A NEW THEORY FOR FREQUENCIES COMPUTATION OF OVERHEAD LINES WITH BUNDLE CONDUCTORS.

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Keywords :

power networks, high voltage overhead lines, galloping, torsional frequencies, torsional stiffness, vertical frequencies.

1. INTRODUCTION

Some galloping are closely related to the relative values of vertical and torsional frequencies, for a single span as well as for a whole section. The existing theory doesn't take into account the tension variation due to torsion in bundle configuration. This leads to significant discrepancies between experimental and theoretical results. Due to these discrepancies , an unadequate design of antigalloping devices may occur. This paper describes an improvement of the model currently used and shows the importance of the neglected terms. The new model agrees in a good way with the published experimental results. The frequency formulation is proposed in a closed form easily managed by hand.

It will be put into evidence that, in opposition to previous theories, the bundle geommetry influences the torsional frequencies and this explains why these frequencies are sometimes lower than vertical ones. The fixation to anchoring towers by yoke plates also plays a key role. This has never been investigated until now. The results of the theory may be applied to a proper design of pendulums whose well known effect is to detune vertical and torsional frequencies.

2. GENERAL EQUATIONS

2.1. Basic hypothesis

We are mainly concerned by small amplitude deviations around equilibrium. The value of the eigenfrequencies and the mode shapes so obtained may be used for galloping investigations. Galloping amplitudes may be rather important producing significative tensions variations, nevertheless the non-linear term of tension variations don't have a major influence neighter on eigenfrequencies values nor on modal shapes. Of courses tension variations computation must include after all the non-linear terms.

We shall neglect the propagation time of elastic wave (5000 m/s) , the slope of the catenary and the suspension string oscillations which practically are weak. The *spatial* tension variations all along a multispan section may be so neglected. Tension is independent of abscissa but is *time* dependent. Tension variation is related to the total variation of length induced by the motion of the cable on the whole section.

Assuming a parabola shape for the static catenary of each span, the curvature is constant and given by :

$$a = \frac{\partial^2 y_0}{\partial z^2} = \frac{mg}{T_0} \tag{2.1}$$

2.2. Vertical motion

Let's consider the basic equation for vertical motion :

$$\frac{\partial^2 y}{\partial t^2} - \frac{T}{m}\frac{\partial^2 y}{\partial z^2} = \frac{f_e(z)}{m} \tag{2.2}$$

let us suppose : $T = To + \Delta T$ $\qquad$ $l = lo + \Delta l$

With :

T_o : equilibrium static tension.

l_o : length of the cable at equilibrium conditions related to $y_o(z)$

l : instantaneous length at a given time t, related to y(z,t)

Taken into account the tower anchorings stiffness (K) , The Hooke's law yields for a whole section to :

$$\Delta T = \frac{EA}{L}\left(\Delta l - \frac{\Delta T}{K}\right) = K_v\,\Delta l \qquad L = \sum_{s=1}^{N_s} L_s$$

$$\frac{1}{K_v} = \frac{L}{EA} + \frac{1}{K} \tag{2.3}$$

For small variations around equilibrium position equations (2.2) and (2.3) may be written in the form :

$$\frac{\partial^2 \Delta y}{\partial t^2} - \frac{1}{m}\left[T_o\frac{\partial^2 \Delta y}{\partial z^2} + a\,\Delta T\right] = 0$$

$$\Delta T = -\,a\,K_v \int_0^L \Delta y\,dz \tag{2.4}$$

2.3. Torsional motion

The well known torsional dynamic equilibrium condition is for free vibration :

$$I\frac{\partial^2 \vartheta}{\partial t^2} - \frac{\partial C}{\partial z} = 0 \tag{2.5}$$

According to (1.21 in appendix 1) and assuming for a bundle $I=mr^2$ (m is the total mass of the bundle per unit length and r the bundle radius) we have(see appendix) for small variations :

$$\frac{\partial^2 \Delta\vartheta}{\partial t^2} - \frac{1}{m}\left[\left(\frac{\tau}{r^2} + T_o\right)\frac{\partial^2 \Delta\vartheta}{\partial z^2} + a\,\Delta H\right] = 0$$

$$\Delta H = -\,a\,K_\vartheta \int_0^L \Delta\vartheta\,dz \tag{2.6}$$

The previous equations concern each span (s = 1, ... , Ns) of a whole section. Coupling between spans arises from ΔT and ΔH terms which are only *time dependent.*

2.4. Analytical solution

The solutions of the previous equations may be found assuming the following decompositions :

$$\Delta y_s = \sum_k \Delta y_{s,k}\,\sin\frac{k\pi z}{L_s}$$

$$\Delta\vartheta_s = \sum_k \Delta\vartheta_{s,k}\,\sin\frac{k\pi z}{L_s} \tag{2.7}$$

where :
s = span index
k = mode number

substituting the expression (2.7) into (2.4) , multiplying by $\sin(k\pi z/L_S)$ and integrating between 0 and L_S the partial differential equation splits into a set of ordinary differential equations :

$$\frac{d^2\Delta y_{k,s}}{dt^2} + \left(\frac{k\pi}{L_s}\right)^2 \frac{T_0}{m}\Delta y_{k,s} = 0 \qquad k \text{ even} \tag{2.8}$$

$$\frac{d^2\Delta y_{k,s}}{dt^2} + \left(\frac{k\pi}{L_s}\right)^2 \frac{T_0}{m}\Delta y_{k,s} + \frac{4}{k\pi}\frac{a\,\Delta T}{m} = 0$$

$$\Delta T = \frac{2\,a\,K_v}{\pi}\sum_s L_s \sum_k \frac{\Delta y_{s,k}}{k} \qquad k \text{ odd} \tag{2.9}$$

Substituting expression of ΔT into left equation of (2.9) we get :

$$\frac{d^2\Delta y_{k,s}}{dt^2} + \left(\frac{k\pi}{L_s}\right)^2\frac{T_0}{m}\Delta y_{k,s} + b\,\frac{1}{k}\left\{\frac{L_1}{L}\sum_{j\,od}\frac{\Delta y_{1,j}}{j} + \frac{L_2}{L}\sum_{j\,odd}\frac{\Delta y_{2,j}}{j} + \dots + \frac{L_{N_s}}{L}\sum_{j\,odd}\frac{\Delta y_{N_s,j}}{j}\right\} = 0 \tag{2.10}$$

with :

$$b = \frac{8\,a^2\,K_v\,L}{\pi^2\,m} \qquad (2.11)$$

Equation (2.8-10) concern vertical motion, same equations hold for torsional motion mutatis mutandis, replacing :

$$y \text{ by } \vartheta,\ K_v \text{ by } K_\vartheta \text{ and } T_0 \text{ by } T_0 + \tau / r^2 \qquad (2.12)$$

Strictly speaking this system is infinite. But for practical cases, coupling between modes becomes weak for k greater than 3. (the coefficients of the matrix of the system are divided by k and j , so the matrix tends to become diagonal as k and j increase.) Computation of modal shape and corresponding frequencies implies the computation of the eigenvalues and the eigen vectors of the matrix associated to those coupled equations.

3. FREQUENCIES DISCUSSION

3.1. Torsional frequencies of single conductors.

Generally speaking equation (2.6) holds for bundle conductors and its structure is similar to (2.4) holding for vertical motion. In the case of bundle conductor it must be expected (see later) that vertical frequencies are closely related to the torsional one. But for single conductor equation (2.6) reduces to :

$$\frac{\partial^2 \Delta\vartheta}{\partial t^2} - \frac{\tau}{I}\frac{\partial^2 \Delta\vartheta}{\partial z^2} = 0 \qquad (3.0)$$

in which torsional frequencies depend only of intrinsic stiffness τ . This intrinsic stiffness is related to the internal elastic properties of the material.
With a good approximation one may admit the following formulas :

$$\tau = k_\tau\ k_{ageing}\ k_{constitution}\ \phi^4$$

$$(\ \phi \text{ in mm },\ \tau \text{ in N m}^2\ ) \qquad (3.1)$$

- k_τ : 0.00027
- k_{ageing} : between 1 and 3
- $k_{constitution}$: 1 for stranded; between 1.5 and 2.5 for smooth

Fig. 1 has been build after collecting more than 50 experimental measurement performed in different countries (Ontario Hydro [1] and [2], EDF [3], Laborelec [4], Nigol et al [5 and 6], Richardson [7], Iowa state University [8] , Leppers et al [9] , Kaiser Aluminium[10]).
It appears that there is no need to try to refine the diameter exponent, as proposed by some authors, mainly due to the influence of ageing .

