A level set approach for the optimal design of flexible components in multibody systems

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UNIVERSITY OF TWENTE
Euromech Colloquium 524
Enschede, Netherlands
February 27-29, 2012

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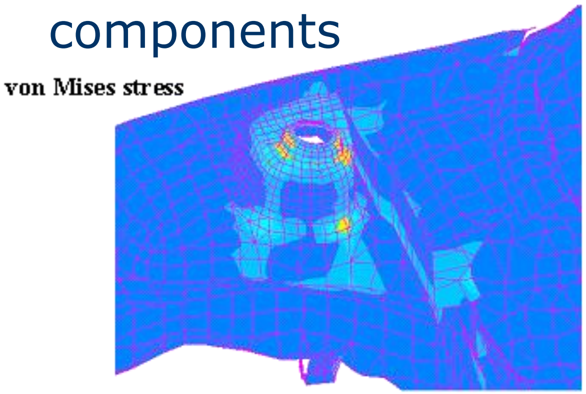
Outline

- Introduction & Motivations
- Level Set description and the proposed method
- Formulation of the flexible multibody system optimization problem
- Numerical applications
- Conclusions & Perspectives

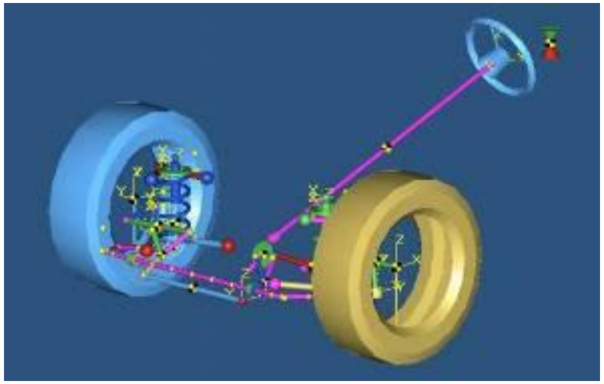
Introduction

Evolution of virtual prototyping

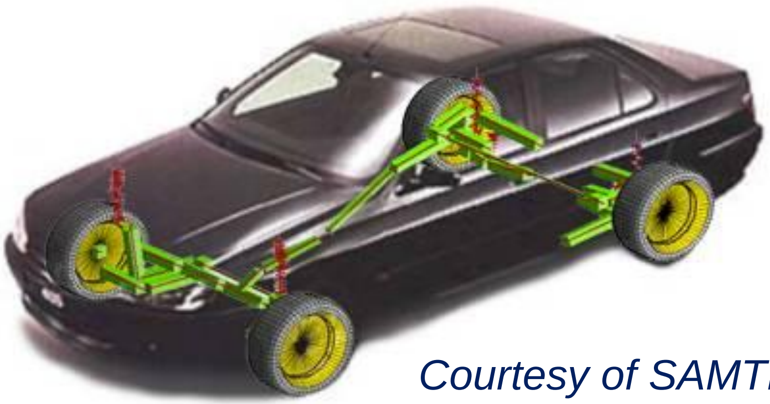
- Finite Element Method: Structural analysis of components



- Rigid Multibody Systems: Simulation of mechanisms



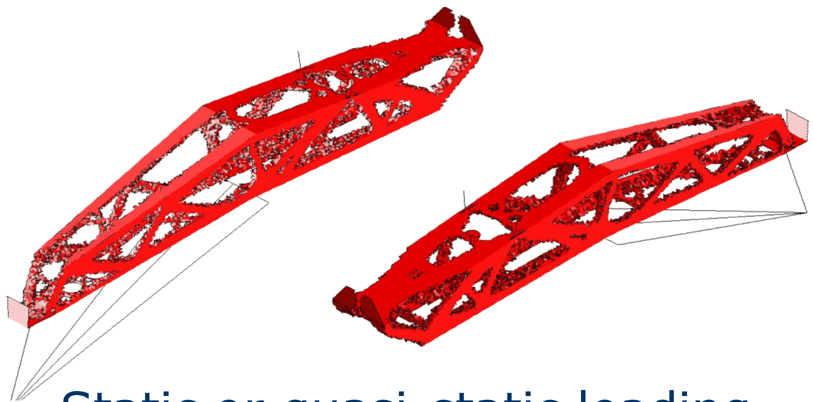
- Flexible Multibody Systems: System approach (MBS) & Structural dynamics (FEM)



Courtesy of SAMTECH

Evolution of virtual prototyping

- Structural optimization

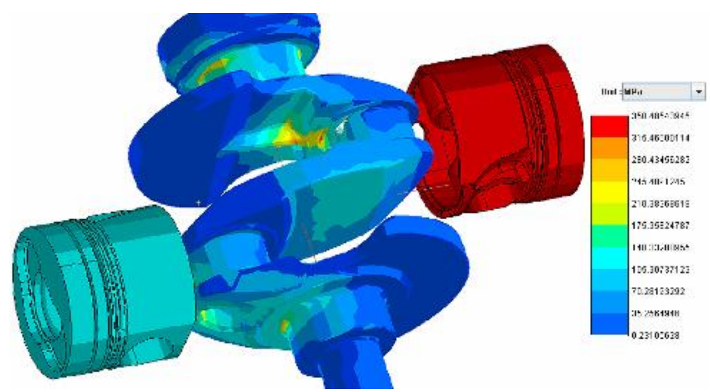


Static or quasi-static loading

- Flexible multibody systems

Dynamic loading

- Optimization of flexible components in multibody systems



Motivations

- Optimization of flexible components in multibody systems
  - Define realistic dynamic loadings
  - Take care of the coupling between large overall rigid-body motions and deformations
- Common approach: Equivalent static loads approach + Rigid (or component mode approach) MBS
  - Component interactions are ignored
  - Global vibration behavior and modeling of high frequency loadings are poor
- Here « Fully Integrated Method »
  - MBS approach based on non-linear FEM (SAMCEF Mecano)
  - Coupling with an optimization shell (Boss Quattro)

Finite Element Approach Of Multibody Systems Dynamics

Equation of FEM-MBS dynamics

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates **q** (Geradin & Cardona, 2001)
- Dynamic equations of multibody system

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\text{ext}} - \mathbf{g}^{\text{int}}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = 0$$

- Solution based on an augmented Lagrangian approach of total energy

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}^T(k\boldsymbol{\lambda} + p\mathbf{\Phi}) = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ k\mathbf{\Phi}(\mathbf{q}, t) = 0 \end{cases} \qquad \mathbf{B} = \frac{\partial \mathbf{\Phi}}{\partial \mathbf{q}}$$

$$\mathbf{q}'(0) = \mathbf{q}'_0 \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_0$$

Time Integration

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- The set of nonlinear DAE solved using the generalized-α method by Chung and Hulbert (1993)
- Define pseudo acceleration **a**:

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma \mathbf{a}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1} + h^2(1/2-\beta)\mathbf{a}_n + h\beta \mathbf{a}_{n+1}$$

- Solve iteratively the dynamic equation system (Newton-Raphson)

$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\boldsymbol{\lambda} = \Delta\mathbf{r} \\ \mathbf{B} = \mathbf{0} \end{cases} \qquad \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\boldsymbol{\lambda}$$

