VALIDATION OF A FEM MODEL COUPLED WITH TEXTURE APPLIED TO DEEP DRAWING PROCESS

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Abstract: This paper presents a constitutive law based on the Taylor model implemented in our non linear finite element code LAGAMINE. The yield locus is only locally described and the interpolation method is presented. This local yield locus model uses a discrete representation of the material's texture. The influence of this discrete representation and a deep-drawing simulation are presented.

Keywords: finite element simulation, deep-drawing, texture, yield locus

1. Introduction

Various constitutive laws based on texture analysis have been implemented in the non linear finite element code LAGAMINE. Our first step in the integration of the texture effects was the use of a 6^{th} order serie yield locus defined by a least square fitting on a large number of points (typically 70300) in the deviatoric stress space [3]. Those points were calculated by a Taylor model based on an assumed constant texture of the material. This method, i.e. a global description of the yield locus, is actually used in the FEM code.
Unfortunately, taking into account the texture evolution effects with this yield locus would imply the computation of the 210 serie coefficients for each integration point, each time a texture updating is necessary. This would require an impressive amount of computation and memory storage (210 coefficients for each integration point) which is only partially useful as generally the stress state remains in a local zone of the yield locus. So, two new approaches, where the whole yield locus is unknown, have been investigated.
In the first case, the interesting part of the yield locus is represented by an hyperplane which is a plane defined in the five-dimensional deviatoric stress space. As it has been shown [2], the yield locus discontinuities bred by this very simple interpolation method give rise to convergence problems in the finite element code. That is the reason why a second method has been developed.
For that second approach, no yield locus is defined and a direct stress-strain interpolation between Taylor points is achieved. In this case, the stress and the plastic strain rate continuity conditions are fulfilled but, as there is no yield locus formulation, a particular stress integration scheme has to be used.
This stress-strain interpolation method and the implementation of the Taylor texture updating model are briefly described. As the number of crystals in the discrete set representing the texture is a very important parameter for the Taylor model, the influence of this parameter on the accuracy of FEM simulations is investigated. A deep-drawing simulation is also presented.

2. Description of the constitutive law

We will not describe further the 6^{th} order serie yield locus model nor the hyperplane model. The interested readers can refer to [1], [2], [3] and [4].

2.1 The stress-strain interpolation model

This model is particular in the sense that it does not use a yield locus formulation neither for the interpolation nor in the stress integration scheme.
We use a linear stress-strain interpolation described by Equation 1.

$$\underline{\sigma} = \tau \cdot \underline{\underline{C}} \bullet \underline{u} \tag{1}$$

In this equation, $\underline{\sigma}$ is a 5-D vector containing the deviatoric part of the stress; the hydrostatic part being elastically computed according to Hooke's law. The 5-D vector $\underline{u}$ is the deviatoric plastic strain rate direction; it is a unit vector. τ is a scalar describing the work hardening according to the exponential relationship of Equation 2 where the strength coefficient K, the offset Γ^0 and the hardening exponent n are material parameters fitted to experimental data and Γ is the polycrystal induced slip.

$$\tau = K \cdot \left(\Gamma^0 + \Gamma\right)^n \tag{2}$$

The interpolation is included in the matrix C of Equation 1 and is based on the following concepts. We assume 5 directions: $\underline{u}_i$ (i=1...5) advisedly chosen in the deviatoric strain rate space and the associated deviatoric stress: $\underline{\sigma}_i$ (i=1...5) lying on the yield surface according to the Taylor model. These points will define the interpolation domain and will be called stress nodes. Additionally, we compute the contravariant vectors $\underline{ss}_i$ and $\underline{uu}_i$ defined by Equations 3 and 4.

$$\underline{ss}_i \bullet \underline{\sigma}_j = \delta_{ij} \tag{3}$$

$$\underline{uu}_i \bullet \underline{u}_j = \delta_{ij} \tag{4}$$

With the use of those contravariant vectors we define intrinsic co-ordinates in the interpolation domain for any stress vector $\underline{\sigma}^*$ by projection according to Equation 5 and for any plastic strain rate direction $\underline{u}^*$ with Equation 6.