The second parameter which influences the torsional frequencies is the moment of inertia .This parameter depends on :

- type of conductor (number of wire)
- mass of the conductor

It is the moment of inertia referenced to the centre of the conductor (single) . It is simply given by :

$$\rho_{cable}\frac{\pi\phi^4}{32} f_{strand} \qquad (3.2)$$

where f_{strand} is a function of the strand in the cable usually close to 0.7.

For a given ageing the ratio τ/I is merely a constant for all kind of conductors whatever the diameter is (about $10^6\ m^2/s^2$). There is no evidence for the torsional frequencies to be close to the vertical ones. For instance the fundamental torsional frequency of a single conductor is about 5 to 10 times the vertical one.

3.2 Comparison of torsional and vertical frequencies for bundle conductors

General considerations.

Looking at the equations (2.4) and (2.6) it appears that their structure is quite similar. It is due to the fact that when computing the torsional torque we consider tension variations both in direction and magnitude.

Nigol et al[5] , when computing the torsional stiffness discarded the magnitude variation induced by length variations of the subconductors. The so drop term is not negligible and could explain why Nigol et al's analytical predeterminations are smaller than experimental measurements, with discrepancies reaching 50% in some cases. In all cases it would be necessary to know exactly what kind of anchoring device was used to be able to appreciate in what extend our theory fits with experiences. Anyway our additional term increases the torsional stiffness.

It will appears further on that, for this kind of conductor, some torsional frequencies may be close to the vertical ones. We shall consider two cases: the antisymmetric modes (even number of "loops" in each span) and the symmetric modes (odd number of "loops" in each span).

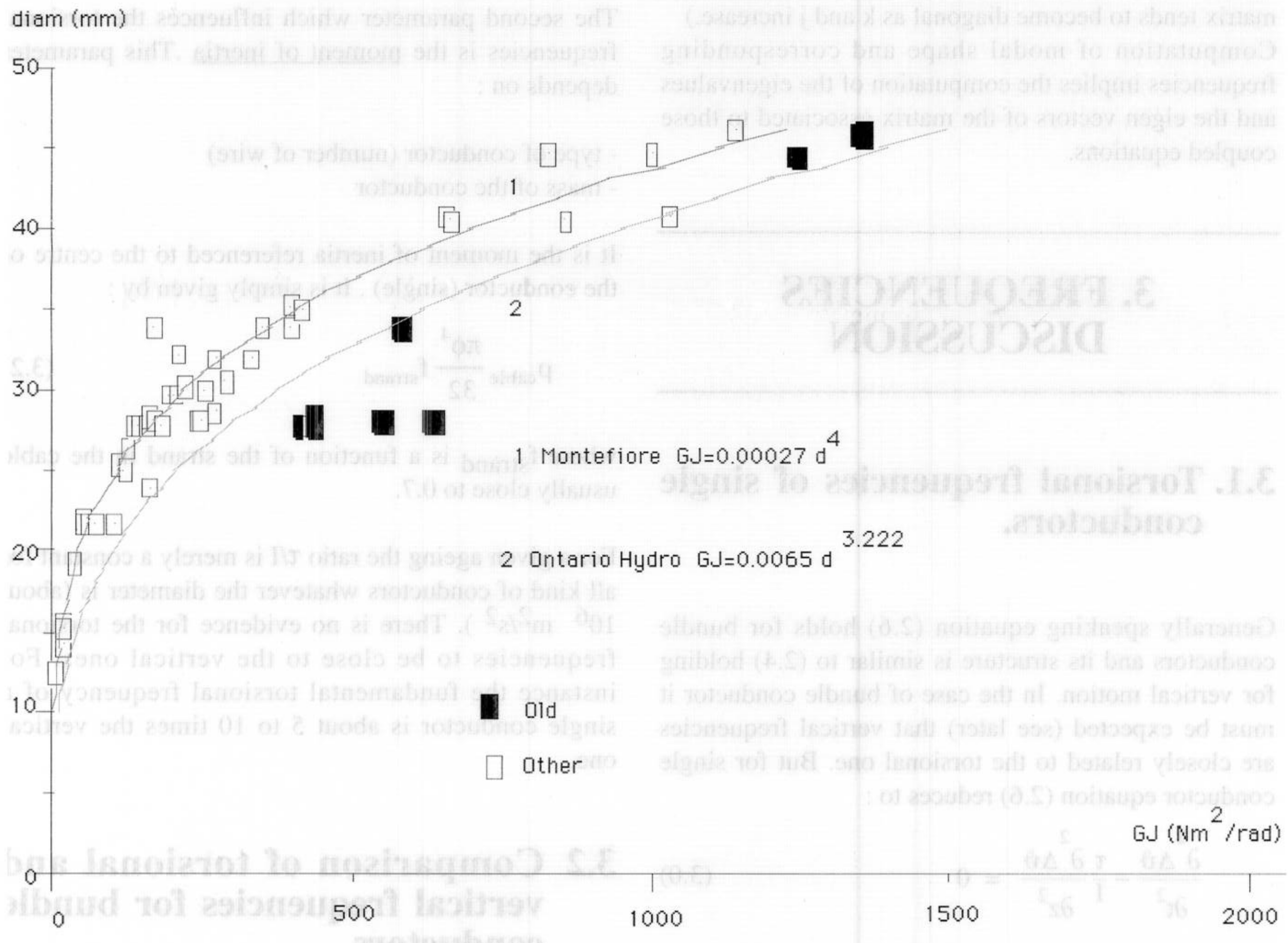


Fig.1 : Fittings of measured torsional stiffness on single conductors

a) Antisymmetric modes

The antisymmetric modes correspond to even values of 'k' in equations (2.7). The eigenfrequencies are given by equations (see 2.8) :

$$\frac{d^2 \Delta y_{k,s}}{d t^2} + \left(\frac{k\pi}{L_s}\right)^2 \frac{T_0}{m} \Delta y_{k,s} = 0 \quad k \text{ even}$$

$$\frac{d^2 \Delta \vartheta_{k,s}}{d t^2} + \left(\frac{k\pi}{L_s}\right)^2 \frac{1}{m}\left(T_0 + \frac{\tau}{r^2}\right) \Delta \vartheta_{k,s} = 0 \tag{3.3}$$

The eigen pulsations and the corresponding modal shape for a whole section are given by the characteristic values of the previous system .

It appears that the antisymmetric modes (k even) are *totally decoupled* (diagonal matrix) either inside a span either between spans. It is due to the fact that no first order tension variation is induced by even number of loops per span. This is obviously related to the fact that such deformations do not induce any first order length variation. For those antisymmetric modes the eigen pulsations are simply given by :

$$\omega^2_{v,k} = \left(\frac{k\pi}{L_s}\right)^2 \frac{T_0}{m} \qquad k \text{ even}$$

$$\omega^2_{\vartheta,k} = \left(\frac{k\pi}{L_s}\right)^2 \frac{1}{m}\left(T_0 + \frac{\tau}{r^2}\right) \tag{3.4}$$

and the modal shape given by :

$$\Delta y_s = \Delta y_{s,k} \sin \frac{k\pi z}{L_s}$$

$$\Delta y_s = \Delta y_{s,k} \sin \frac{k\pi z}{L_s} \tag{3.5}$$

These modes are easy to manage, each span may be studied separately for itself.