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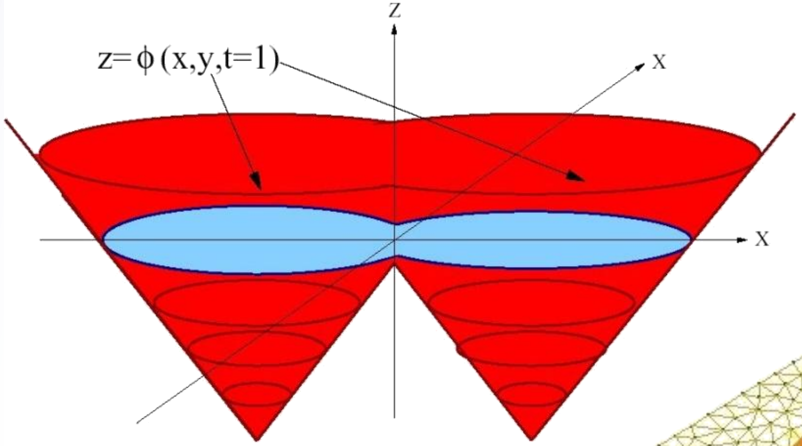
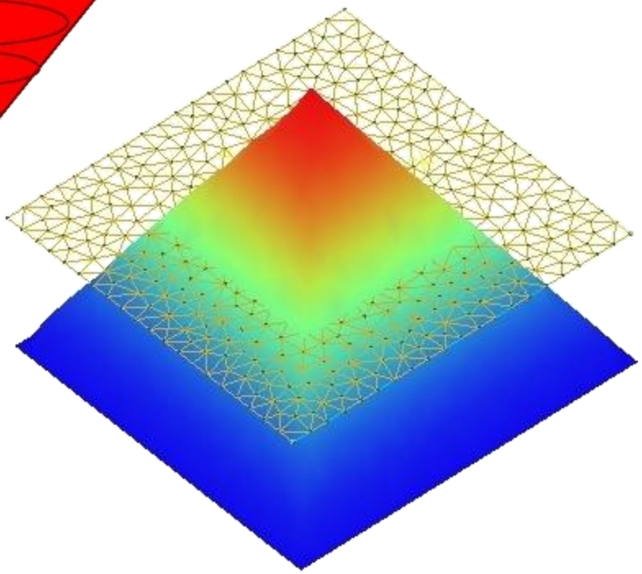
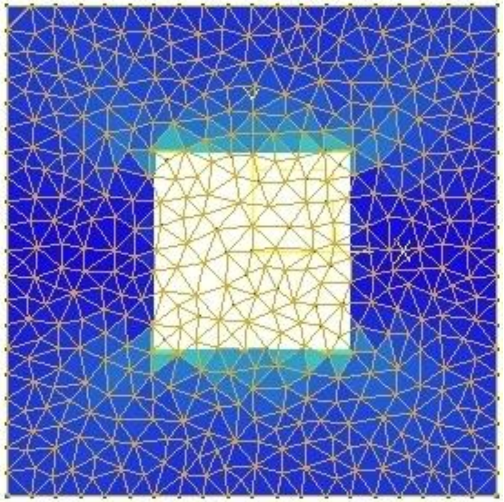
The Level Set Description

Principle (Sethian & Osher, 1988)

- Numerical technique for tracking interfaces
  - Introduce a higher dimension function
  - Implicit boundary representation $\psi(x,t)=0$
  - Interface = the zero level of the function







Advantages - Drawbacks

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Advantages

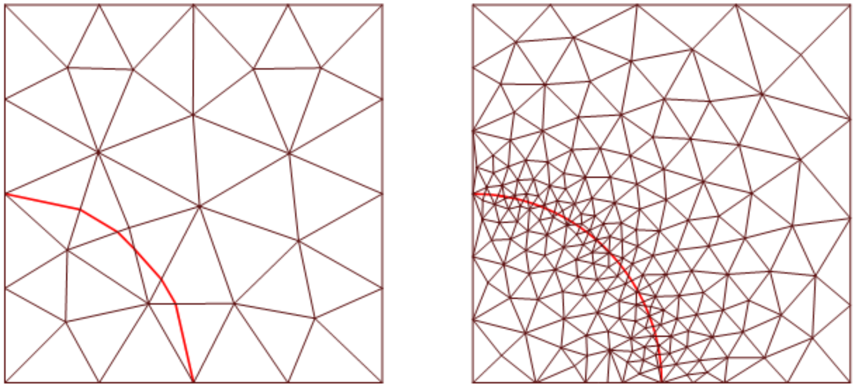
- Combination of entities (min, max,…)
  - Remove entities
  - Separate entities
  - Merge entities

➔ Topology modifications

- Extension 2D/3D
- Useful with Extended - FEM

Drawbacks

- Construction:
  - Specific tools
  - Analytical functions
  - Point set – Nurbs
- Mesh "adaptation" necessary but not in the method proposed here



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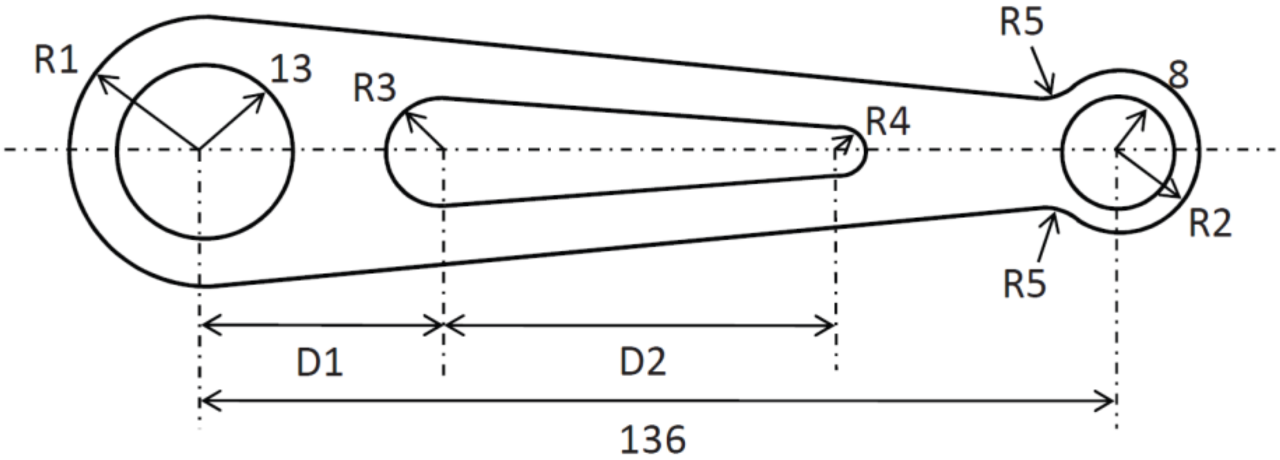
Shape optimization

