

# ***Topology Optimization of Compliant Mechanisms: Application to vehicle suspensions.***

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# Outline

- Introduction to compliant mechanisms
- Design of compliant mechanisms with topology optimization
- A robust and efficient method to design a compliant suspension for vehicles
- Numerical applications
- Conclusions & Perspectives



# Introduction



# Definition of a mechanism

- A mechanism is a mechanical device used to transfer or transform motion, force or energy.
- Traditional rigid-body mechanisms consist of rigid links connected by joints.

Universal tongs:



-Energy transfer: from input (hand) to output (workpiece)

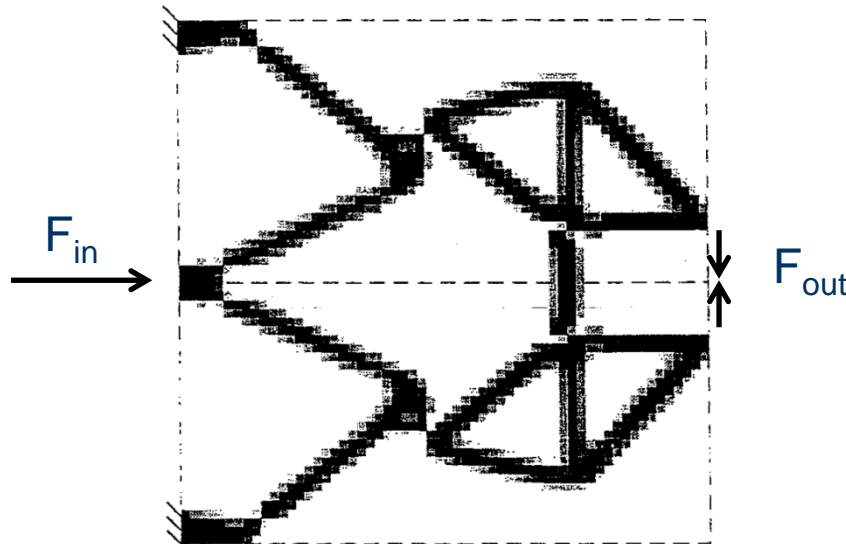
-Energy conservation: can lead to  $\rightarrow$  Output force  $>$  Input force  
and

Output displacement  $<$  Input displacement

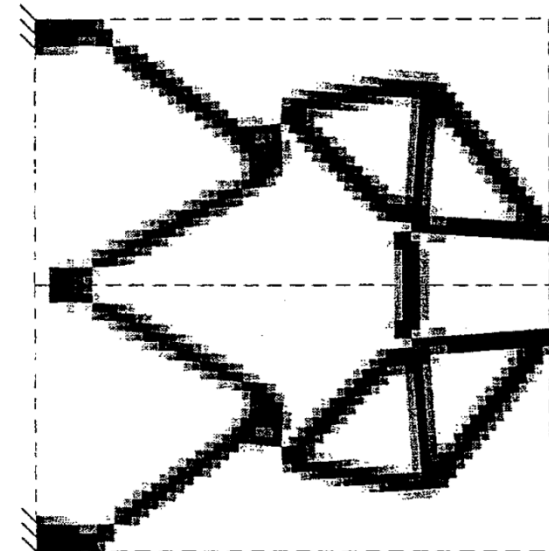
# Definition of a compliant mechanism

- A compliant mechanism is a mechanism that gains its mobility from the flexibility of some or all of its members.
- Some energy is here stored in the form of strain energy in the flexible members.

## Micro-gripping mechanism



Optimal micro-gripper  
topology



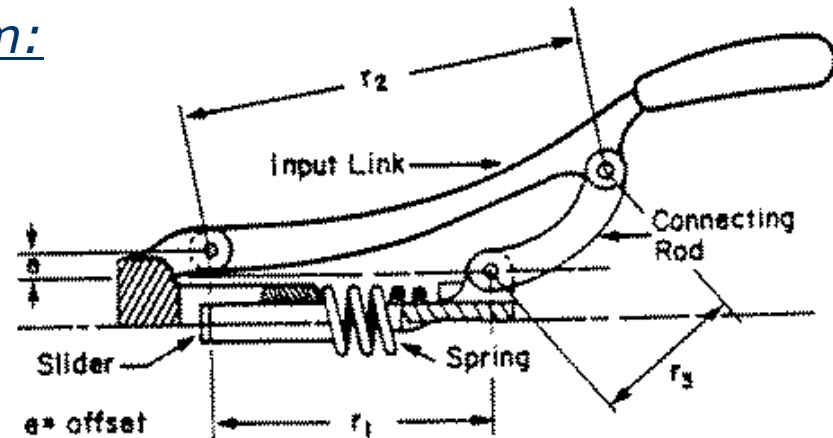
Displacement pattern

O. Sigmund. On the design of compliant mechanisms using topology optimization. Mechanics of structures and machines, 25(4) :493-524, 1997.