As it was previously announced the torsional and vertical frequencies for antisymmetric modes are close to each others, this is due to the fact that usually the intrinsic stiffness term is weak before T_0 (excepted for high subconductors diameter). The torsional frequencies are slightly higher than the vertical one.

b) Symmetric mode on a dead-ended single span(Ns = 1)

The symmetric deformations (k odd, equations 1.10 and 1.11) are coupled inside a span and between spans. It is due to the fact that tension variation associated with length variation occured when an odd number of loops appear in a span. The associated matrix is no longer diagonal and the mode shapes are the combination of weighted sine functions. Due to the structure of the matrix it can be seen that the eigenpulsations may be expressed by :

$$\omega_{v,k} = \Omega_v \, f_k(M'_v) \qquad \Omega^2_v = \left(\frac{\pi}{L_s}\right)^2 \frac{T_0}{m}$$

$$M'_v = \frac{b}{\Omega^2_v} = \frac{8 a^2 K_v}{\pi^2 \Omega^2_v m} L_s \tag{3.6}$$

$$\omega_{\vartheta,k} = \Omega_\vartheta \, f_k(M'_\vartheta) \qquad \Omega^2_\vartheta = \left(\frac{\pi}{L_s}\right)^2 \frac{1}{m}\left(T_0 + \frac{\tau}{r^2}\right)$$

$$M'_\vartheta = \frac{b}{\Omega^2_\vartheta} = \frac{8 a^2 K'_\vartheta}{\pi^2 \Omega^2_\vartheta m} L_s \tag{3.6bis}$$

With Ω the basic pulsation which is also the lowest antisymmetric eigenfrequency, and K_V and K_ϑ defined by :

$$\frac{1}{K_v} = \frac{L_s}{EA} + \frac{1}{K} \qquad K_\vartheta \text{ defined in appendix} \tag{3.7}$$

Recall that for antisymmetric modes the only difference between torsional and vertical frequencies for bundle configurations comes from the intrinsic stiffness of each subconductor. For symmetric modes the anchoring stiffnesses may induce significant difference between the two type of frequencies.

For vertical frequencies K_V depends on the anchoring tower stiffness K (about 500000 N/m). For multi-span section K may be consider as infinite compared with EA/L_S.

For torsional frequencies K_ϑ depends on the geommetry of the bundle and on the device by which the bundle is anchored to the tower (shape of the yoke plates). See appendix for special cases formulas. The influence of yoke plates on torsional frequencies has never been investigated by other authors. Its influence can be so important that it can produce a torsional frequency lower than the vertical one. This is due to the fact that K_ϑ may be significantly smaller than K_v . Firstly by the term taking into account the yoke plate design (possible value for h is 0.1m), secondly the number of subconductors. This last point may be understood if we consider an infinite anchoring stiffnesses both for the tower (K infinite) and for the yoke plate (h very large or for yoke plate designed for double anchoring insulators chain) :

$$M_v = \frac{8 E g^2 \rho_c^2}{\pi^4 \sigma^3} L_s^2$$

$$M_\vartheta = \left(\begin{array}{l} \text{for horizontal twin } : M_v \\ \text{for quad bundle } : 0.5\, M_v \end{array} \right) \qquad (3.8)$$

This fact might be taken into account for pendulum design.

Roughly speaking 'M' is proportional to the square of the span length, other terms are rather constant (E , Young's modulus about 6 10^{10} N/m^2 ; $g = 9.81\ m/s^2$; ρ_c , specific mass, about 3000 kg/m^3 , σ the every day stress =T/A , about 55 N/mm^2). The functions $f_k(M)$ are plotted on fig. 2 for k=1 and 3.

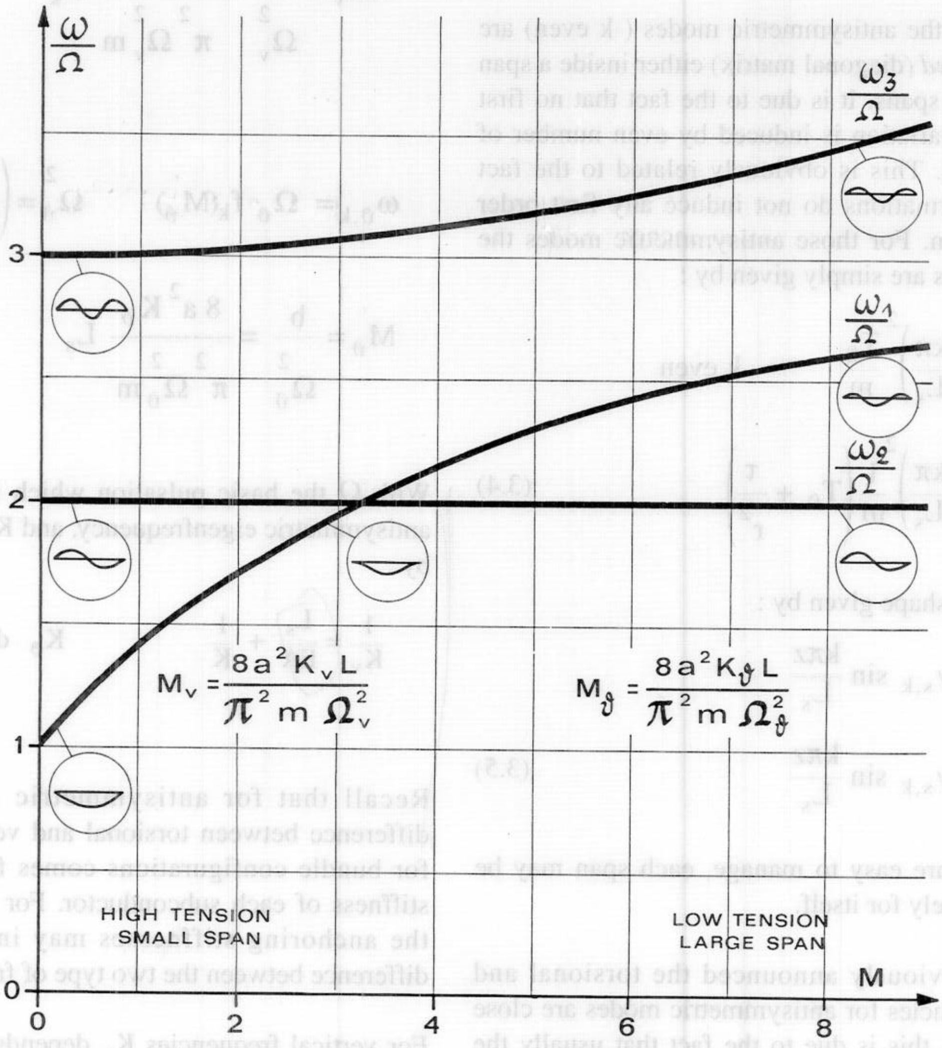


Fig.2 : Giving the ratio betwen the symetric mode frequency (kodd) and the basic frequency. Valuable both for vertical and torsional motion (see formula 3.6).

For practical value of 'M' (M<10) , $f_k(M)$ is about 1 for k>4. This parameter has already been discussed by some authors [11,12,13]

The functions $f_k(M)$ are such that $f_k(0) = k$ and increase with M in such a way that for some values of M (>3) we have $f_1(M) > 2$ and the lowest symmetric frequency is higher than the first antisymmetric one(2 loops). Let's notice that the shape of this mode may be called a *pseudo one-loop* due to the fact that its shape may not be properly approximate by a pure sine wave as it may be accepted for the higher ones.(See Fig. 3). This fact has already been pointed out by other authors [12]

c) Symmetric modes for equal span section

Let's consider the special case of equally span section. In this case the matrix has the form of a matrix of submatrices with equal diagonal submatrices and equal off-diagonal submatrices. According to this it may be shown that eigen pulsations would be those of a span of the section, isolated from the others, and supposed to be dead-ended one time with an anchoring stiffness (N_S x K) and another time with zero anchoring stiffnesses .

For zero stiffness the eigen pulsations are given by :

$$\omega_k^2 = k^2 \Omega^2 \qquad \text{k odd} \qquad (3.9)$$

and the corresponding mode shapes for the whole section :

$$\Delta y_s = A_s \sin \frac{k\pi z}{L_s} \qquad s = 1,N_s \qquad \text{such that} \quad \sum_s A_s = 0 \qquad (3.10)$$

Expression (3.9) implies that no first order tension variations occurs. It must be pointed out that this shape and those eigen pulsations are not met in a single span as long as the anchoring stiffness is different from zero. Let's notice that anti-symmetric mode could be considered as included in the last one , the right condition (3.10) being omitted.

These modes are the so called "up and down galloping" the most frequently observed.

