

## An Analytical-Numerical Method Based on "Static Equivalent Load" for the Evaluation of Maximum Dynamic Stresses in HV Supporting Structures (Portals)

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### Abstract

The static equivalent load is a load which causes at an considered particular part of a mechanical structure the same maximum constraint as the dynamic load. This paper presents the theoretical investigation of the procedure to calculate the maximum dynamic stresses in high-voltage substation equipment and steel supporting structures (portals) due to any kind of loading —magnetic (short-circuit current forces) and mechanical (conductor dropping) —.

The investigated outdoor substation is a 110 kV substation consisting of two consecutive spans with single stranded conductors. Two loading cases, short circuit and conductor dropping, are considered. The results of tests and calculations for both loading cases are compared. There exists a very good agreement between the respective results.

This study is an important point of discussion within the CIGRE task force on the effects of short-circuit currents assigned to Working Group 23-11.

### INTRODUCTION

The maximum mechanical constraint of a substation component —switchgear or other equipment— due to short-circuit does not depend only on the amplitude of dynamic loading, but on its whole time function. The determination of the maximum dynamic constraint, which is a necessity for the designer of the substation as well as of the equipment itself, can be done on an experimental or a calculatory basis. Experimental investigations are expensive, labor intensive and not always practicable. Experience has shown that for the calculation of the dynamic stresses the method of the finite elements is the most important and efficient numerical method. The disadvan-

tage of this method is the difficult handling and hence needs a lot of experience. Therefore there is also a need for a simpler method, which gives quick information about the maximum dynamic constraints of the components of the substations. The occurring dynamic stresses as short circuit, conductor dropping, earthquake and wind should be considered.

In this study substation components like post insulators, current transformer and HV steel supporting structures are considered. These apparatus will be modelised by simple beam models. In this case it is the clamped/free beam. It will be shown how the behaviour of substation apparatus, which are stressed by dynamic loads, can be modelised. The mathematical method, which allows this, is the modal analysis. It must be distinguished between the different kinds of dynamic loads, short-circuit and conductor dropping.

### THEORETICAL BACKGROUND

Based on the theory of a simple beam a static equivalent force is defined, which causes the same maximum material stress at a particular point under consideration as an acting dynamic force [1], [2]. A clamped beam is a simple geometric structure which can be described with a partial differential equation (eq. 1) [5].

$$\frac{d^2}{dx^2} \left[ EI(x) \frac{d^2 y}{dx^2} \right] + m(x) \cdot \frac{d^2 y}{dt^2} = p(x, t) \quad (1)$$

with

x:	Coordinate of length of beam
y:	Coordinate of deflection
EI(x)=EI:	flexural rigidity
m(x)=m:	Mass per unit area
p(x,t):	Excitation

Applying the approach method of Bernoulli and the modal analysis (eq. 2) for solving the partial differential equation, first the eigenvalue problem and afterwards the decoupled equation of motion and its superposition have to be solved.

$$y(x, t) = \sum_{i=1}^n y_i(x) \cdot g_i(t) \quad (2)$$

with:

$$y_i(x) = \sinh(\lambda_i \cdot \eta) - \sin(\lambda_i \cdot \eta) - C_i \cdot (\cosh(\lambda_i \cdot \eta) - \cos(\lambda_i \cdot \eta)) \quad (3)$$

where:

$$C_i = \frac{[\sin(\lambda_i) + \sinh(\lambda_i)]}{[\cos(\lambda_i) + \cosh(\lambda_i)]} \quad (4)$$

As mentioned  $y_i(x)$  is a function dependent on the location and describes the form of oscillation for each mode.  $\lambda_i$  is given by the solution of a transcendent equation considering a concentrated mass at the top of the beam.

$$g_i(t) = \frac{\Gamma_i}{\omega_{Di} \cdot \mu_i} \cdot$$

$$\int_0^t f(\tau) \sin(\omega_{Di}(t - \tau)) \cdot e^{-\zeta_i \cdot \omega_i \cdot (t - \tau)} d\tau \quad (5)$$

$$\omega_{Di} = \omega_i \cdot \sqrt{1 - \zeta_i^2} \quad (6)$$

To get to the general solution, the time function  $g_i(t)$  has to be considered. It describes the movement of the head of the beam in dependence of time.