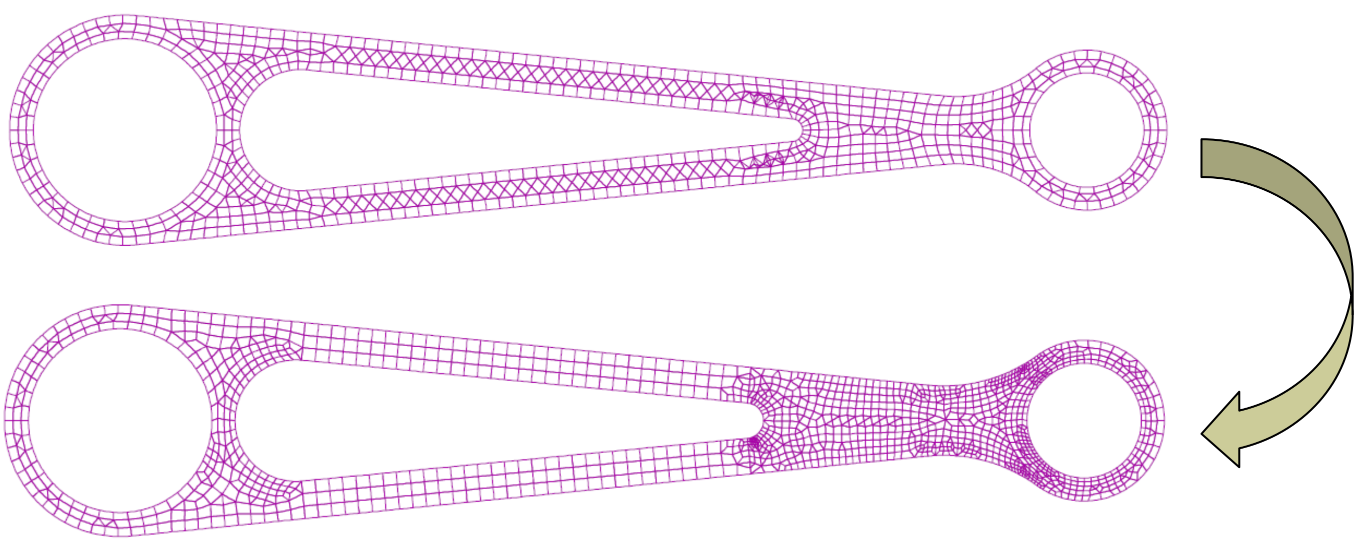
- Necessary to have an initial design of the component
- Parametric model
- Shape variables: Geometrical parameters of flexible body shape



Shape optimization

- The finite element mesh moves according to shape modifications.
  - It leads to mesh distortion. Major Problem!
  - The quality of the mesh decreases and the solution accuracy of the FEA decreases after the first iteration.

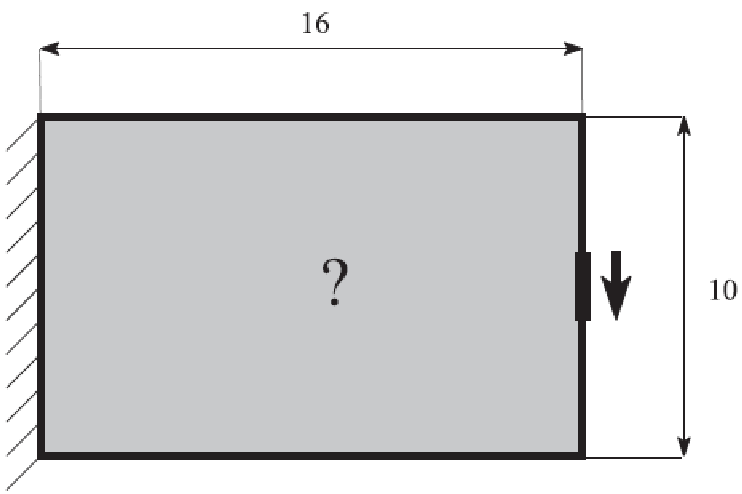
Re-meshing techniques exist to avoid this problem.

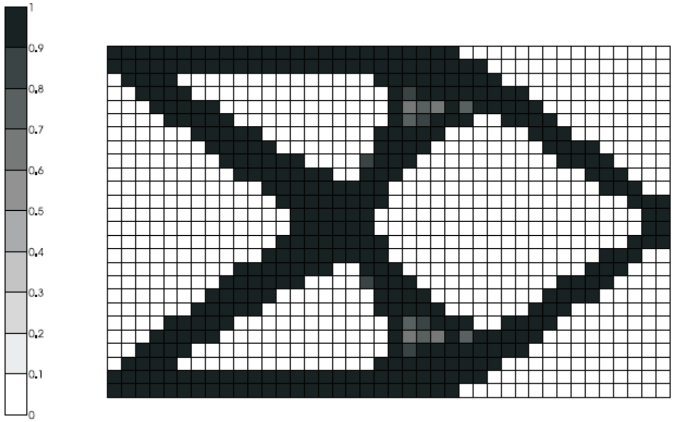


Re-meshing with error control strategy

Topology optimization

- Can be seen as an optimal material distribution within a design domain
- No initial knowledge on the component
  - Only have to define:
    - The design domain
    - The loading
    - The boundary conditions
    - A volume constraint
- The optimization process gives the best design for these information.



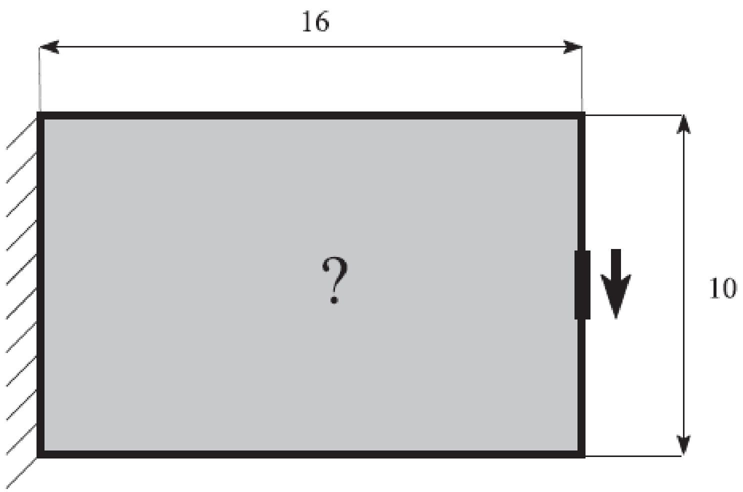


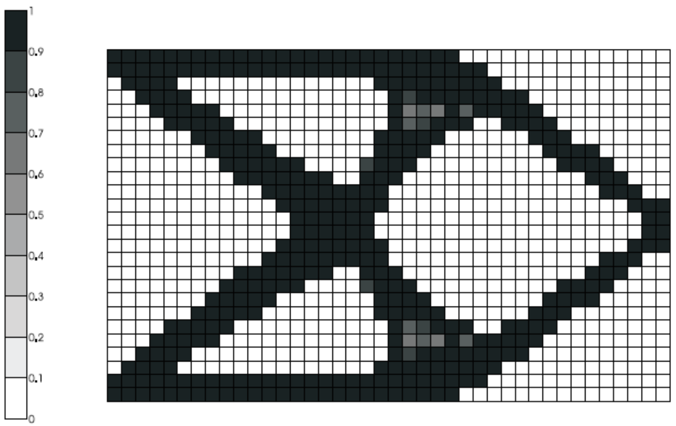
Topology optimization

- The design variables are the density of each finite element.
  - ➔ Large number of variables – local optimum
- Feasibility of manufacturing:
  - Difficulties to determine structural boundary shape from the topology optimization results.