# Advantages of compliant mechanisms (1/2)

- Usually monolithic (single-piece)
  - ➔ Reduce time for manufacturing
  - ➔ Reduce time for assembly
  - ➔ Reduction of the costs

## Fully compliant crimping mechanism:



<http://compliantmechanisms.byu.edu>

- Significant reduction in weight
- Compliant mechanisms can easily be miniaturized. (MEMS)

# Advantages of compliant mechanisms (2/2)

- Since there are no (less) joint:
  - Less wear
  - Less friction
  - Less backlash → increase the mechanism precision
  - Less need for lubrication
- Valuable characteristics for applications where the mechanism is not easily accessible, or for operation in harsh environments that may adversely affect joints.
- Vibration and noise caused by the turning and sliding joints of rigid-body mechanisms may also be reduced.
- Have built-in restoring force
  - Similar to the potential energy in a deflected spring
  - Can store energy and releases it at a later time and/or in a different manner

# *Drawbacks of compliant mechanisms*

- Fatigue analysis is critical. (Cyclic loading)
- The motion coming from the deflection of the mechanism is limited to the strength of its members.
- A compliant link cannot produce a continuous rotational motion as the one produces by an hinge for instance.





# The challenges of compliant mechanisms

- The largest challenge :  
*Analysis and design of compliant mechanisms*
- Require the knowledge of :
  - 1) Mechanism analysis methods
  - 2) The deflection of flexible members
- ➔ Not only an understanding of both, but also an understanding of the interactions of the two methods in a complex system.

# Goals and Motivations

- Define a robust and efficient method to design a compliant suspension
  - Reduction of the weight
  - Reduction of the cost
  - Improvement of the reliability and design flexibility
- “In a world where environmental awareness and oil price raise...”
  - Improvement of the fuel efficiency
- In the past, Kobayashi (2009) used a combination of topology and shape optimizations. Topology optimization was used to obtain a first topology with requirements only on the stiffness and the flexibility.
- We developed a method to design the compliant suspension fully based on topology optimization for all the different criteria.



# Topology optimization



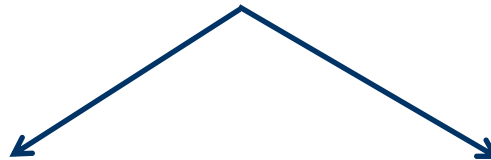
# How to design?

- Due to the complexity of the elastic behavior, trials and errors methods were used in the past.
- Nowadays

## *Two design methods*

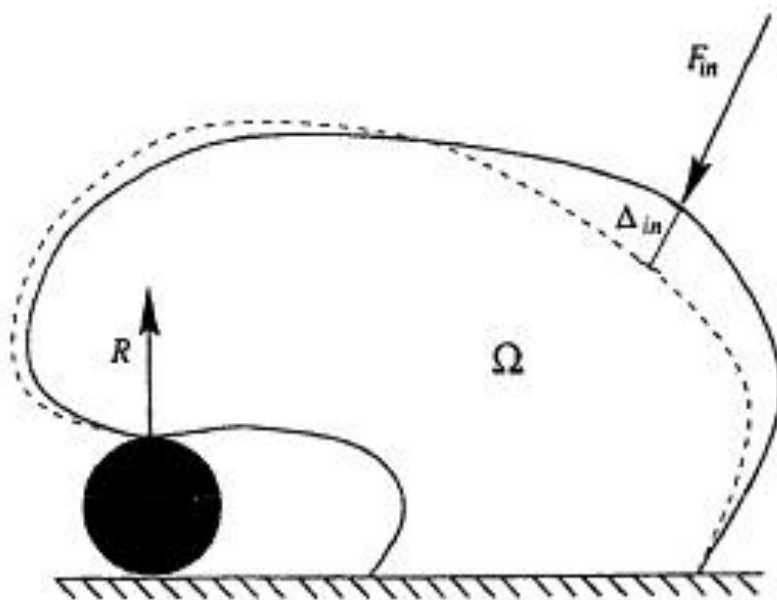
Pseudo  
rigid-body  
mechanism  
method

Topology  
optimization  
methods



# The design of compliant mechanism: a crunching mechanism

- In the case of a crunching mechanism, the major goal is to maximize the output force for a given input force.
- The Mechanical Advantage is defined as the ratio between the output and the input force.