The insertion of the boundary and starting conditions leads to a static equivalent force:

$$F_{steq} = \sum k_i \cdot n_i \quad (7)$$

with:

$$k_i = \frac{EI \cdot \lambda_i^2 \cdot C_i \cdot y_i(L) \cdot (-2)}{L^3 \cdot \mu_i \cdot \omega_{Di}^2} \quad (8)$$

and

$$n_i = p(L) \omega_{Di} \cdot \left[ \int_0^t f(\tau) \sin(\omega_{Di}(t - \tau)) \cdot e^{-\zeta_i \cdot \omega_i \cdot (t - \tau)} d\tau \right]_{max} \quad (9)$$

The static equivalent force (eq. 7) consists of the sum of the products  $k_i$  and  $n_i$  which are determined in two separate calculations. Phy-

sical structure data, as length  $L$ , modal generalized mass  $\mu_i$  and damping dependent eigenfrequency  $\omega_{Di}$  are contained in the factor  $k_i$ . The index  $i$  gives the order (1,2,3,..) of the eigenfrequencies. The factors  $n_i$  are determined by the force analysis. As input parameter for the force analysis the dynamic force characteristic  $f(t)$  as well as two additional structure data, eigenfrequency and the corresponding damping are used.

An analytical numerical method requires all input parameters in closed form. Since the force excitation is not given in analytical form, but as a measured or calculated time function, an analytical function can be derived with the equation (eq.10) using the values of the equidistant interpolation points in the time range.

$$f(t) = \sum_{n=0}^m f(n \cdot \tau) \cdot \frac{\sin\left[\pi\left(\frac{t}{\tau} - n\right)\right]}{\pi\left(\frac{t}{\tau} - n\right)} \quad (10)$$

In equation 10  $f(n\tau)$  is the value of the interpolation point,  $\sin(\pi(t/\tau - n))/(t/\tau - n)$  the si- or interpolation function and  $m$  the number of interpolation points.

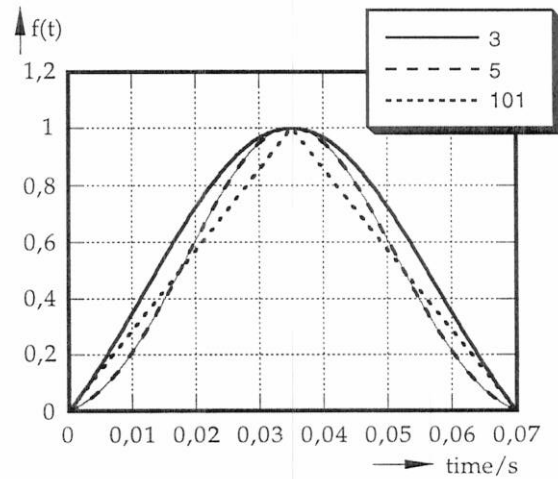


Fig. 1: Approximation of a triangular force impulse by using a si-function. (3, 5, 101 interpolation points)

Fig. 1 shows an exemplary approximation of a triangular force impulse with 3, 5 and 101 points. For each point  $t=kn$  the si-function with  $n=k$  has the value 1, while all other functions with  $n \neq k$  disappear. This leads to an

When we record for each frequency the maximum value of  $n$  during the excitation time, we get the participation factors  $n_i$ , see Fig 5. A damping of 7% was assumed for the first three modes.