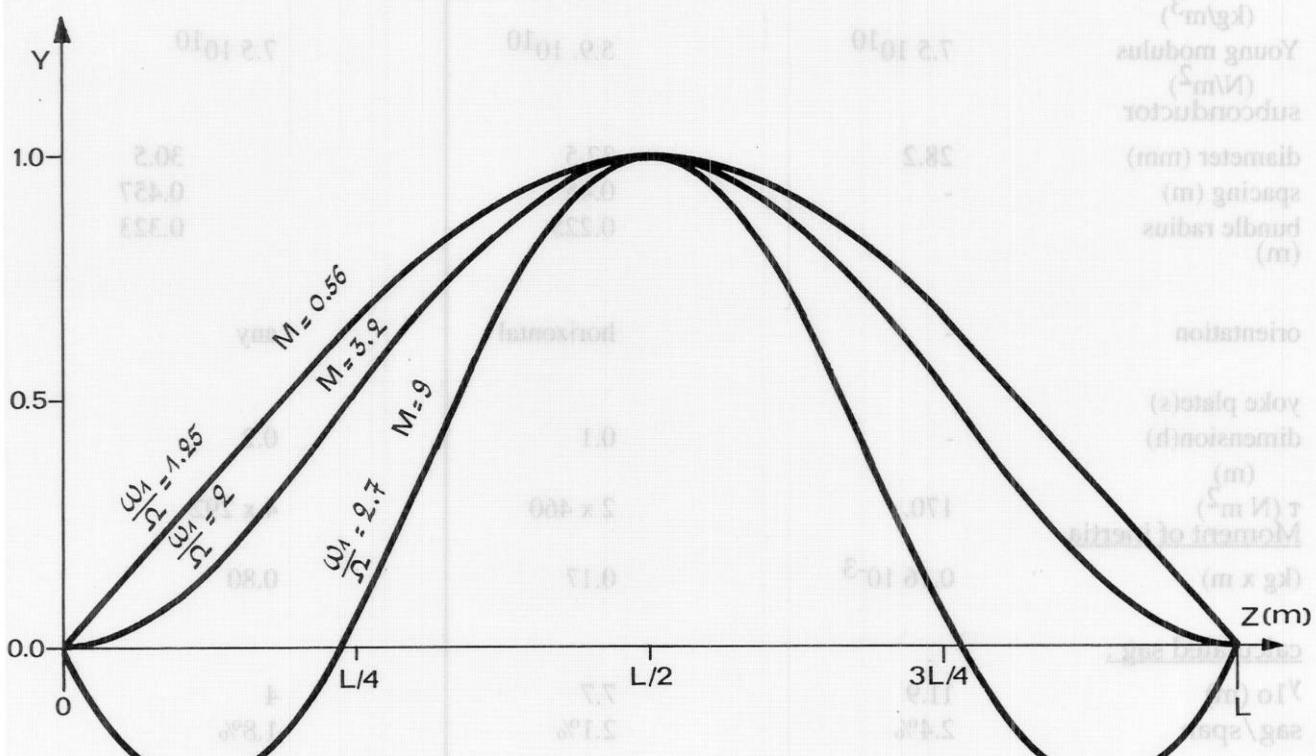


Fig.3 : Shape of the pseudo-one loop mode for different M values.

For the $N_S K$ stiffness the corresponding mode obtained for the dead-ended single span is simply repeated in phase all along the section (in phase mode).

d) Symmetric modes in the general case

For non equal span section eigenvalue of the complete system (2.10) have to be computed for each case.

4. ORDER OF MAGNITUDE

Let's consider three kinds of cable whose characteristics follows :

	ACSR 470^2 (drake)	AMS 2 x 620^2	ACSR 4 x 550^2
span length (m)	488	360	225
tension (kN)	40	2 x 35	4 x 29.25
stress (N/mm^2)	85	56	53
mass (kg/m)	1.63	2 x 1.7	4 x 1.9
catenary parameter (T/mg) (m)	2500.	2100.	1570.
volumic mass (kg/m^3)	3460	2740	3470
Young modulus (N/m^2)	$7.5\ 10^{10}$	$5.9.\ 10^{10}$	$7.5\ 10^{10}$
subconductor			
diameter (mm)	28.2	32.5	30.5
spacing (m)	-	0.45	0.457
bundle radius (m)		0.225	0.323
orientation	-	horizontal	any
yoke plate(s) dimension(h) (m)	-	0.1	0.2
τ ($N\ m^2$)	170.	2 x 460	4 x 292
Moment of inertia (kg x m)	$0.16\ 10^{-3}$	0.17	0.80
calculated sag :			
y_{1o} (m)	11.9	7.7	4
sag/span	2.4%	2.1%	1.8%
Basic pulsations			
Ω_V(rad/s)	2π x 0.16	2π x 0.20	2π x 0.28
Ω_ϑ(rad/s)	2π x 1.07	2π x 0.22	2π x 0.29

deadended span :

bundle stiffness (torque at mid-span)

(N m/rad)	-	105.(h=∞)	410. (h=∞)
	-	70. (h=0.1)	290. (h=0.2)
	-	50. (h=0)	240. (h=0)

frequencies

a) $K_v = K_\vartheta = 0$ (hypothetic case)

	$M'_v = 0$	$M'_\vartheta = 0$	$M'_v = 0$	$M'_\vartheta = 0$	$M'_v = 0$	$M'_\vartheta = 0$
	v	ϑ	v	ϑ	v	ϑ
1 loop	0.16	1.07	0.20	0.22	0.28	0.29
2 loops	0.32	2.14	0.40	0.44	0.55	0.58
3 loops	0.48	3.21	0.60	0.66	0.83	0.87
4 loops	0.64	4.28	0.80	0.88	1.10	1.15

b) K=250000. N/m and yoke plate of dimension h

	$M'_v = 2.2$	$M'_\vartheta = 0$	$M'_v = 1.4$	$M'_\vartheta = 0.51$	$M'_v = 0.60$	$M'_\vartheta = 0.25$
stiffness (N/m)	$K_v = 6\ 10^4$ $K_\vartheta = 0.$		$K_v = 11\ 10^4$ $K_\vartheta = 5\ 10^4$		$K_v = 19\ 10^4$ $K_\vartheta = 8.5\ 10^4$	
	v	ϑ	v	ϑ	v	ϑ
pseudo-1 loop	0.28	1.07	0.31	0.27	0.35	0.32
2 loops	0.32	2.14	0.40	0.45	0.55	0.58
pseudo-3 loops	0.49	3.21	0.60	0.67	0.83	0.87
4 loops	0.48	4.28	0.80	0.89	1.10	1.15
5 loops	0.80	5.35	1.0	1.12	1.38	1.44

c) K = ∞ and h = ∞

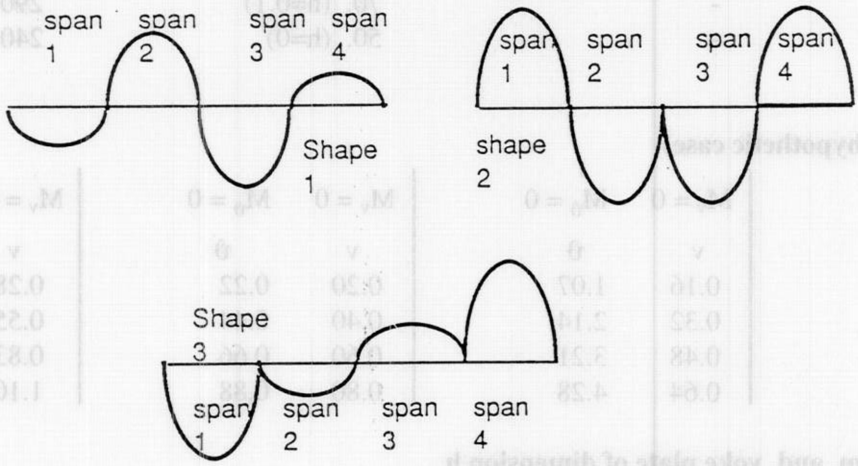
	$M'_v = 2.78$	$M'_\vartheta = 0$	$M'_v = 2.5$	$M'_\vartheta = 2.0$	$M'_v = 2.38$	$M'_\vartheta = 1.08$
stiffness (N/m)	$K_v = 7\ 10^4$ $K_\vartheta = 0.$		$K_v = 20\ 10^4$ $K_\vartheta = 20\ 10^4$		$K_v = 73\ 10^4$ $K_\vartheta = 36\ 10^4$	
	v	ϑ	v	ϑ	v	ϑ
pseudo-1 loop	0.31	1.07	0.37	0.38	0.50	0.41
2 loops	0.32	2.14	0.40	0.45	0.55	0.58
pseudo-3 loops	0.49	3.21	0.61	0.68	0.84	0.87
4 loops	0.64	4.28	0.80	0.89	1.10	1.15
5 loops	0.80	5.35	1.0	1.12	1.38	1.44

four-spans section frequencies (equal spans) :

bundle stiffness (torque at mid-span of the first span)

(N m/rad)	-	60. (h=∞)	270. (h=∞)
	-	57. (h=0.1)	260. (h=0.2)
	-	50. (h=0)	240. (h=0)

frequencies
K=250000. N/m and yoke plate of dimension h
remark : "up and down" mode corresponds to any shape of he whole section satisfying the condition : see fig. as an example of 1-loop(up and down) mode



	v	ϑ	v	ϑ	v	ϑ
1 loop (up and down)	0.16	1.07	0.20	0.22	0.28	0.29
pseudo-1 loop (in phase)	0.3	1.07	0.35	0.33	0.42	0.36
2 loops	0.32	2.14	0.40	0.44	0.56	0.58
3 loops (up and down)	0.48	3.21	0.60	0.67	0.83	0.87
pseudo-3 loops (in phase)	0.49	3.21	0.61	0.68	0.84	0.87
4 loops	0.48	4.28	0.80	0.88	1.12	1.16
5 loops (any type)	0.80	5.35	1.0	1.12	1.38	1.44

5. EXPERIMENTAL COMPARISONS

5.1 Bundle stiffness

For a single span configuration we will refer to Nigol and Havard [5] experimental results.