But…

- Fixed mesh grid





Goals of this work

- The *Level Set Description* of the geometry leads to an intermediate type of optimization between the shape optimization and the topology optimization.
  - Fixed mesh grid: No mesh distortion
  - The geometry is based on CAD entities: can easily be manufactured.
  - Remove, separate, merge entities: Modification of the topology

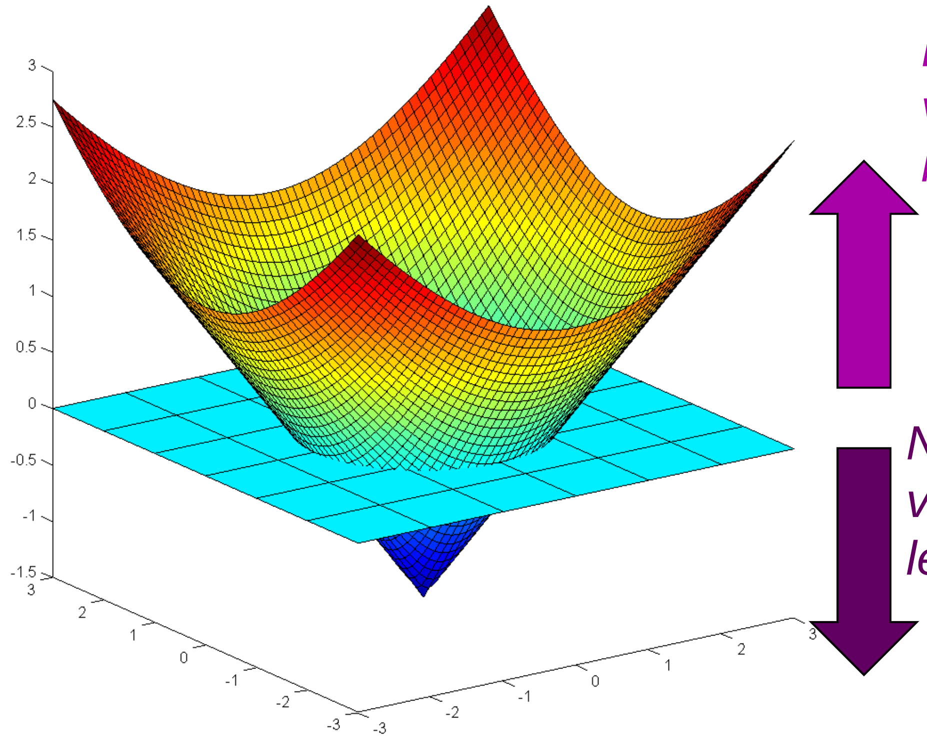
Remark: It is not a "full" topology optimization because the level set description does not allow the creation of new holes, they must be introduced a priori.

Topology optimization can be realized with a level set approach, see G. Allaire.

The method

The method: Square plate with a hole

- Mesh definition (fixed during all the process) + Level Set definition:
  Mesh: 6*6 elements
  Level Set: a cone
- Any element is removed to create the hole but the properties of elements are modified: the density and the Young modulus.



The method: Square plate with a hole

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- For each node: Computation of the level set value.

- Different possibilities can happen for each element:

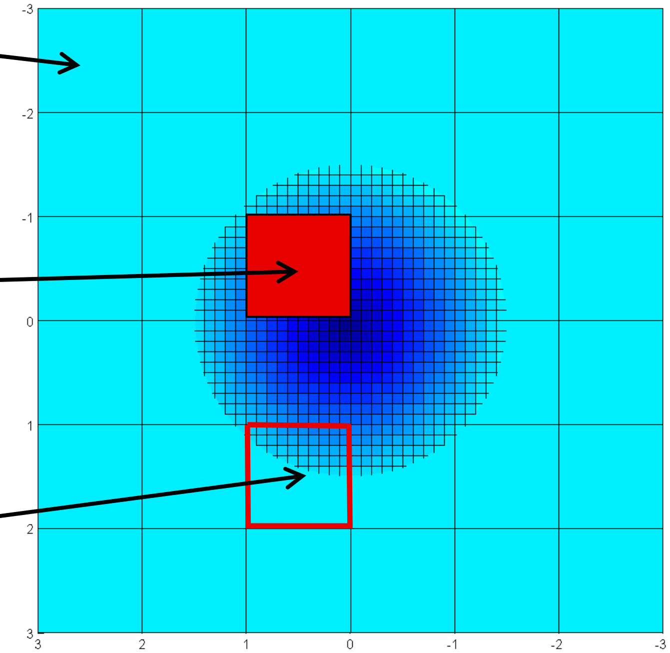
  - 4 positive nodal values: full material

$$\rho = \rho_0 \text{ and } E = E_0$$

  - 4 negative nodal values: void

$$\rho = 10^{-3}\rho_0 \text{ and } E = 10^{-9}E_0$$

  - Positive and negative nodal values = boundary element



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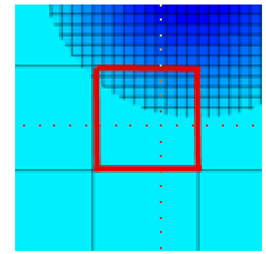
The method: Square plate with a hole

- For the boundary elements ➔ SIMP law
  - Introduction of a pseudo-density

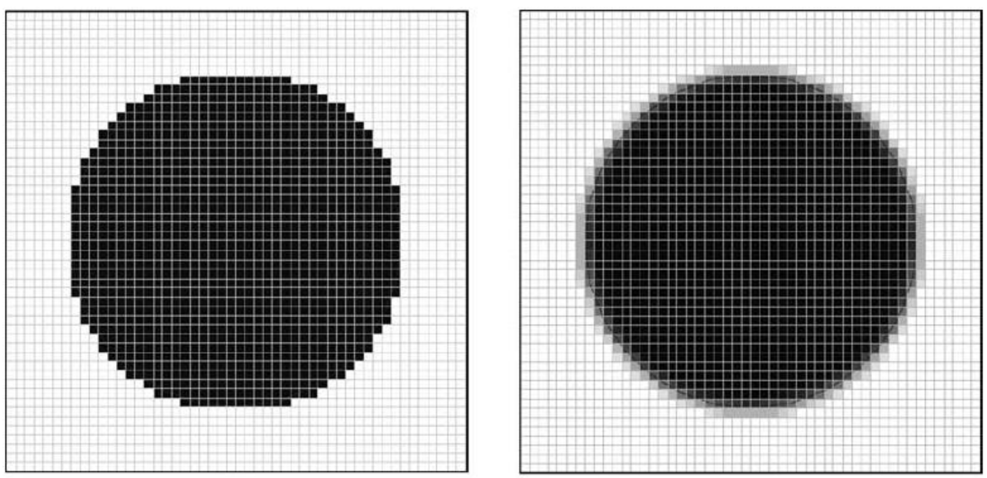
$$\mu = \frac{\text{Volume of material}}{\text{Volume of the element}}$$



  - SIMP law

$$\rho = \mu \rho_0 \text{ and } E = \mu^3 E_0$$

- Consequence:



(a) (b)

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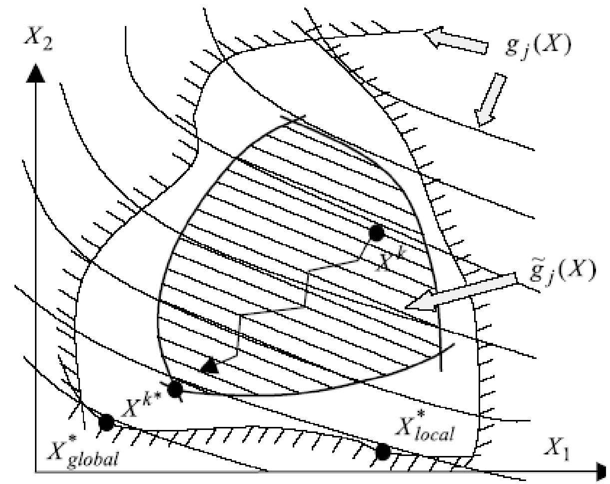
Formulations Of Flexible Multibody Systems Optimization Problem

General form of the optimization problem

- Design problem is cast into a mathematical programming problem



$$\min_{\mathbf{x}} g_0(\mathbf{x})$$

$$s.t. \begin{cases} g_j(\mathbf{x}) \le \overline{g}_j, & j = 1, \ldots, m \\ \underline{x}_i \le x_i \le \overline{x}_i, & i = 1, \ldots, n \end{cases}$$

- Provides a general and robust framework to the solution procedure

- Efficient solver :
  - Sequential Convex Programming (Gradient based algorithm)