$$\eta_i = \underline{\sigma}^* \bullet \underline{ss}_i \tag{5}$$

$$\eta_i = \underline{u}^* \bullet \underline{uu}_i \tag{6}$$

The most important property of our stress-strain interpolation states that if the stress $\underline{\sigma}^*$ and the plastic strain rate direction $\underline{u}^*$ physically correspond to the same point, then the intrinsic co-ordinates η_i computed for $\underline{\sigma}^*$ (Eqn. 5) or for $\underline{u}^*$ (Eqn. 6) are equal.
The interpolation is achieved with the use of those intrinsic co-ordinates to compute the stress or the strain rate with a common formulation:

$$\underline{\sigma} = \sum_i \eta_i \cdot \underline{\sigma}_i \tag{7}$$

$$\underline{u} = \sum_i \eta_i \cdot \underline{u}_i \tag{8}$$

Putting together Equations 6 and 7, we can compute the stress associated to a plastic strain rate direction and get the expression of the interpolation matrix $\underline{\underline{C}}$ (the hardening not being taken into account here):

$$\underline{\sigma} = \underbrace{\sum_i \underline{\sigma}_i \cdot \underline{uu}_i}_{=\underline{\underline{C}}} \bullet \underline{u} \tag{9}$$

As long as the interpolation is achieved in the domain delimited by the 5 stress nodes, all the 5 η_i must remain between 0 and 1. When one of the 5 η_i becomes negative, it means that the current stress is out of the domain and an updating of the stress nodes must take place. The classical updating method consists in finding 5 new stress nodes defining a new domain containing the current stress direction. The developed improved updating method makes use of the adjacent domain. Therefore, only 1 new stress node is computed with the Taylor model and 4 of the 5 old stress nodes are kept for the interpolation. The main advantages of this method are that it requires only 1 (instead of 5) Taylor model call for an updating and it improves the continuity of the resulting yield locus and the continuity of its normal.

2.2 Implementation of the texture updating

The principal characteristic of our texture updating model is that it is based on the application of the Taylor's assumptions with a full constraint method on a discrete set of orientations representing the material's texture (computed according to [5]). It should be pointed out that the constitutive law is based on the interpolation method described earlier and on the Taylor model applied on the actual set of crystallographic orientations. These orientations are represented with the help of the Euler angles ranging from 0° to 360° for φ_1 and from 0° to 90° for ϕ and φ_2 so as to take crystal cubic symmetry into account but not the sample symmetry. In the code, a loop on the elements especially dedicated to texture evolution has been added in order to achieve the orientation set updating only after time step convergence. The lattice rotation of each crystal is computed with the same Taylor model by subtracting the slip induced rotation from the rigid body rotation included in the strain history.