$$M = \frac{F_{out}}{F_{in}}$$

# *The design of compliant mechanism: a crunching mechanism*

- The objectives of the design of the compliant crunching mechanism are conflicting:
  - The mechanism should be stiff to be able to transmit a high force.
  - But the mechanism should be soft enough to deflect and make contact with the workpiece.
- The design problem can be defined as the problem of finding the optimal mechanism topology within a design domain that satisfies the goals and some constraints  
*or*
- *The optimal compliant mechanism may be found, in an optimal way by distributing a limited amount of material in the design domain = topology optimization problem*



# Topology optimization: formulation

- Initially, topology optimization was used to optimize the stiffness of elastic structure.
- In a finite element formulation, the topology optimization problem has the following form:

$$\begin{array}{ll} \min_{\mathbf{u}, \mathbf{E}} \mathbf{g}^T \mathbf{q} & \longrightarrow \text{Objective function: Compliance} \\ \text{such that } \left\{ \begin{array}{ll} \cdot \mathbf{K}(\mathbf{E}_e) \mathbf{q} = \mathbf{g} & \longrightarrow \text{Equilibrium equation} \\ \cdot \text{Volume} \leq \bar{V} & \longrightarrow \text{Restriction on the volume} \\ \cdot \mathbf{E} \in \mathbf{E}_{ad} & \longrightarrow \text{Admissibility of the elasticity tensor} \\ \cdot \text{Others constraints.} & \end{array} \right. \end{array}$$

- The design variables are the density of each finite element. (Large numbers)



# Topology optimization: SIMP law

- A density variable ( $\mu$ ) is associated with each finite element:  
 $\mu = 0 \rightarrow$  Void and  $\mu = 1 \rightarrow$  Full density
- Discrete optimization problem  $\rightarrow$  too complex to solve
- Relaxation  $\rightarrow$  intermediate value for  $\mu$  between  $\mu_{\min}$  and 1.
  - Allows the use of mathematical programming
- In order to force the value of  $\mu$  towards 0 or 1: SIMP law
  - With  $n > 1$ , the stiffness of elements with intermediate densities is lowered, thus making “uneconomical” to have intermediate values.

$$E^e = (\mu^e)^n E^0$$

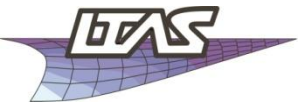
- Filtering techniques to avoid checkerboarder pattern



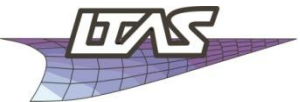


# Topology optimization: algorithm

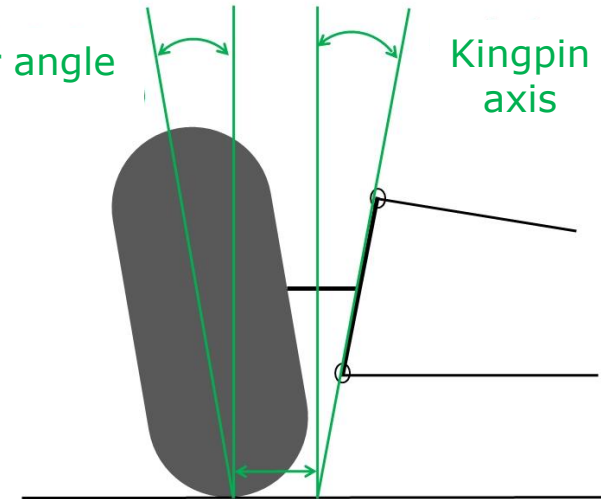
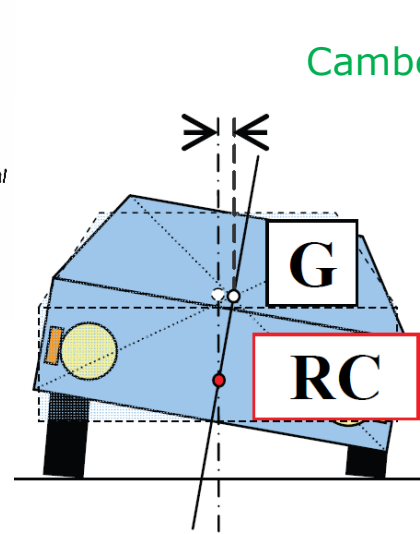
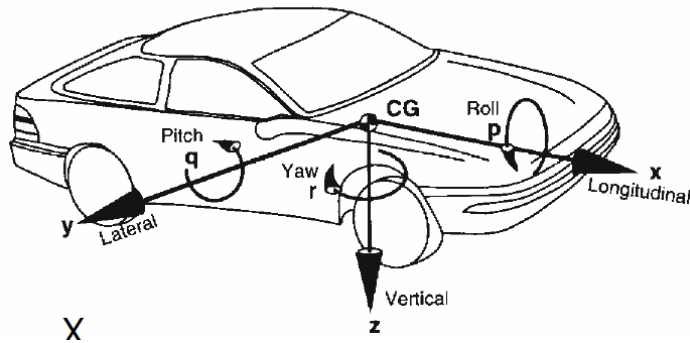
- General and robust framework for the optimization problem
  - Possibility to use different optimization algorithms.
- We use gradient-based methods:
  - High convergence speed
  - Limited number of iterations and function evaluations
  - But local optima.
- CONLIN algorithm is used in the present study. It is based on the so-called sequential convex programming approach. (Fleury and Braibant, 1986)