    ➔GCM (Bruyneel et al. 2002)

Sensitivity analysis

- Gradient-based optimization methods require the first order derivatives of the responses

- Finite differences $$\frac{\partial \mathrm{f}}{\partial x} \approx \frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}$$

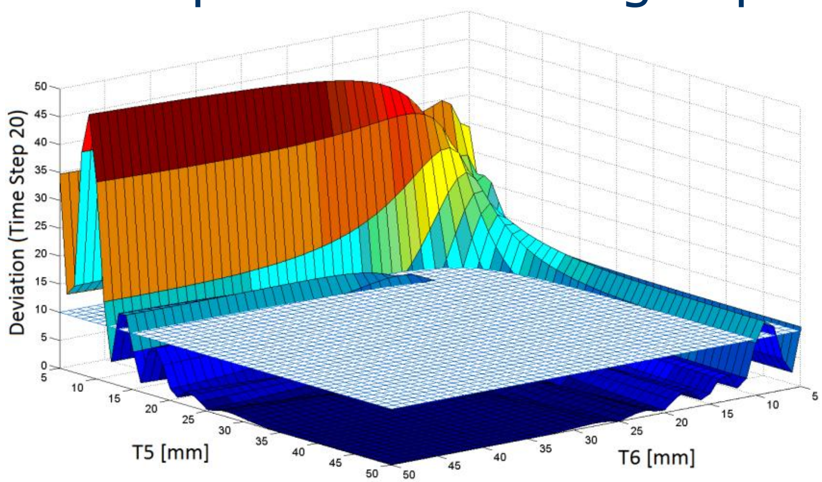
  Perturbation of design variable
  → Additional call to MBS code

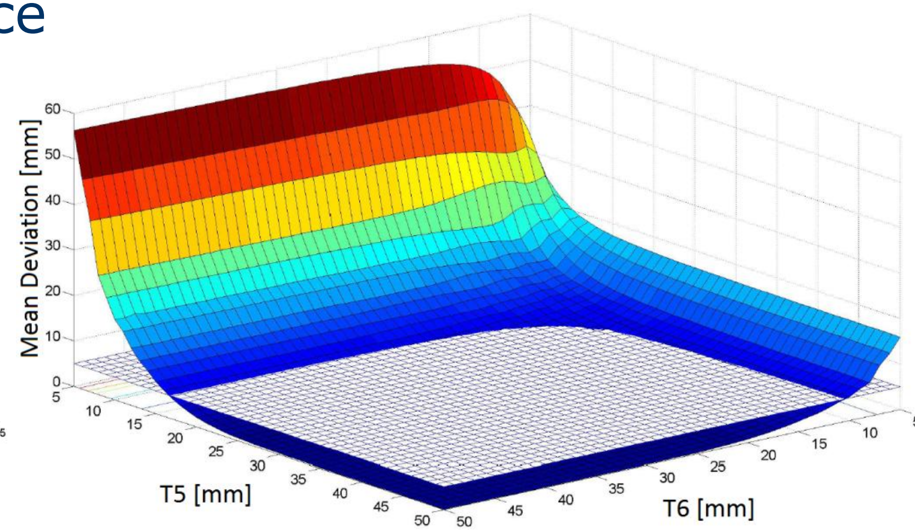
- Semi-analytical approach (Not yet developed)

$$\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x+\delta x)-\mathbf{r}(x)}{\delta x} \qquad \frac{\partial \mathbf{\Phi}}{\partial x} \approx \frac{\mathbf{\Phi}(x+\delta x)-\mathbf{\Phi}(x)}{\delta x}$$

The formulation

- The formulation is a key point for this type of problems:
  Very complex nonlinear behavior
- Impact on the design space



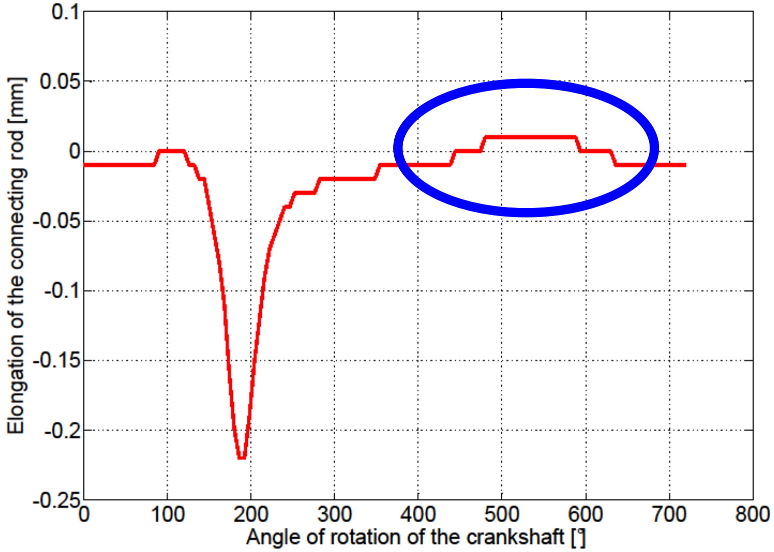


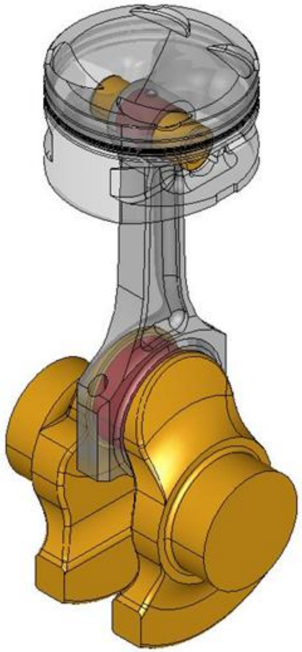
- Extremely important for gradient based algorithm
- Genetic algorithm
  - Do not necessary give better results
  - Computation time much more important

Numerical Applications

Connecting rod optimization

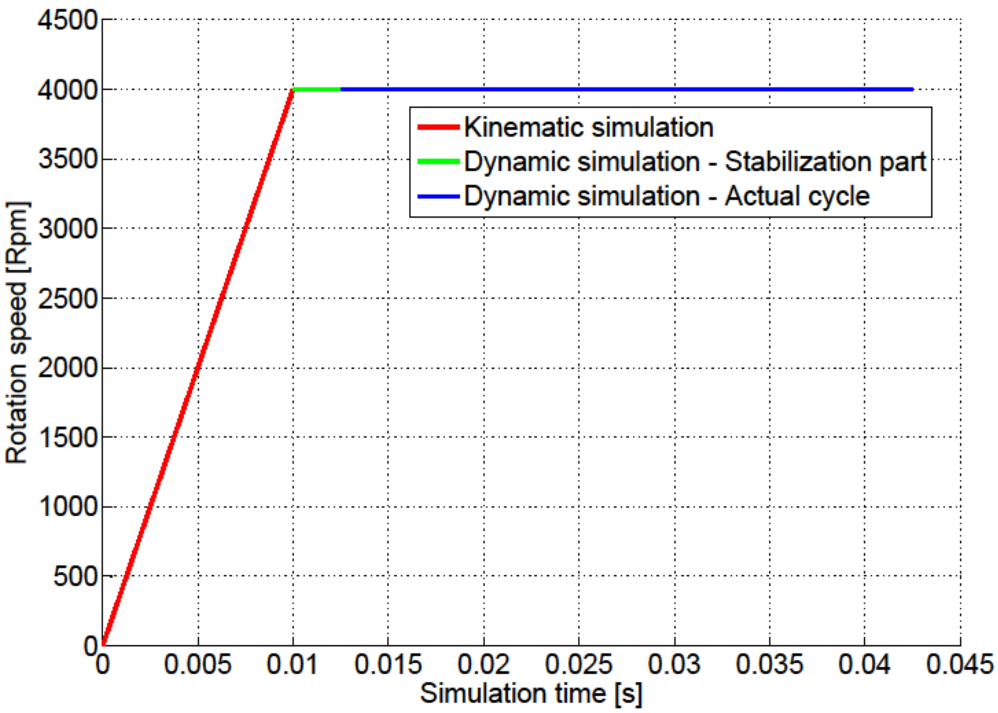
- The link between the piston and the crankshaft in a combustion engine.
- During the exhaust phase, the connecting rod elongates which can destroy the engine.
  ➔Collision between the piston and the valves.
- Minimization of the elongation





Modeling of the connecting rod

- Simulation of a single complete cycle as the behavior is cyclic (720°)
- Rotation speed 4000 Rpm
- Gas pressure taken into account.