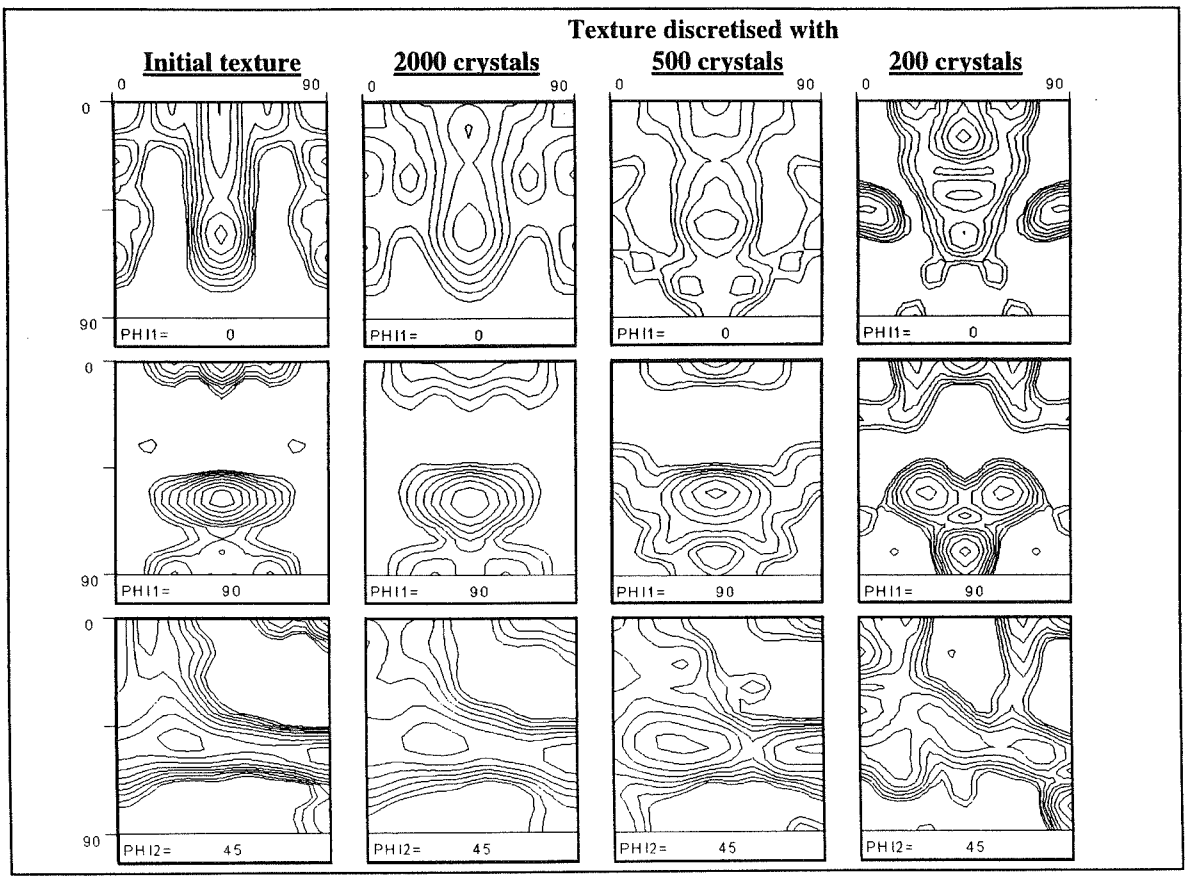

Fig. 1: Sections of the ODF before and after discretisation.

3. The discrete set of crystallographic orientations

As it has already been mentioned, the number of crystal in the discrete set representing the texture is a very important parameter for our model. The choice must results from a compromise between accuracy and CPU time. The influence of the number of crystals on the accuracy can be analysed from different point of view.
In order to have a qualitative representation of the influence on the resulting texture, we have first compared sections of the original ODF of a high tensile steel with the corresponding sections of the discretised ODF (see Figure 1). On one hand, for the best case (2000 crystals), the shape of the sections of the ODF and the locations of the maximum values are quite good. On the other hand, with only 200 crystals, there is still a similitude between the graphs but large differences appear. From this analysis, it is clear that a low number of orientations can significantly reduce the accuracy on the represented texture.

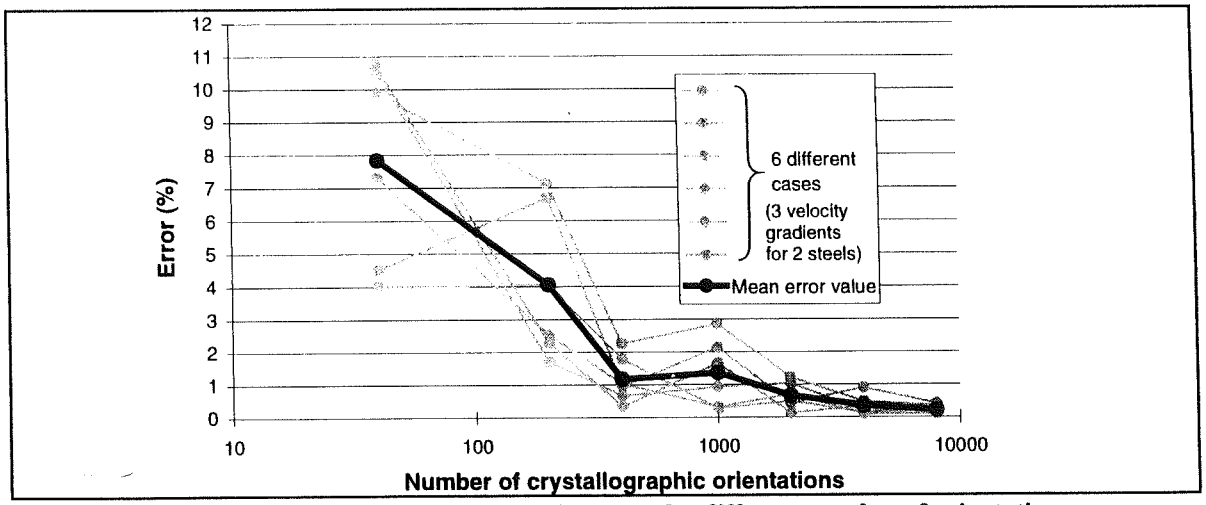

Fig. 2: Evolution of the error on computed stresses for different number of orientations (the reference result is taken as the one computed with 40000 orientations).