# Compliant suspension for vehicles



# *A few definitions in the front plane*



- Roll center (RC): Considering a front plane, it is the point around which the chassis rotates.
- Roll center height: Distance between the ground and the RC.
- Camber angle: Angle between the centerline of the tire and a perpendicular line to the ground.
- Track width: Distance between the centerline of the left tire and the center line of the right tire (ground level).

# *The design criteria of a vehicle suspension*

1. Stroke length:  
Flexibility, the fundamental criterion.
2. Rigidity of the suspension:  
Must support the different loads and reactions.
3. Roll center height :  
It influences the dynamic behavior.
4. Bounce and Roll movement:  
Modification of the track width and the camber angle.

The design is restricted to the **front plane**.



# Multi-Objective formulation?

- A lot of criteria, how to take them into account?
- The order of magnitude of the numerical values can be very different.

- Multi-objective formulation:

$$\text{Objective function} = w_1 fct_1 + w_2 fct_2 + \dots$$

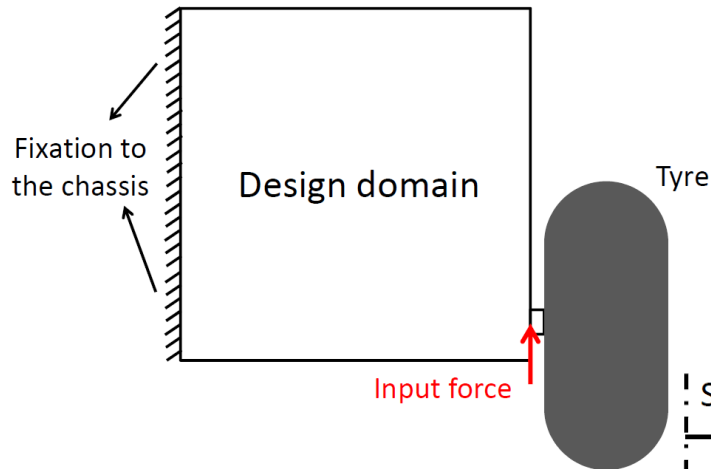
Weighting coefficient

- Scaling:

$$\text{Objective function} = \frac{fct_1}{target_{fct_1}} + \frac{fct_2}{target_{fct_2}} + \dots$$

- As our optimizer has been build to work with one objective function and constraints, we work with a function similar to the “mechanical advantage” as the objective function and the others criteria are considered as constraints.