Local formulation

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t.\;\; k\left(\Delta l\left(\mathbf{x}, t_i\right) \leq \Delta l_{max}\right)$$

$$\text{with } i = 1, \ldots, \text{nbr time step}$$

- The constraint on the elongation $\Delta l\left(\mathbf{x}, t_i\right)$ is considered at each time step.

First application – 1 level set

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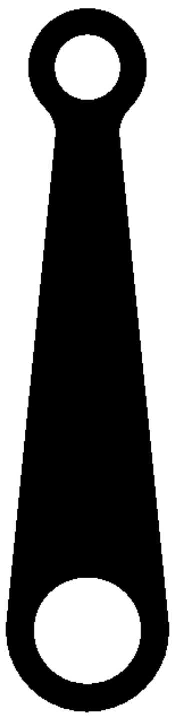
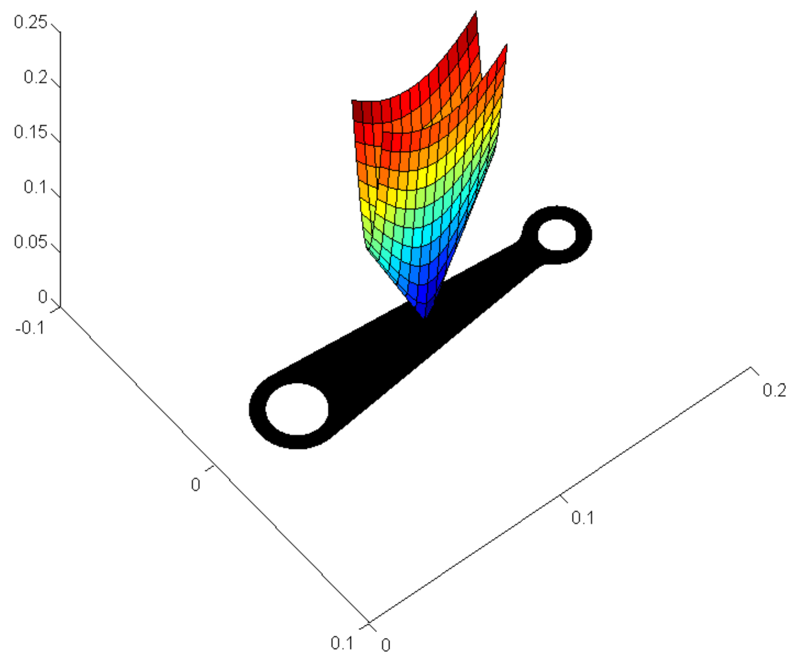
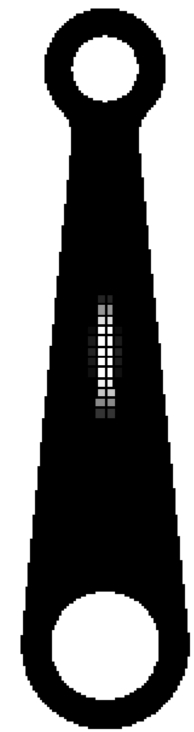
- The level set is defined in order to have an ellipse as interface.
- 3 different design variables :a, b, d. Here only c is chosen.

