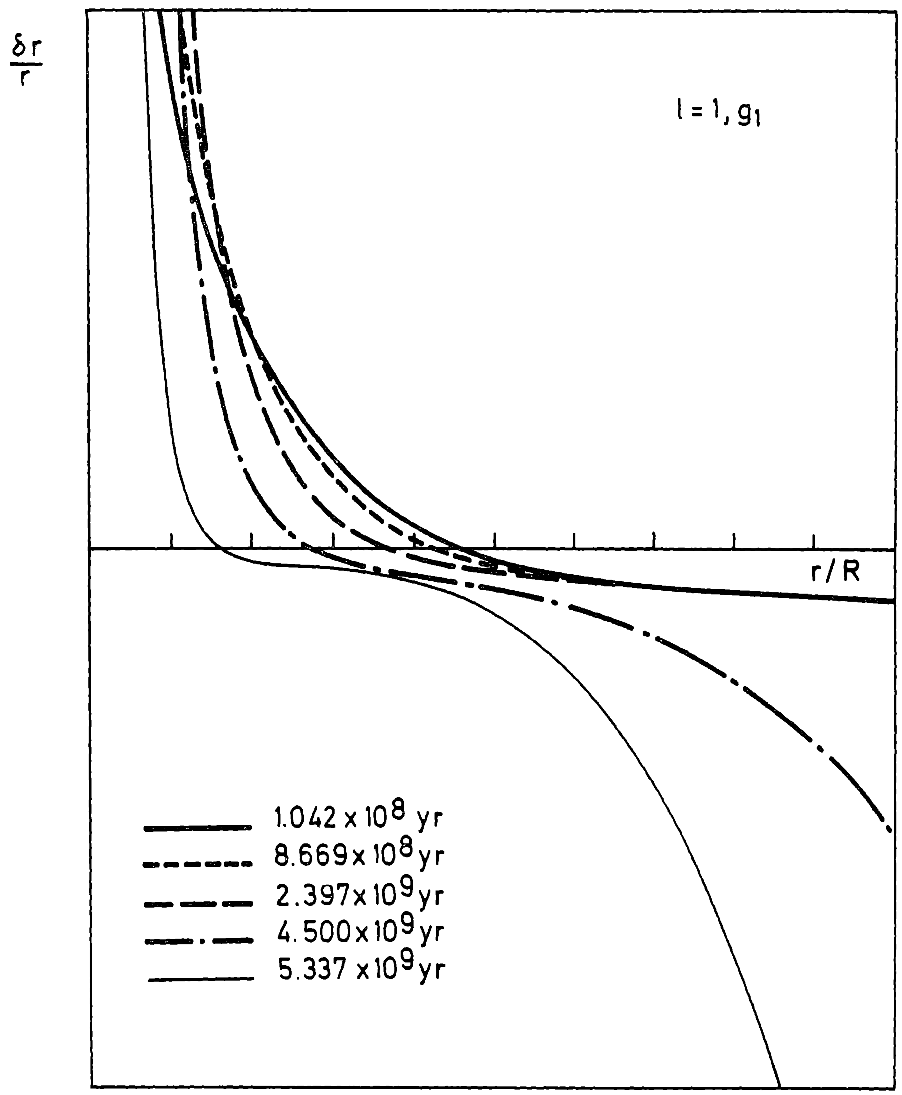

Fig. 2, — $\frac{\delta r}{r}$ as a function of r/R in the case $l = 1$, g_1.

The different lines have the same meaning as in figure 1.

adiabatic frequency of oscillation and M_a is the value of the mass where we have stopped the integration. It corresponds to the point where the non-adiabatic correction to $\frac{\delta T}{T}$ becomes of the same order as the adiabatic value.