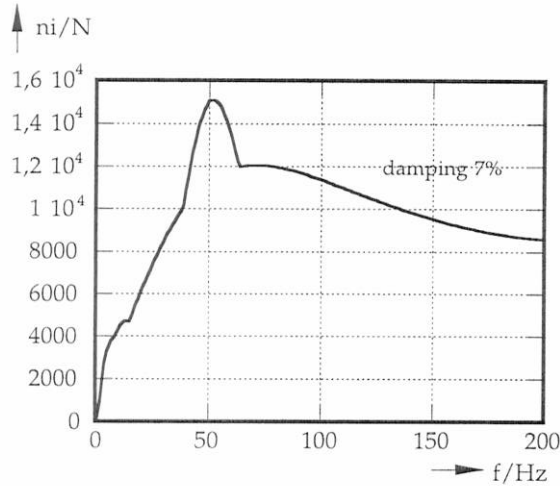


Fig.5: Participation function at a damping of 7%.

As result of the Force- and Structure analysis a static equivalent force is calculated:

$$F_{\text{steq}} = n_1(44\text{Hz}) \cdot k_1(0.0174) + n_2(276\text{Hz}) \cdot k_2(0.0174) + n_3(773\text{Hz}) \cdot k_3(0.0174)$$

$$F_{\text{steq}} = (13156.28 \cdot 1.1407)\text{N} + (8268.52 \cdot -0.1716)\text{N} + (7970.7932 \cdot 0.058)\text{N} = 14050.8\text{N}$$

*14997*

For comparison a calculation was done with the finite element method on a real discretized insulator and on a reference beam model for the identical dynamic force excitation of Fig.2. The maximum constraint of the real insulator with sheds appears near the transition porcelain/socket and that of the reference insulator at the bottom end. The finite element method calculates a static equivalent force of 14990N, which causes the same maximum insulator constraint as the time function of Fig. 2. The comparison with the result of the Force- and Structure Analysis above of 14050,8N gives a deviation of only 6.3%.

*14997*  
0 %

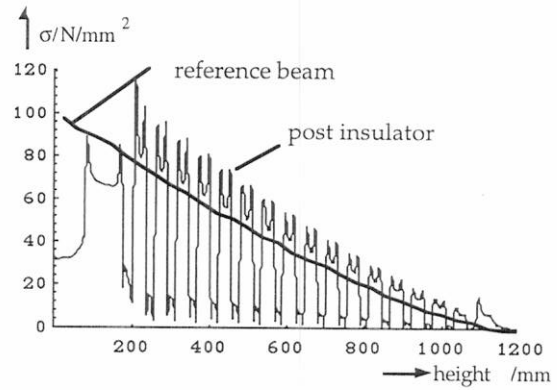


Fig.6: FEM-calculation of the maximum mechanical constraints of the completely discretized post insulator and of the reference beam model insulator

### MODELLING OF A CURRENT TRANSFORMER

For a clamped/free beam with a mass at the head of 1000 kg and an evenly distributed mass of 1000 kg along the 3m high insulator, typical for a 400 kV current transformer, a triangular force impulse with an amplitude of 1000 N [1] was considered as excitation, Fig. 7.

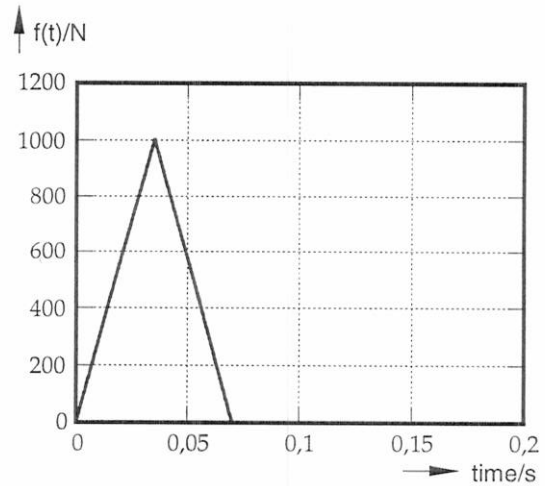


Fig. 7: Triangular force excitation with an amplitude of 1000 N

The first three eigenfrequencies of  $f_1=5.87\text{Hz}$ ,  $f_2=61.2\text{Hz}$ ,  $f_3=191.7\text{Hz}$  were calculated on the basis of the finite element method.