$$\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - d = 0$$

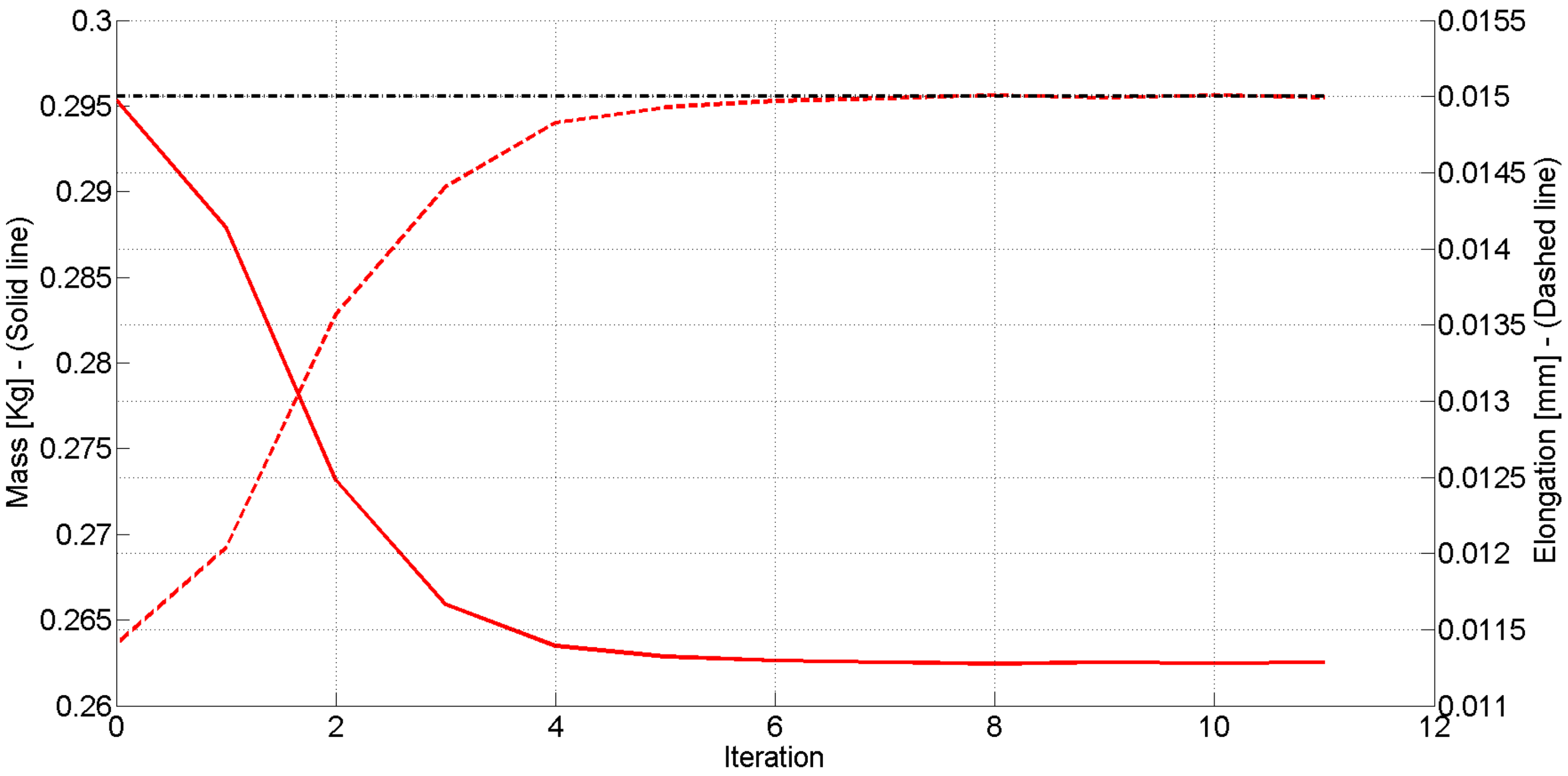






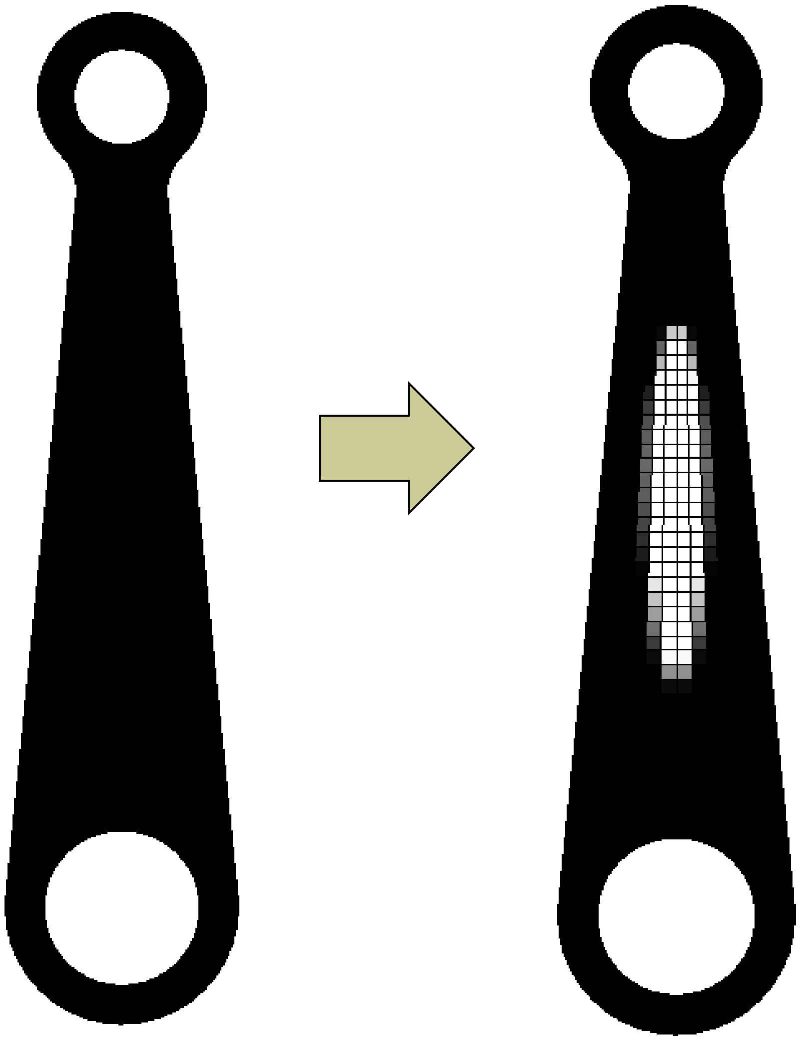
Results

- Convergence obtained after 12 iterations
- Monotonous behavior of the optimization process



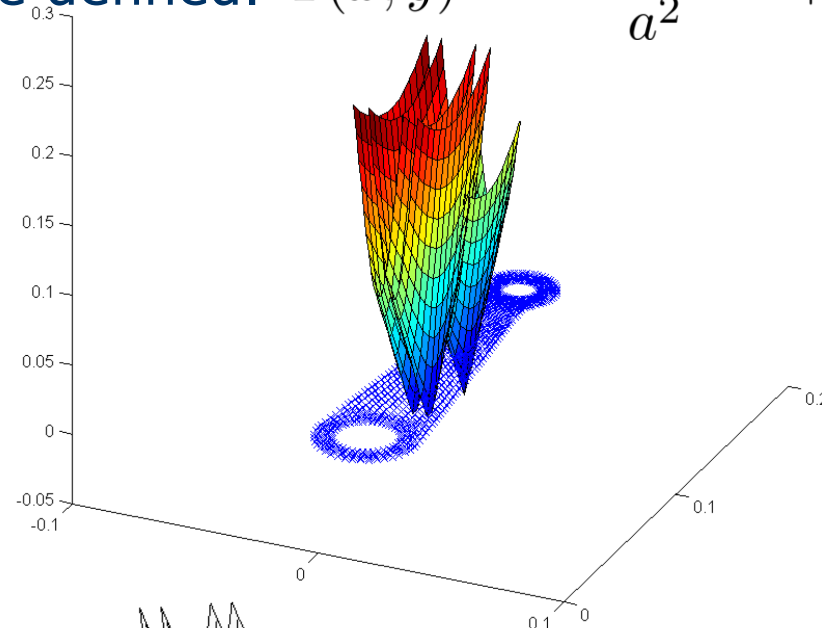
Results – Optimal design

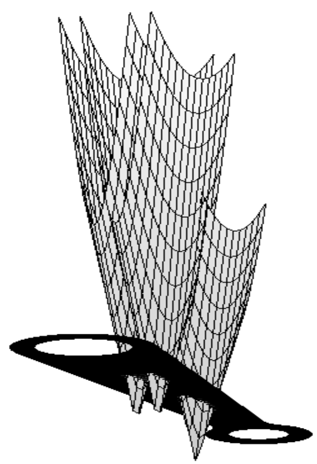
- Even if the boundary of the hole is not clear on the mesh, the boundary is defined by a CAD entity and the connecting rod can then be manufactured without any post processing.

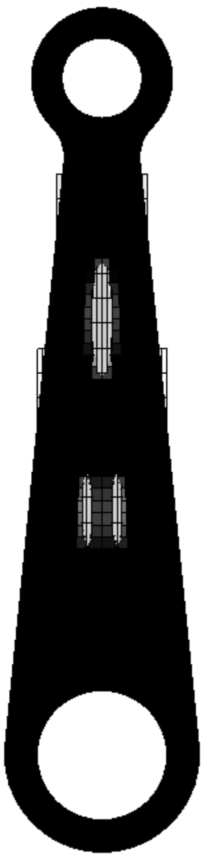


Second application – 3 level sets

- 3 ellipses are defined. $\Phi(x,y) = \frac{(x-c_x)^2}{a^2} + \frac{(y-c_y)^2}{b^2} - c = 0$