In the first term, $\delta\varepsilon$ has been computed using the effective sensitivity to density and temperature of the nuclear rate

$$\delta\varepsilon = \sum_i \left[\frac{\partial\, \varepsilon_i}{\partial\, ln\, \rho}\frac{\delta\rho}{\rho} + \frac{\partial\, \varepsilon_i}{\partial\, ln\, T}\frac{\delta T}{T} + \sum_j \frac{\partial\, \varepsilon_i}{\partial\, ln\, x_j}\frac{\delta x_j}{x_j}\right] \tag{2}$$

$$= \varepsilon\left(\mu_{\text{eff}}\frac{\delta\rho}{\rho} + \nu_{\text{eff}}\frac{\delta T}{T}\right)$$

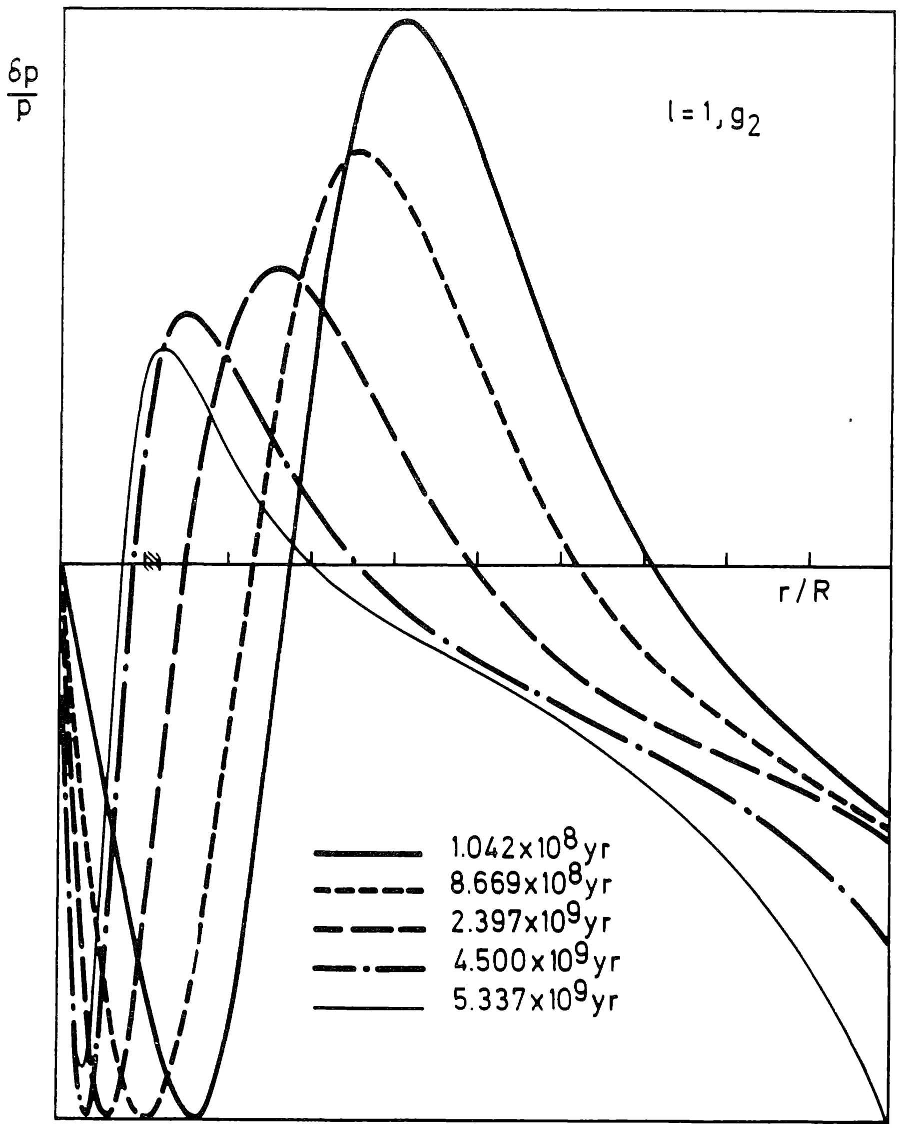

Fig. 3. — Same as Figure 1 in the case $l = 1$, g_1.

This led us to solve the perturbed equations of the kinetics of nuclear reactions. In the simple case where the only nuclear reactions are those of the PPI chain, ν_{eff} is given by

$$\nu_{eff} = \frac{(\varepsilon - \varepsilon_{33})\,\nu_{11} + \varepsilon_{33}\,\nu_{33}}{\varepsilon} \simeq \frac{1}{2}(\nu_{11} + \nu_{33}) \tag{3}$$

where subscripts 11 and 33 refer respectively to the reactions (p, p) and (He^3, He^3). This expression shows that for such stars the effective value of ν is very different from the static one ($= \nu_{11}$). For example, at the centre of model 4 (present sun), ν_{eff} is of the order of 10 while ν_{11} is of the order of 4.