As we pointed out before it must be expected that application of Nigol et al formula will give stiffness values lower than experimental one, especially for "M" values greater than 1. In this last eventuality there is a major influence of tension variations between subconductors, that means *a major influence of subconductor fixation on the anchoring insulator.*

Comparison with our theoretical formula is made for a yoke plate dimension corresponding to the value of "h" (see appendix 3) equal to 0.2 meter. Moreover we have indicated the stiffness value for h=∞, which , for a twin, would correspond to subconductors connected to two separate anchoring insulator string. Let's notice that Nigol's theoretical values correponds to a value of "h" equal to zero.

bundle 2x470 ACSR ($\tau = 2 \times 164\ Nm^2$) - torque applied at mid-span

spacing (m)	tension (N)	span length (m)	h=0 [5]	h=0.2	h=∞	Exper. [5]	(test N°)
			Stiffness in Nm/deg				
0.305	2x35600	244	0.57	0.76	0.89	0.76	(9)
0.457	2x40000	488	0.64	1.18	1.49	1.30	(31)
0.610	2x35600	244	2.0	2.3	3.26	2.1	(35)

bundle 4x550 ACSR ($\tau = 4 \times 292\ Nm^2$)- torque applied at mid-span

spacing (m)	tension (N)	span length (m)	h=0 [5]	h=0.2	h=∞	Exper. [5]	(test N°)
			Stiffness in Nm/deg				
0.457	4x29250	225	4.0	5.06	7.14	5.08	(46)

Tests on a four spans section of Belgium experimental line [4]

The subconductors are fixed to the anchoring towers by two insulator chains, in such a way "h" can be consider as infinite.

Special spacers were used (5 per span) whose mass were about 7.5 kg each. The center of gravity of those spacers were situated below the subconductors plane (of about 0.13m). These spacers act like pendulums and thus add a significant contribution to bundle stiffness. This has been taken into account in the computation of stiffness and later on in the computation of frequencies.

twin 2x620 AMS ($\tau = 2x460\ Nm^2$) horizontal

spacing (m)	tension (N)	spans length	h=0	h=∞	experience
0.450	2x35200	361	1.2	1.33	1.34
		361			
		397	torque applied at the middle		
		429	of the first span (about 50Nm)		

Lets notice that the same structure without pendulum effect of the spacers would give a stiffness of 1 Nm/deg (for h=∞) in the same conditions of testing. Thus a reduction of about 30% of the bundle stiffness !!

5.2 Bundle frequencies (Hz)

The same four-span section of the belgium experimental line has been tested. We first propose the results of computations :

mode	torsional ϑ	vertical v
1. one loop	0.24	0.17
2. one loop	0.26	0.19
3. one loop	0.27	0.20
4. pseudo-one loop	0.40	0.36
5. two loops	0.40	0.33
6. two loops	0.43	0.36
7. two loops	0.47	0.39
8. two loops	0.47	0.39
9. three loops	0.58	0.50
10. three loops	0.63	0.54
11. three loops	0.68	0.59
12. pseudo three loops	0.69	0.59

Experimental measurements :

The measurement of the frequencies has been made as explained in [4].

The following frequencies has been found :

for vertical motion : 0.40 (test serie 30, subtest 31) and 0.19 (all other tests) Hz (one loop)

for torsional tests : 0.22 (test N°60) and 0.36 (test N°150) Hz (one loop)

It is important to notice that for vertical tests, recording of the mechanical tension shows a very clear one frequency oscillation of 0.37 Hz for all tests

Those frequencies must be included in our 4 first modal proposition (one loop) and it is the case with a very good correlation.

6. CONCLUSIONS

A more general theory for overhead lines torsional stiffnesses has been established. This theory takes into account the tension variations in the bundle induced by its torsion. We must point out , in spite of its weakness , the tension variation between subconductors may increase the bundle stiffness to more than 50%. The new resulting mathematical terms leads to torsional eigenvalues equations structurally similar to the vertical ones. The difference between torsional and vertical frequencies comes in a minor way from intrinsic stiffness of the subconductors and essentially from bundle geommetry and equivalent stiffnesses of anchoring at both dead-end towers. For vertical motion the stiffness of the dead-end towers is to be considered. For the torsional motion the "equivalent stiffness" decribes the behaviour of anchoring device (yoke plates) which moves trying to balance the tension variations between subconductors. The importance of these two stiffnesses decrease as the number of spans increases in the section. With the introduction of tension variation a coupling between spans appears in the section.

Although not explained in the paper we have introduce pendulum effects (static and dynamic) in the equations in order to study their detuning effect. A proper choice of pendulums equipment may be delicate. Our theory shows that ,for a given same mode, torsional frequency may be significantly smaller than vertical one, especially for three and quad bundle, and not systematically greater as it is commonly admitted. Pendulums tend to increase torsional frequencies so it is worthwhile to be sure that a tuning effect is not obtained instead of a detuning one.

A last refinement has been introduce to take into account the motion of the suspension insulators for vertical motion. This makes appear small tension variations between spans and doesn't affect significantly the eigenfrequencies.

Let's recall that our theory has been established for small displacements around equilibrium position, the non-linear terms appearing in tension expression have been dropped. This is perfectly acceptable for frequencies evaluation. To some extend this would be unvaluable for studying the free response of a section when the amplitude is not strictly negligeable compared with the sag (about 5%). Addition of non linear terms show that due to a parametric excitation unexisting modes in initial conditions may arise whose amplitudes are comparable to the initially excited modes.

This suggest that a proper modelization of galloping must consider the phenomenon as a whole and that physical simplifications have to be carefully introduced step by step.

For instance, up to now we supposed a very short longitudinal propagation time compared with galloping period allowing us to suppose that tension is spatially constant all along the section. When observing some oscillogram of tension recorded during galloping , a significant and sometimes dominant "high frequency" (about 1 to 2 Hz) component appears. Curiously this frequency is very close to the first *longitudinal* eigen frequency. Reasonably it could be suspected that an unfortunate tuning exists between this last frequency and a high harmonic of vertical motion. This phenomenon has been reproduced by a general simulation program based on finite elements method. This phenomenon would have to be investigated because when occuring it could be very dangerous for the anchoring towers whose first frequency is around 163 Hz.