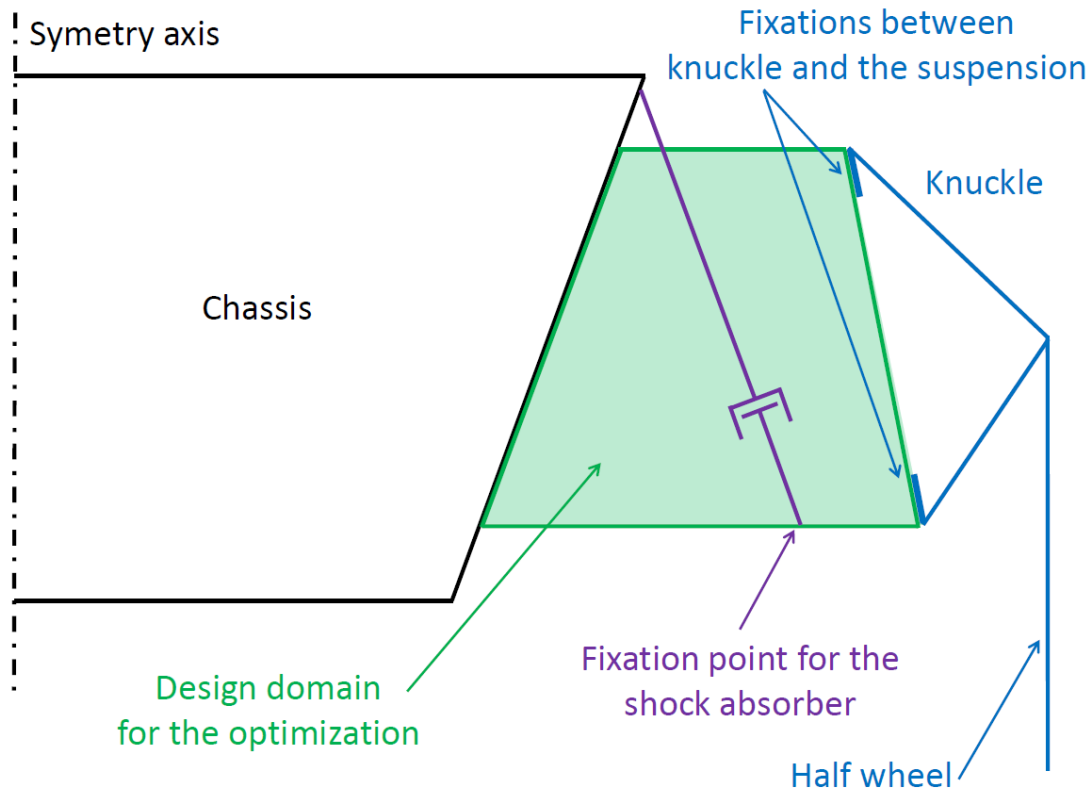
# The modeling



## The situation

- Chassis slope:  $20^\circ$
- Kingpin inclination axis:  $11^\circ$
- Aluminum

## The modeling



# Step 1: Flexibility

- Mobility of the wheel with regard to the chassis
- The objective function:  
During the displacement of the wheel, we maximize the displacement in the direction of the shock absorber.  
(with a mutual mean compliance formulation)
- Restrictions:
  - Under the estimate *max* load, the stroke length is max 80 mm.
  - During the wheel travel, the camber angle variation must be less than  $1^\circ$ .
  - During the wheel travel, the track width variation must be less than 1 mm.



# Mutual mean compliance

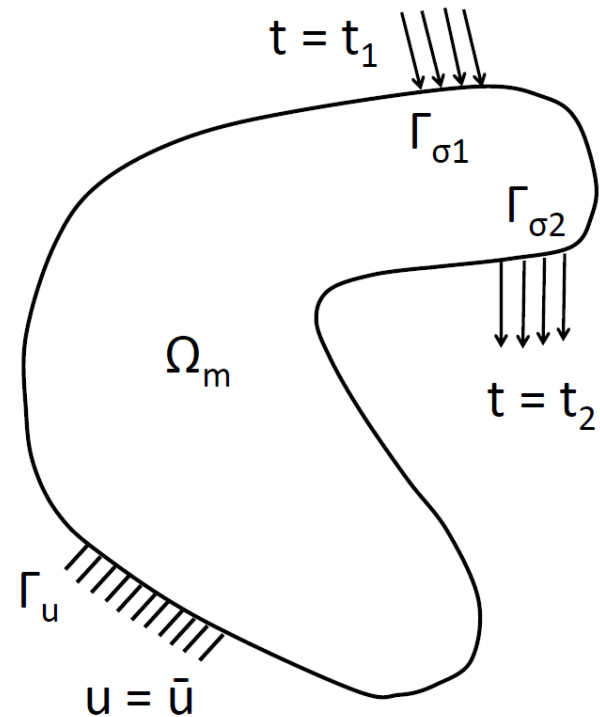
The flexibility is introduced using a *mutual mean compliance* formulation.

$$l_2(\mathbf{u}_1) = \int_{\Gamma_{\sigma 2}} t_{2i}^T u_{1i} d\Gamma, \quad \mathbf{u}_1 \in U_1$$

where

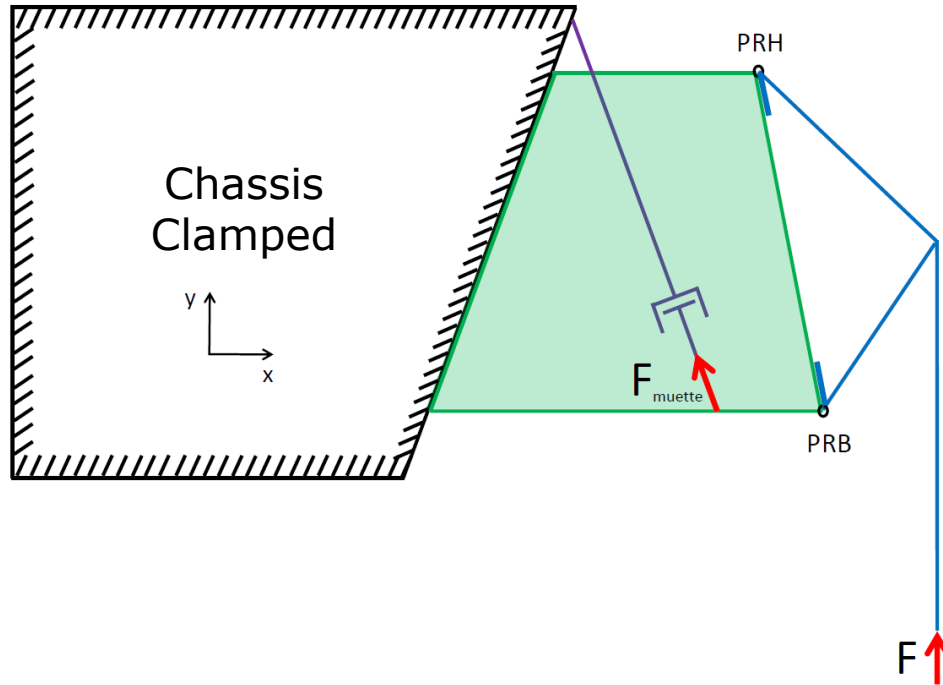
$$U_1 = \{\mathbf{v} = v_i \mathbf{e}_i : \mathbf{v} = 0 \text{ on } \Gamma_u\}$$