It should be noticed that, in this Lagrangian formalism, the He^3 gradient does not appear anywhere; it is the value of the He^3 abundance itself which is important since it has a direct effect on ν_{eff} through ε_{33}.

The perturbation of the term of flux is given by the following expression, valid in a radiative case as well as in a convective one :

$$\delta\left(\frac{1}{\rho}\,\vec{\nabla}.\vec{F}\right) = \frac{d\delta L}{dm} - \frac{l\,(l+1)}{r^2}\left[\frac{\delta F^h}{\rho} + \frac{\chi}{\sigma_a^2}\frac{dm}{dL} - \frac{F}{\sigma_a^2\rho}\frac{\chi}{r}\right] \tag{4}$$

where $\dfrac{\delta L}{L} = \dfrac{\delta F^r}{F} + 2\,\dfrac{\delta r}{r}$ and $\chi = \dfrac{p'}{\rho} + \Phi'$, upperscripts r and h referring respectively to the radial and horizontal components of the total flux (convective plus radiative). The perturbation of the convective flux comes from an extension of the theory of Unno (1967) to non-radial oscillations. All details can be found in Gabriel et al. (1974).

The last term in eq. (1) gives the influence of the mechanical effect of convection. Because of the great uncertainty in the theory, it has been neglected in the results presented here. However, we can say that, with our theory of convection, this term would always be destabilizing and all the more important as the amplitude in the outer layers is large. In fact, all the models considered here would be found unstable (though not all the modes). The values obtained for the e-folding time, σ'^{-1} (in yr) are listed in tables I ($l = 1$) and II ($l = 2$). A negative sign for σ'^{-1} means instability. In the case $l = 1$, the ZAMS model is stable. An instability appears between models 1 and 2, at an age which can be evaluated at 2.4 10^9 yr. Stabilization then occurs at about 3 10^2 yr and models 4 (present sun) and 5 are stable. For $l = 2$, all the models tested are stable.

If we now come to the interpretation of this instability, it should be noticed first that, written in the Lagrangian formalism, the nuclear term is always destabilizing.

As to the term of flux, its influence depends on the exact form of the eigenfunction but in the present case, and most often, it is stabilizing and its main contribution comes from the outer layers. The influence of an increasing mass concentration on the behaviour of the eigenfunction is illustrated in figure 1. Two competing features show up. First, the ratio of the surface value of $\dfrac{\delta p}{p}$ to the value at the minimum increases with time. That means that the stabilizing influence of the outer layers increases accordingly. Second, the node and the extremum come closer and closer to the centre in the course of evolution. We have represented by hatched lines the layers where the energy generation is equal to half its central value. As long as the node is sufficiently outside that region, the evolution favours the destabilizing nuclear term, since larger and larger values of $\frac{\delta T}{T}$ occur in the nuclear burning core. On the contrary, once the node is too close to the centre, the effect is reversed.

A similar behaviour is of course present in $\frac{\delta r}{r}$ shown in figure 2. For higher modes, although those features can again be found (fig. 3), the increase in the number of nodes gives less and less weight to the destabilizing central region. In the case $l = 2$, or for higher l values, the ratio of the surface value to the value at the extremum is larger than for $l = 1$. This situation is accordingly less favourable for the occurence of an instability.

V. CONCLUSION

After about 2.4 10^8 yr on the main sequence, the sun becomes unstable towards low order g^+ modes. On the basis of this study, it is however impossible to predict the influence of such an instability on the subsequent evolution of the star, particularly with regard to a possible mixing of the material, should it be complete or partial, instantaneous or rather slow. It must however be pointed out that the e-folding time is never short compared to the Kelvin-Helmholtz time scale. This would seem to favour a slow mixing rather than the instantaneous one required for example by Ezer and Cameron (1972) to match the solar neutrimo measurements.

REFERENCES

BAHCALL, J. N. and MAY, R. M., *Astrophys. J.*, **155**, 501, 1969.