Appendix 1 Torsional stiffness for bundle conductors

if for instance :

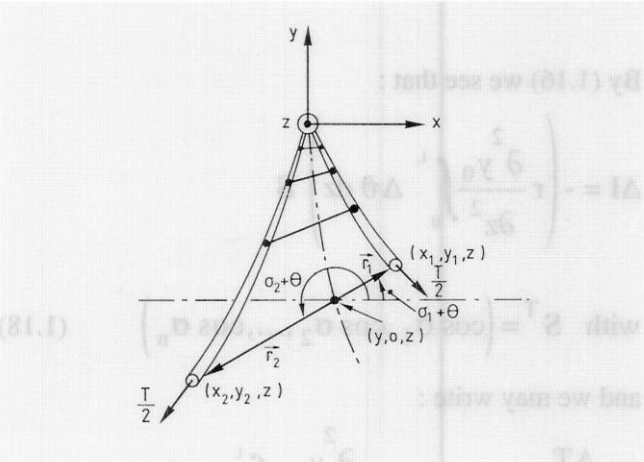


Fig.A.1 bundle torsion situation

$x_i(z)$, $y_i(z)$ coordinates of subconductor i in the bundle (fig A.1)

T_i the tension in the subconductor i

C the centre of the bundle

r the radius of the bundle

σ_i the angular position of subconductor i in static position

the vectorial expression for the tension is :

$$T_i \mathbf{e_t} = T_i \begin{pmatrix} \frac{\partial x_i}{\partial z} \\ \frac{\partial y_i}{\partial z} \\ 1 \end{pmatrix} \quad \text{with} \quad \frac{\partial x_i}{\partial z} << 1 \quad \frac{\partial x_i}{\partial z} << 1 \qquad (1.1)$$

Discarding temporarily the torque due to intrinsic stiffness, the torque with regard to the centre of the bundle is given by :

$$C_i = r_i \wedge T_i \, e_t \qquad (1.2)$$

where the vector **r** is given by (see fig A.1)

$$\mathbf{r_i} = \begin{pmatrix} r\cos(\sigma_i + \Delta\vartheta) \\ r\sin(\sigma_i + \Delta\vartheta) \\ 0 \end{pmatrix} \qquad (1.3)$$

We are only interested by the z component of the torque , which can be written:

$$C_{z,i} = T_i \, r \left(-\frac{\partial x_i}{\partial z} \sin(\sigma_i + \Delta\vartheta) + \frac{\partial y_i}{\partial z} \cos(\sigma_i + \Delta\vartheta) \right) \qquad (1.4)$$

with :

$$x_i = r\cos(\sigma_i + \Delta\upsilon) \qquad y_i = y + r\sin(\sigma_i + \Delta\upsilon) \qquad (1.5)$$

$$\frac{\partial x_i}{\partial z} = -r\sin(\sigma_i + \Delta\vartheta)\frac{\partial \Delta\vartheta}{\partial z}$$

$$\frac{\partial y_i}{\partial z} = \frac{\partial y}{\partial z} + r\cos(\sigma_i + \Delta\vartheta)\frac{\partial \Delta\vartheta}{\partial z} \qquad (1.6)$$

Substitution into (1.34) yields to :

$$\frac{\partial y_i}{\partial z} = \frac{\partial y}{\partial z} + r\cos(\sigma_i + \Delta\vartheta)\frac{\partial \Delta\vartheta}{\partial z} C_{z,i}$$

$$= T_i \, r \left(r\frac{\partial \Delta\vartheta}{\partial z} + \frac{\partial y}{\partial z}\cos(\sigma_i + \Delta\vartheta) \right) \qquad (1.7)$$

Let's

$$T_i = \frac{T_0}{n} + \Delta T_i \qquad (1.8)$$

So we have for the total torque :

$$C_z = r^2 \left(T_0 + \sum \Delta T_i \right) \frac{\partial \Delta\vartheta}{\partial z} +$$

$$r\frac{\partial y}{\partial z}\left[T_0 \sum \cos(\sigma_i + \Delta\vartheta) + \sum \Delta T_i \cos(\sigma_i + \Delta\vartheta) \right] \qquad (1.9)$$

Neglecting $\sum \Delta T_i$ compared with T_0 and considering only symmetric bundle ($\sigma_i = \sigma_{i-1} + \pi/n$) equation (1.9) may be written :

$$C_z = r^2 T_0 \frac{\partial \Delta\vartheta}{\partial z} + r\frac{\partial y}{\partial z} \sum \Delta T_i \cos(\sigma_i + \Delta\vartheta) \qquad (1.10)$$

whose first derivative may be written , after dropping sine term which is either nul or negligible :

$$\frac{\partial C_z}{\partial z} = r^2 \left[T_0 \frac{\partial^2 \Delta\vartheta}{\partial z^2} + \frac{\partial^2 y}{\partial z^2} \sum \frac{\Delta T_i}{r} \cos\sigma_i \right] \qquad (1.11)$$

We have still to compute the ΔT_i values according to the elastic properties of the section (L/EA) , and the flexibility matrix ($\mathbb{F}$) associated to the device by which subconductors are fixed to the dead-end towers of the section. We are mainly concerned with dead-end towers fixations due to the fact that we admit free horizontal motions of subconductors at their fixations on suspension towers.

$$\begin{pmatrix} \Delta T_1 \\ \Delta T_2 \\ \dots \\ \Delta T_n \end{pmatrix} = \frac{EA}{nL} \left[\begin{pmatrix} \Delta l_1 \\ \Delta l_2 \\ \dots \\ \Delta l_n \end{pmatrix} -2 \begin{pmatrix} F_{1,1} F_{1,2} & \dots & F_{1,n} \\ F_{2,1} F_{2,2} & \dots & F_{2,n} \\ \dots & \dots & \dots \\ F_{n,1} F_{n,2} & \dots & F_{n,n} \end{pmatrix} \begin{pmatrix} \Delta T_1 \\ \Delta T_2 \\ \dots \\ \Delta T_n \end{pmatrix} \right] \quad (1.12)$$

or with matricial notations :

$$\Delta T = [\Delta l - 2\, \mathbb{F}\, \Delta T\,] \quad (1.13)$$

as $\Delta y_i \ll y$ we have by application of rectification formula :

$$\Delta l_i \cong \frac{1}{2} \int_0^L \left[\left(\frac{\partial (y + \Delta y_i)}{\partial z} \right)^2 - \left(\frac{\partial y}{\partial z} \right)^2 \right] dz =$$

$$\int_0^L \frac{\partial y}{\partial z} \frac{\partial \Delta y_i}{\partial z} dz = - \int_0^L \frac{\partial^2 y}{\partial z^2} \Delta y_i \, dz \quad (1.14)$$

with :

$$\Delta y_i = r\,(\sin(\sigma_i + \Delta \upsilon) - \sin \upsilon_i) \quad (1.15)$$

For torsional motions around equilibrium position or if vertical motion is relatively small coupling between vertical and torsional motion may be neglected and we may replace (1.14) and (1.15) by :

$$\Delta l_i = - \frac{\partial^2 y_0}{\partial z^2} \int_0^L \Delta y_i \, dz = - \left(r \frac{\partial^2 y_0}{\partial z^2} \int_0^L \Delta\vartheta \, dz \right) \cos \sigma_i \quad (1.16)$$

Let's recall :

$$\frac{\partial^2 y_0}{\partial z^2} = \frac{mg}{T_0} \quad (1.17)$$

By (1.16) we see that :

$$\mathbf{\Delta l} = - \left(r \frac{\partial^2 y_0}{\partial z^2} \int_0^L \Delta\vartheta \, dz \right) \mathbf{S}$$

$$\text{with} \quad \mathbf{S}^T = \left(\cos \sigma_1, \cos \sigma_2, \dots, \cos \sigma_n \right) \quad (1.18)$$

and we may write :

$$\sum \frac{\Delta T_i}{r} \cos \sigma_i = -K_\vartheta \frac{\partial^2 y_0}{\partial z^2} \int_0^L \Delta\vartheta \, dz = \Delta H \quad (1.19)$$

$$K_\vartheta = + \mathbf{S}^T \left[\frac{nL}{EA} \mathbb{U} + 2\, \mathbb{F} \right]^{-1} \mathbf{S} \qquad \text{where } \mathbb{U} = \text{diag}(1, \dots) \quad (1.20)$$

Reintroducing the torque due to total intrinsic stiffness ($\tau = n\tau_c$), and according to (1.19) equation (1.11) becomes :

$$\frac{\partial C}{\partial z} = \tau \frac{\partial^2 \Delta\vartheta}{\partial z^2} + r^2 \left[T_0 \frac{\partial^2 \Delta\vartheta}{\partial z^2} - K_\vartheta \left(\frac{\partial^2 y_0}{\partial z^2} \right)^2 \int_0^L \Delta\vartheta \, dz \right] \quad (1.21)$$

Appendix 2 Flexibility matrix

Case 1 twin bundle

For the yoke plate shown on fig. A.2 we have :

$$F_{1,1} = F_{2,2} = -F_{1,2} = -F_{2,1} = d^2 / 4hT = q$$

For twin bundle of any orientation : $\sigma_1 = \sigma$ and $\sigma_2 = \sigma + \pi$

Expression (1.20, appendix 1) is easy to compute. The vector **S** is an eigen vector of $\mathbb{F}$ associated to the eigenvalues 2q. It is allways an eigen vector of the matrix (1.20, appendix 1) associated to the eigenvalue " 4q + nL/EA" .We have so for K_ϑ :

$$K_\vartheta = |\mathbf{S}|^2 \left(\frac{d^2}{hT} + \frac{nL}{EA} \right)^{-1}$$

and
$$\frac{1}{K_\vartheta} = \frac{1}{\cos^2 \sigma} \left(\frac{d^2}{2hT} + \frac{L}{EA} \right) \qquad \text{(A2.1)}$$