$u_1$  displacement for the load  $t_1$   
 $t_2$  dummy load





# Step 1: Load case



$$\min_{\mathbf{u}} \frac{1}{\text{Flexibility}}$$

such that

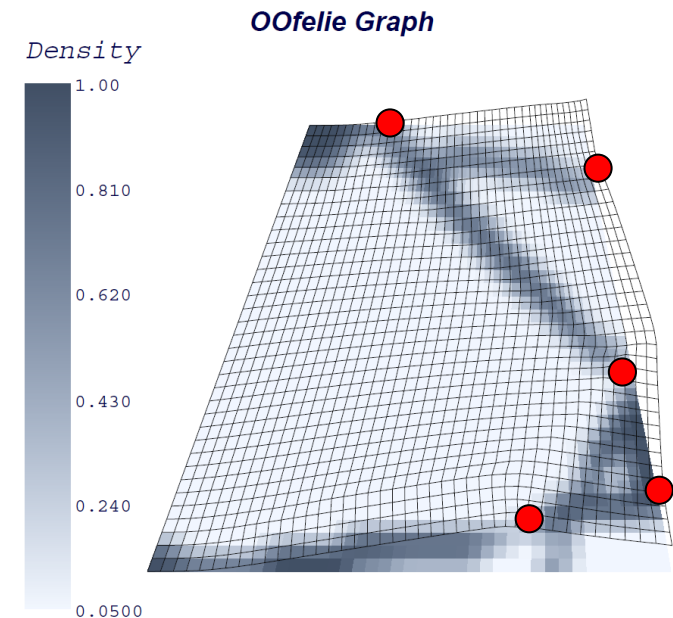
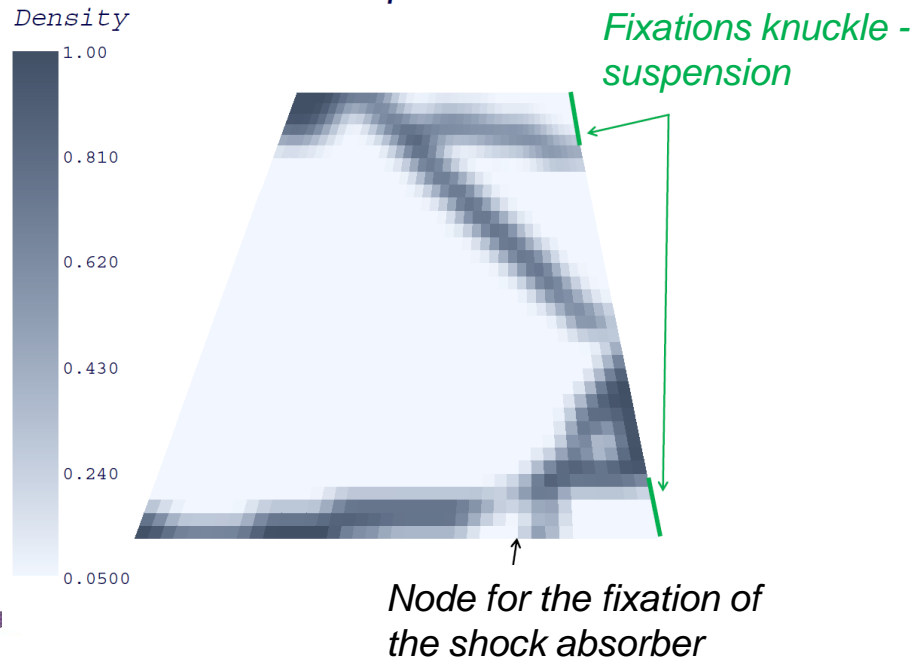
- Max Displ  $\leq 80mm$
- Camber angle  $\leq 1deg$
- Track width  $\leq 1mm$
- Volume  $\leq 0.25$