COX A. N. and STEWART, J. N., *Astrophys. J. Suppl.*, **19**, 261, 1970.

DAVIS, R. Jr., *Bull. Am. Astron. Soc.*, **17**, 527, 1972.

DILKE, F. W. W. and GOUGH, D. O., *Nature*, **240**, 262, 1972.

DWARAKANATH, M. R. and WINKLER, H. C., *Phys. Rev. C*, **4**, 1532, 1971.

DZIEMBOWSKI, W. and SIENKIEWICZ, R., *Acta Astron.*, **23**, 273, 1973.

EZER, D. and CAMERON, A. G. W., *Nature Phys. Sci.*, **240**, 180, 1972.

FOWLER, W. A., *Nature*, **238**, 1238, 1972.

FOWLER, W. A., CAUGHLAN, G. R. and ZIMMERMAN, B. A., *Ann. Rev. Astron. Astrophys.*, **5**, 525, 1967.

GABRIEL, M., SCUFLAIRE, R., NOELS, A. and BOURY, A., *Acad. Roy. Belg. Bull. Cl. Sci.*, in press, 1974.

LEDOUX, P., in P. Ledoux, A. Noels and A. W. Rodgers (eds.) Stellar Instability and Evolution, IAU Symposium 59, D. Reidel Publishing Company, Dordrecht, Holland, p. 135, 1974.

ROOD, R. J., *Nature Phys. Sci.*, **240**, 178, 1972.

UNNO, W., *Publ. Astron. Soc. Japan*, **19**, 139, 1967.

I. W. Roxburgh (33). — Could the previous speakers tell me what the e-folding time was?

A. Noels (33), — The e-folding time is of the order of 10^7 years, But, an instantaneous mixing would require a much sorter time-scale,

R. T. Rood. — To eliminate the neutrinos expected from the sun via mixing, the mixing must be much faster than the Kelvin time of the energy producing region, i.e. $t_{\text{mix}} < 10^6$ years. The mixing must also be much more rapid than the ^{3}He lifetime (several 10^5 years).

J. Christensen-Dalsgaard, — The idea of the Gough and Dilke paper is that the onset of pulsational instability triggers on finite amplitude convective instability because of mixing of ^{3}He towards the center and the resulting increase in the energy generation.

M. Schwarzschild. — May I make 3 points : 1) The two papers we have just heard were clearly presented and contained an important result. But they hold an absolute record in keeping secret until the last sentence of the second paper their true topic, namely the solar neutrino discrepancy. 2) All of us older ones in this field have now a black mark against us for leaving an instability of a classical type, for a star as simple as the sun, undiscovered for so long. 3) In view of the extreme slowness of the new instability I take it that its consequences are still very uncertain.

T. G. Cowling. — Of course, Prof. Schwarzschild has overlooked the existence of the instability responsible for the hydrogen convection zone. Once usually disregards this because one can smooth out its effects. Similarly it is not serious if there is some unidentified instability unless this leads to effects that cannot be smoothed out.

J. Christensen-Dalsgaard. — We have treated the oscillations in the Eulerian framework and here the gradient in abundance is clearly exibited, but this is not the case when one uses the Lagrangian formalism. I would much appreciate if anybody present would comment our this situation.

M. Gabriel. — We prefer the lagrangian formalism for 2 reasons :

1) Numerically it is easier to handle since we do not need derivatives which are not given by a differential equation such $\frac{dx_i}{dr}, \frac{d\mu}{dr} \cdots$

2) It is easier to make a thermodynamical balance in the lagrangian formalism and to consider in this balance separately the effects of the nuclear reactions and of the divergence of the flux. In the eulerian formation there is a mixing of these 2 effects.

Suppose you mix a very small shell in the region where $\frac{d\mu}{dr} \neq 0$. Two things may happen.

a. The eigenfunction is hardly modify and since $\frac{dx_i}{dr} = 0$ you will find in the eulerian formalism that nuclear reactions have a stabilizing influence. This seems to us unacceptable.

b. The eigenfunction is modified in such a way that nuclear reactions are indeed destabilizing? Then I do not like to work with an eigenfunction so sensitive to small changes in the model.