For a vertical bundle $\sigma = \pi/2$ and $K_\vartheta=0$.
For a horizontal bundle $\sigma = 0$ and
$1/K_\vartheta = d^2/2hT + L/EA$

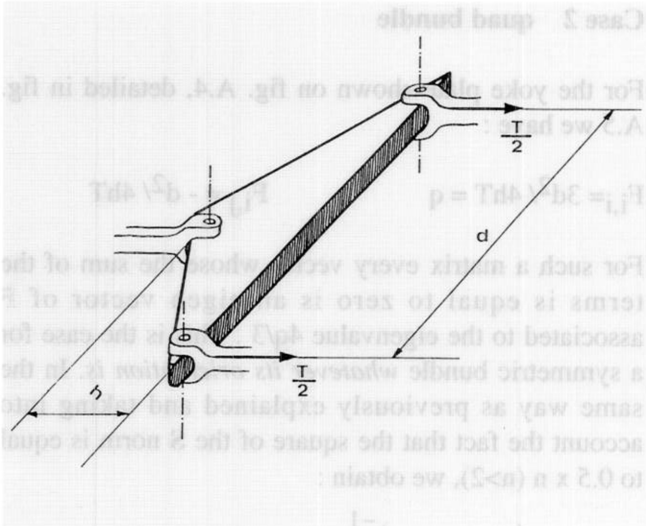


Fig. A.2 twin bundle yoke arrangement, with definition of h

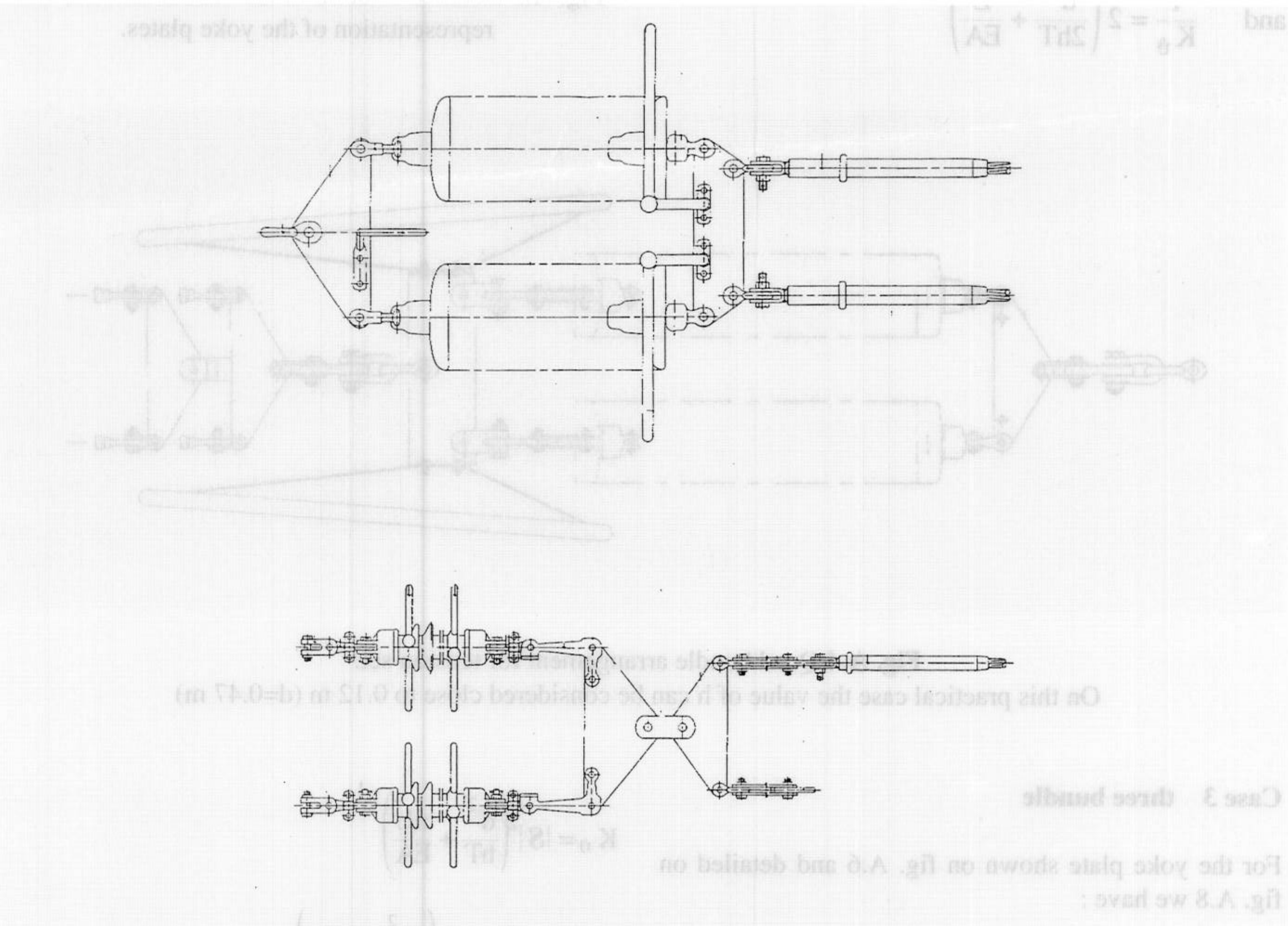


Fig. A.3 Two practical arrangement on twin bundle geommetry
The upper one can be considered with a very large h (h=∞ in equation A2.1)
The lower one can be considered with h = 0.12 m and d=0.4 m.
Means that those two yoke plates arrangement will have very sensible different impact on torsional frequencies, especially for horizontal twin configuration.

Case 2 quad bundle

For the yoke plate shown on fig. A.4, detailed in fig. A.5 we have :

$F_{i,i} = 3d^2/\,4hT = q$ $\qquad$ $F_{i,j} = -\,d^2/\,4hT$

For such a matrix every vector whose the sum of the terms is equal to zero is an eigen vector of $\mathbb{F}$ associated to the eigenvalue 4q/3 . This is the case for a symmetric bundle *whatever its orientation is.* In the same way as previously explained and taking into account the fact that the square of the **S** norm is equal to 0.5 x n (n>2), we obtain :

$$K_\vartheta = |\mathbf{S}|^2 \left(\frac{2d^2}{hT} + \frac{nL}{EA} \right)^{-1}$$

and $$\frac{1}{K_\vartheta} = 2\left(\frac{d^2}{2hT} + \frac{L}{EA} \right)$$

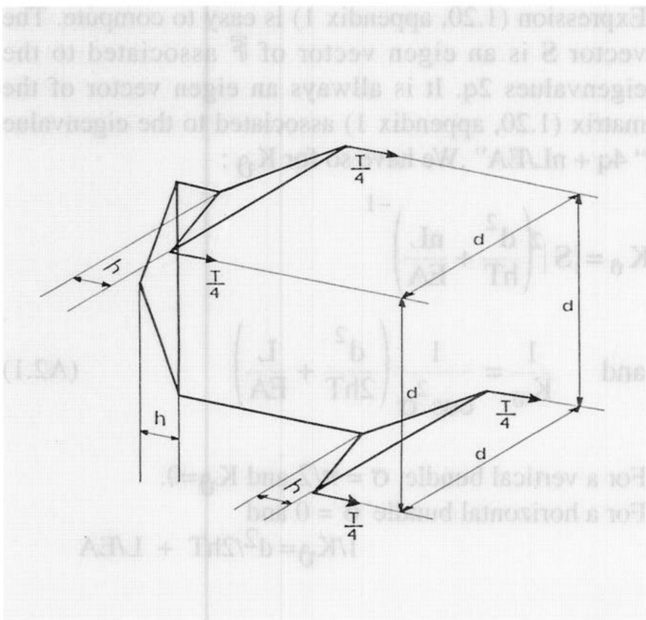


Fig. A.5 Detailed fixation of the subconductors, with representation of the yoke plates.