Applying the Force- and Structure Analysis at a damping of 5% a static equivalent force is determined of:

$$F_{\text{steq}} = n_1(5.87\text{Hz}) \cdot k_1(1.0) + n_2(61.2\text{Hz}) \cdot k_2(1.0) + n_3(191.2\text{Hz}) \cdot k_3(1.0)$$

$$F_{\text{steq}} = (1036.31 \cdot 1.029)\text{N} + (997.51 \cdot -0.0365)\text{N} + (898.59 \cdot 0.0072)\text{N} = 1036.42\text{N}$$

The explicit calculation with finite elements on the beam model with top end mass results in a equivalent static load for the maximum mechanical constraint of 1075 N. The deviation between both results is only 3.6 %.

### MODELLING OF HV SUPPORTING STRUCTURES (PORTALS)

Another example is the measurement on a high voltage test arrangement with single conductors, Fig. 8 [3].

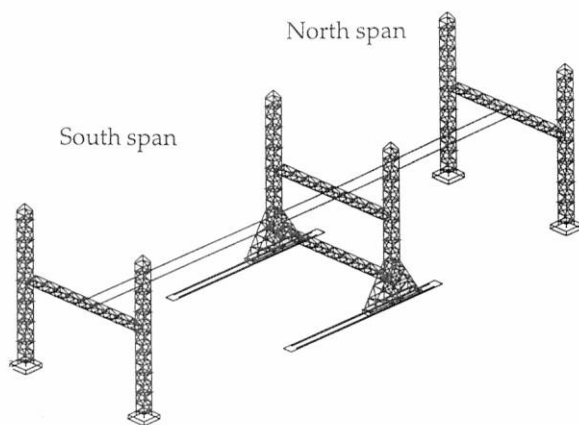


Fig.8: 110kV-test arrangement with the two spans North and South (no short-circuit current in North span)

On this 110kV-substation arrangement short circuit tests as well as conductor dropping tests with and without additional weights were done in the South span. The excitation forces for conductor dropping and short circuit are shown in Fig. 9 and Fig. 10.

For the application of the described method the complete discretized structure has to be transformed into a beam model. The dynamic excitation forces  $F_1$  and  $F_2$  calculated by the detailed FE model [3] serve as excitation forces for the models of Fig. 11.

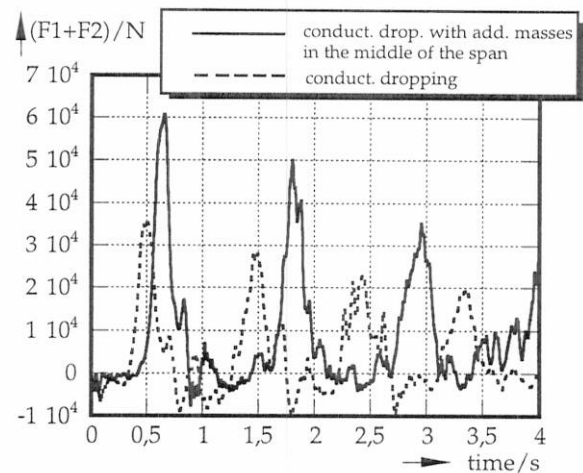


Fig.9: Dynamic excitation forces caused by conductor dropping with and without additional weights