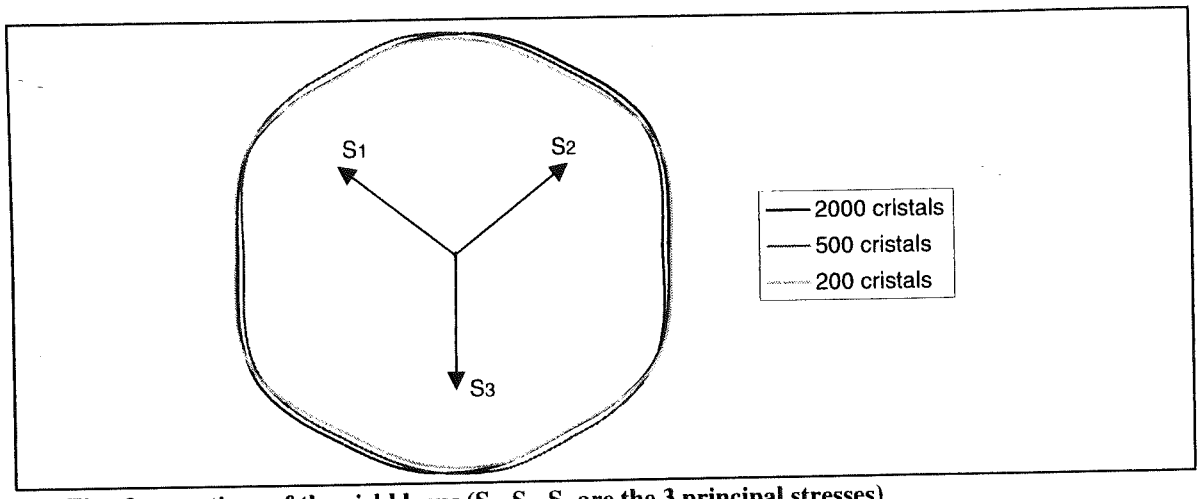

Fig. 3: π-sections of the yield locus (S_1, S_2, S_3 are the 3 principal stresses)

The second method that we have investigated is the comparison of stresses computed from the Taylor model with different number of crystals (Figure 2). This way of comparison is supposed to be more significant because it is closer to what is done in FEM codes. Here again, the number of orientations seems to have a large influence on the accuracy; this number should not be smaller than 2000 (mean error smaller than 1%).
Finally, we have compared the π-sections of the yield locus computed with our micro-macro model for different discretised textures (Figure 3). This third method is directly linked to FEM simulations because the yield locus represents the material's behaviour. The differences observed between the 3 sections of Figure 3 seem small, nevertheless they can affect considerably the results of a FEM simulation. Indeed, not only the scale of the yield stress is important but the normal to the yield locus which is the direction of the strain rate vector.
These considerations show that a particular attention must be paid to the choice of the number of crystallographic orientations.

4. Deep-drawing simulations

In order to validate our model, we have investigated deep-drawing simulations of a cylindrical cup on our finite element code. The results of these simulations will be compared to experimental tests.
A first remark, which is related to the influence of the number of crystallographic orientations, is that we have noticed a better convergence with a higher number of crystals (2000 compared to 200). This is directly linked to the micro-macro approach. Indeed, the yield locus is computed as the mean value of several yield loci of the crystals (which are extremely anisotropic and discontinuous). So a higher number of crystals reduces the discontinuities and then improves the convergence of the FEM simulations.
A first validation was the comparison of our results with FEM results using other models (Hill and 6th order yield locus [3],[4]). The general shape of the deformed cup are quite similar (earing profiles). However, our model can predict and take into account the effects of the texture evolution during FEM simulations. These predicted textures will be compared to experimental ones as soon as the texture measurements on the cup will be available.

5. Conclusions

A first conclusion is that the number of crystallographic orientations is a very important parameter for the accuracy of a micro-macro method. Different applications based on the discrete set of orientations show that good results are obtained with 2000 orientations ranging in the reduced Euler space [0°;360°]x[0°;90°]x[0°;90°]. However, during FEM simulation, we are limited by computation time; indeed, the Taylor model must be applied to each crystal of the representative set.
We have also noticed on several finite element simulations that our stress-strain interpolation method can drive to a good convergence if it is used with enough crystals in order to avoid discontinuities in the yield locus and in the strain rate direction.

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