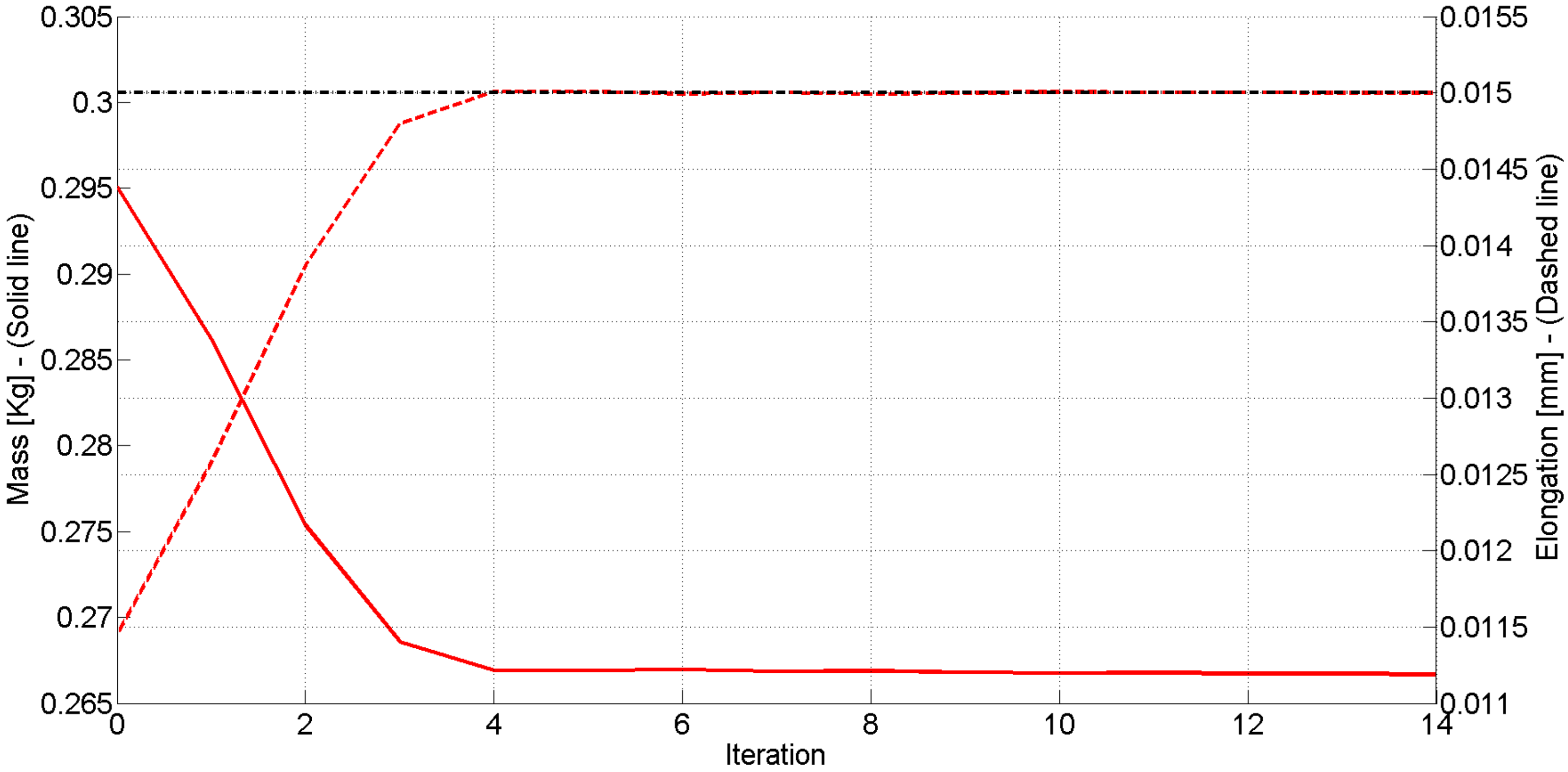






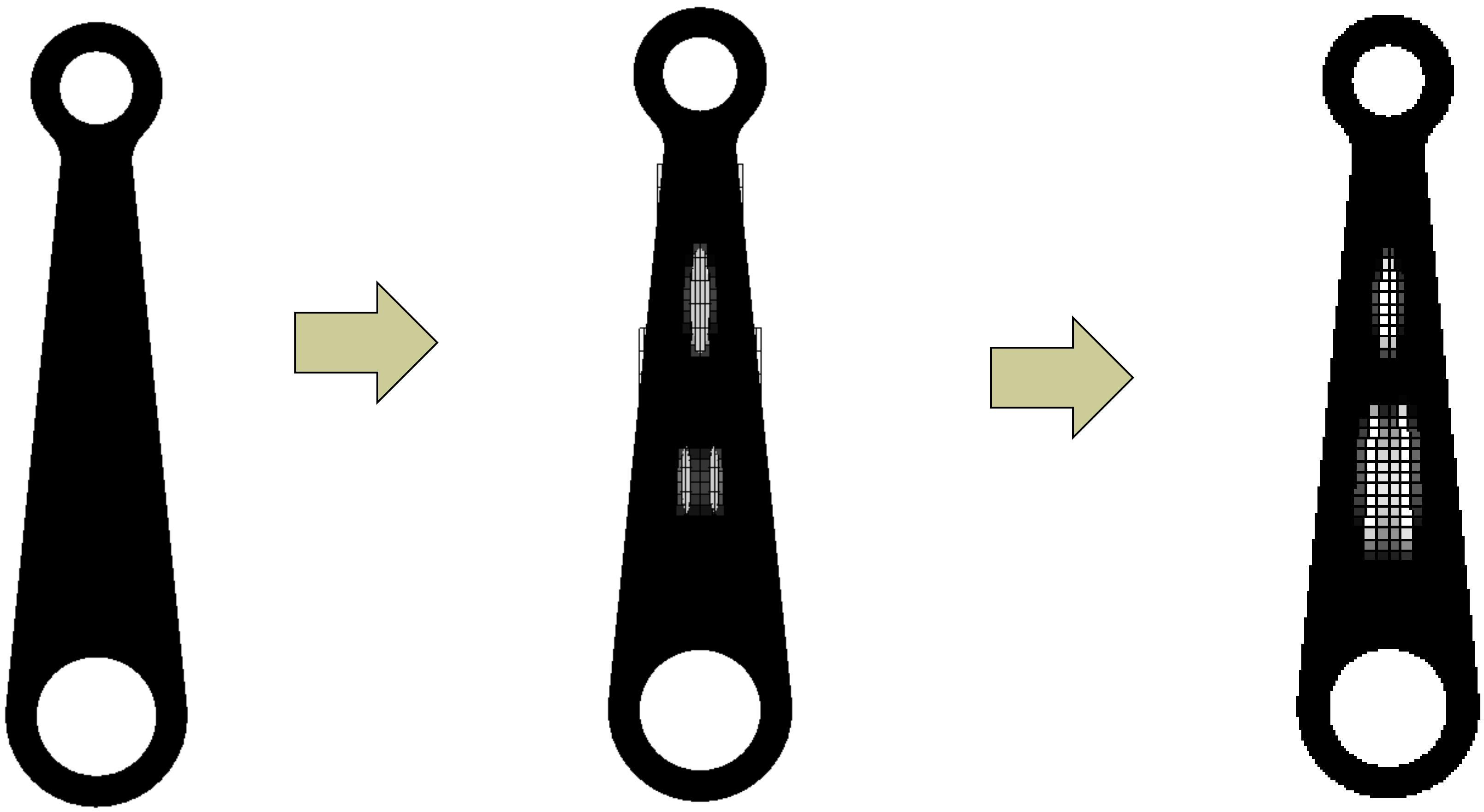
Results

- Convergence obtained after 15 iterations
- Monotonous behavior of the optimization process
- Even better than the simpler case



Results – Optimal design

- Modification of the topology



Conclusions and Perspectives

Conclusions

- Optimization of flexible components carried out in the framework of flexible dynamic multibody systems simulation
- Type of optimization between shape optimization and topology optimization
- Combine the advantages of both methods and try to avoid the drawbacks at best:
  - No mesh problem
  - Possibility of changing the topology but must be introduced before the optimization. Not a real topology optimization!
  - The geometry is expressed by CAD entities
    - ➔ Can be directly manufactured.

Perspectives

- Semi-analytical derivatives

$$\delta u_m = \frac{1}{A_m}\int_C \mathbf{V}^t \mathbf{n}\, d\Gamma$$

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Need to establish the relation between the velocity field and the design variables.

- Formulation based on the dissipated power
  ➔ Extension of the classical compliance formulation

Thank You Very Much
For Your Attention

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