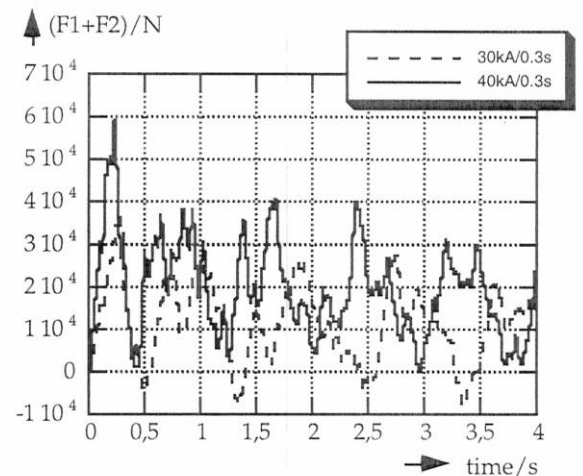


Fig.10: Dynamic excitation forces caused by short circuit

The difficulty of determining the models lies in the required consistency of the 1., 2. and 3. eigenfrequencies. Since structure stiffness shall remain constant, mass and mass distribution is the suitable tool for frequency adaptation. For the adjustment of the eigenfrequencies of the beam model the mass has to be increased. The damping of 12% stays unchanged. The cross-sectional area of the beam elements corresponds to the sum of the vertical corner posts of the tower. Due to the mass corrections the stresses at the bottom of the final beam model correspond to those of the portal in amplitudes and time functions.

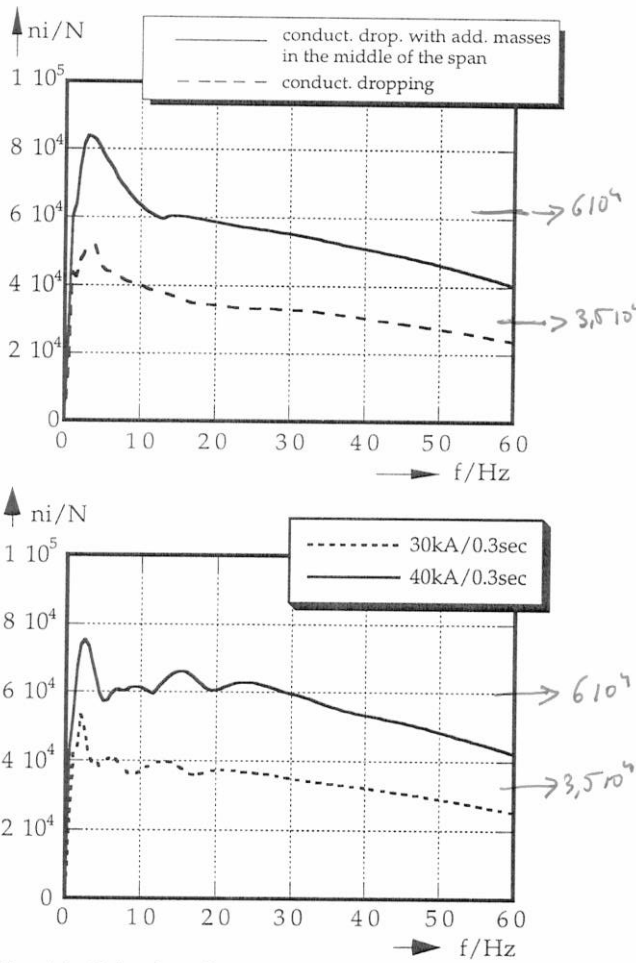


Fig. 10: Calculated participation factors  $n_i$

Table 2: Calculated results of the studies on the portal

excitation	ampl. of total load F1+F2 calculated [kN]	F <sub>streq</sub> FEM-model detailed [kN]	F <sub>streq</sub> beam-model simplified [kN]	F <sub>streq</sub> analy. -num. meth. [kN]	ESL-Factors
					1) measured 2) calculated
s-c current 30kA 0.3sec	37.5	46.0	45.5	43.5	1) 1.2 2) 1.4
s-c current 40kA 0.3sec	61.2	79.0	80.0	74.3	1) 1.2 2) 1.3
conduct .drop.	35.4	49.0	52.5	54.8	1) 1.8 2) 1.8
conduct drop. with add. masses in the middle of the span	60.6	79.0	86.0	89.1	1) 1.7 2) 1.6

## CONCLUSION

In this study an analytical numerical method for the calculation of a static equivalent load (ESL) is presented –Force- and Structure Analysis-. With this static load the design load for the foundations of structures as well as the maximum constraints of structures and equipment, as post-insulators, measuring transformer, can be defined for the dynamic case. Proof of validity of the developed method was led by comparison (ESL-Factors) with measurements on the real structures [3].

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