# Step 1: Results analysis

	« Variable »	Value obtained	Target value
Load case 1	Displacement amplification in the shock absorber	1,772	/
	Vertical displacement of the contact point wheel-ground [mm]	79,998	$\leq 80$
	Camber angle [°]	0,999	$\leq 1$
	Track width [mm]	0,999	$\leq 1$
	Volume constraint	0,25	0,25

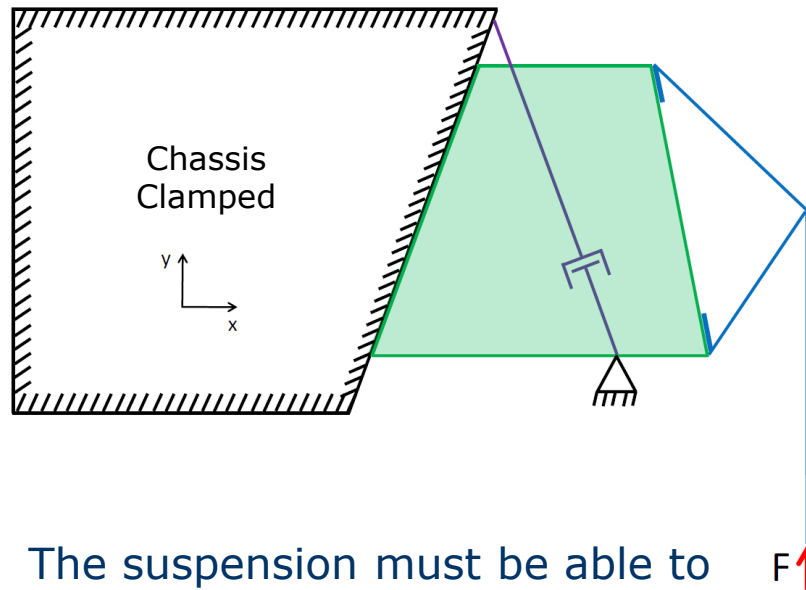
OOfelie Graph



## Step 2: Rigidity

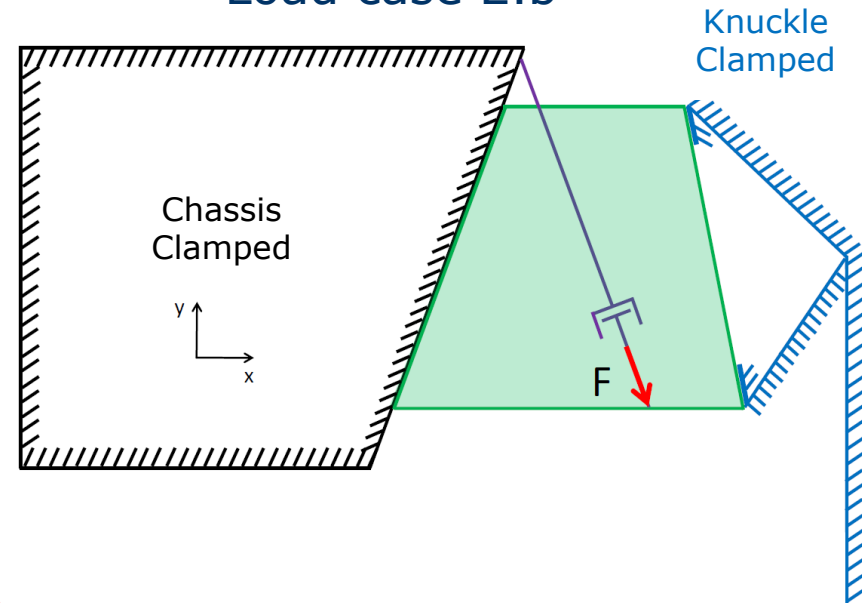
- The mechanism must be stiff enough to be able to support the different loads and reactions.

Load case 2.a



The suspension must be able to transmit the wheel travel into the shock absorber.

Load case 2.b



The suspension must be able to support the reaction loads.

# Compliance

The rigidity is introduced using a *compliance* formulation.