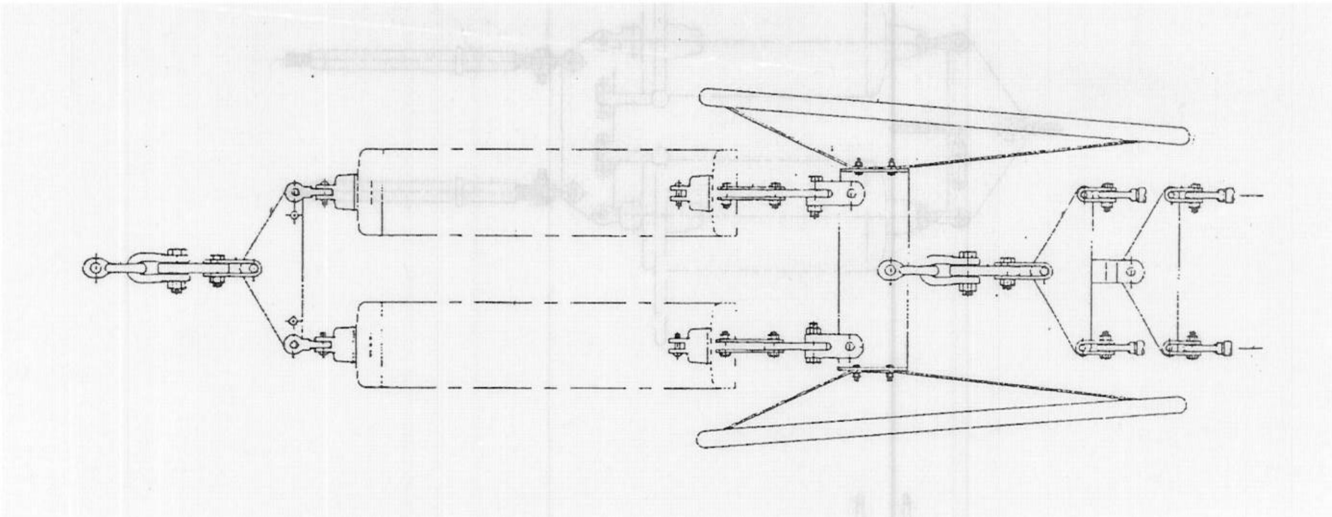


Fig. A.4 Quad bundle arrangement for tension set.
On this practical case the value of h can be considered close to 0.12 m (d=0.47 m)

Case 3 three bundle

For the yoke plate shown on fig. A.6 and detailed on fig. A.8 we have :

$F_{i,i} = +\,d^2/\,3hT = q$ $\qquad$ $F_{i,j} = -\,d^2/\,6hT$

The vector **S** is an eigenvector of $\mathbb{F}$ associated to the eigenvalue 3q/2 . In the same way as previously explained, we obtain :

$$K_\vartheta = |\mathbf{S}|^2 \left(\frac{d^2}{hT} + \frac{nL}{EA} \right)^{-1}$$

and $$\frac{1}{K_\vartheta} = 2\left(\frac{d^2}{3hT} + \frac{L}{EA} \right)$$

Don't forget that A is the total cross section of one phase, T the total tension in one phase and d the subconductor *spacing* .

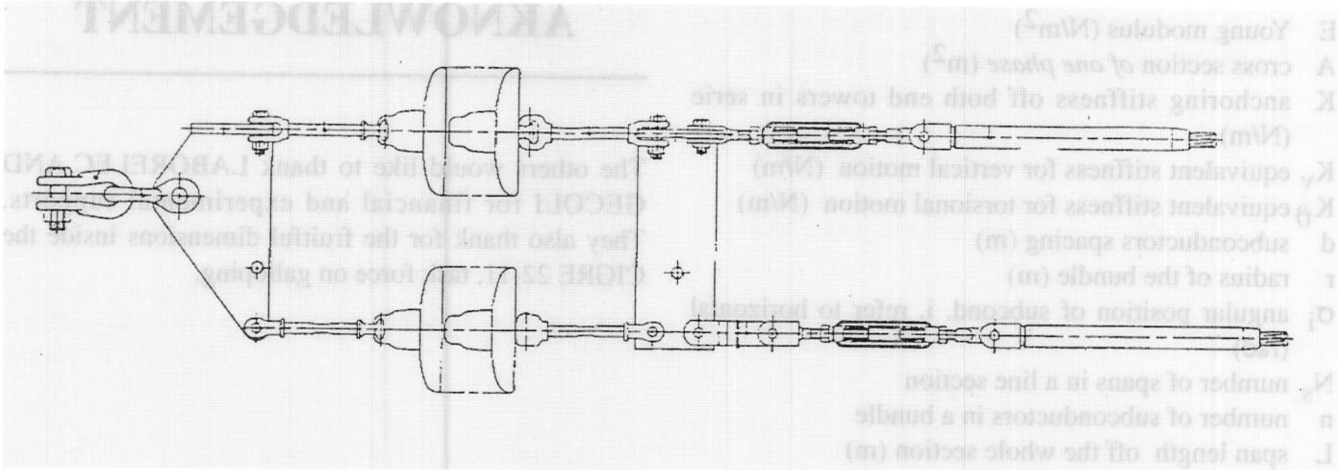


Fig. A.6 Triple tension set : a practical disposition, with h=0.16 m (d=0.47m)

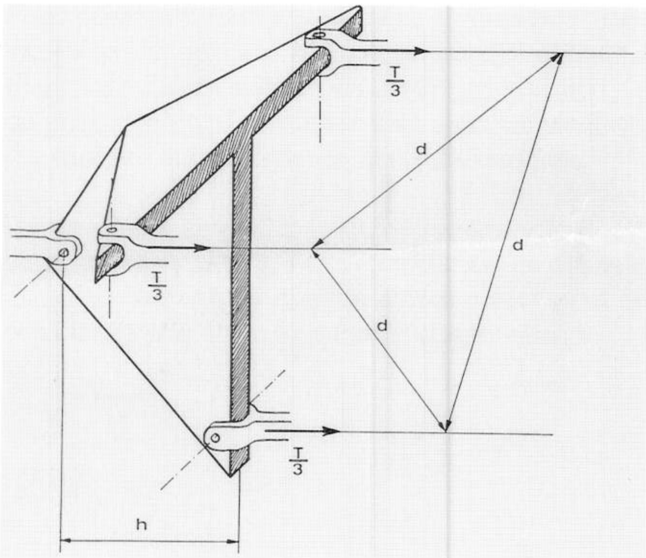


Fig. A.7 Details on the triple yoke plate arrangement.

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NOTATIONS

System characteristics

ρ_c volumic mass of a conductor
a inverse of the catenary parameter (m^{-1}) = mg/T
ϕ (sub)conductor diameter (m)
I moment of inertia of one phase p.u. length (kg x m)
τ intrinsic torsional stiffness of one phase (N m^2)
m mass of one phase p.u. length (kg/m)

E Young modulus (N/m^2)
A cross section *of one phase* (m^2)
K anchoring stiffness off both end towers in serie (N/m)
K_v equivalent stiffness for vertical motion (N/m)
K_ϑ equivalent stiffness for torsional motion (N/m)
d subconductors spacing (m)
r radius of the bundle (m)
σ_i angular position of subcond. i, refer to horizontal (rad)
N_s number of spans in a line section
n number of subconductors in a bundle
L span length off the whole section (m)
L_s span length off span s (m)
M_v structural dimensionless parameter for vertical motion
M_ϑ structural dimensionless parameter for torsional motion

Mathematical and physical quantities

y vertical displacement oriented positive upwards (m) (z and time dependent)
y_k vertical magnitude of mode k (m) ($y_{k,o}$ initial value)
ϑ torsional angle oriented positive sinistrorsum (rad) (z and time dependent)
ϑ_k torsional magnitude of mode k (time dependent)
Ω_v basic pulsation for vertical motion (rad/s)
Ω_ϑ basic pulsation for torsional motion (rad/s)

$\omega_{v,k}$ k^{th} pulsation of the vertical mode (rad/s)
$\omega_{\vartheta,k}$ k^{th} pulsation of the torsional mode (rad/s)
t time
z horizontal position in the span (m)
$\delta = \pi z/L$ auxiliar dimensionless variable
k mode number
T instantaneous mechanical tension *in one phase* (N) (T_o initial static value)
f_e external vertical force p.u. length (N/m)
l instantaneous length of the cable, related to y(z,t) (m) (l_o initial value)

AKNOWLEDGEMENT

The others would like to thank LABORELEC AND GECOLI for financial and experimental supports. They also thank for the fruitful dimensions inside the CIGRE 22-11, task force on galloping.