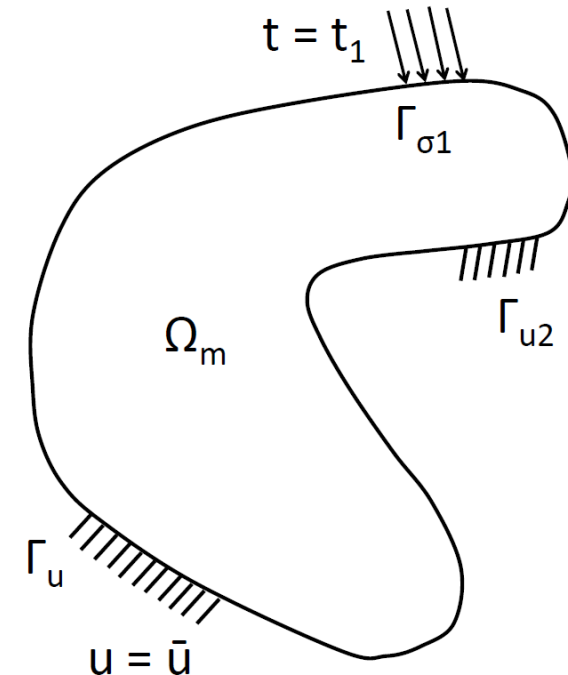
$$l_1(\mathbf{u}_1) = \int_{\Gamma_{\sigma 1}} t_1^T u_{1i} d\Gamma, \quad \mathbf{u}_1 \in U_2$$

where

$$U_2 = \{\mathbf{v} = v_i \mathbf{e}_i : \mathbf{v} = 0 \text{ on } \Gamma_u \text{ and } \Gamma_{u2}\}$$

$u_1$  displacement for the load  $t_1$

The compliance can be seen as the inverse of the stiffness or a quantity of mechanical energy.



## Step 2: Optimization problem formulation

$$\min_{\mathbf{u}} \frac{\text{Compliance}}{\text{Flexibility}}$$

This expression is totally equivalent to the *mechanical advantage* developed for the crunching mechanism by O. Sigmund.

such that

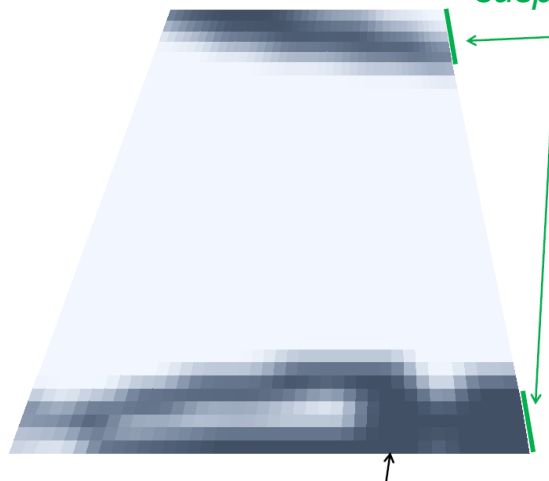
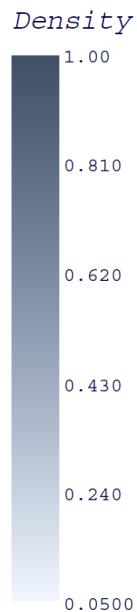
$$\left\{ \begin{array}{l} \cdot \text{Max Displ} \leq 80mm \\ \cdot \text{Camber angle} \leq 1deg \\ \cdot \text{Track width} \leq 1mm \\ \cdot \text{Volume} \leq 0.25 \end{array} \right.$$



# Step 2: Results analysis

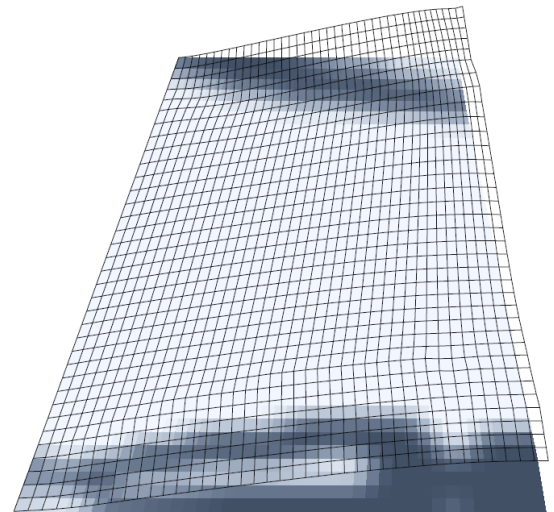
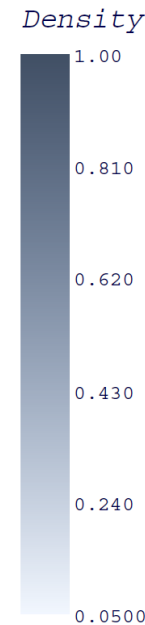
	« Variable »	Value obtained	Target value
Load case 1	Displacement amplification in the shock absorber	0.901	/
	Vertical displacement of the contact point wheel-ground [mm]	79,949	$\leq 80$
	Camber angle [°]	0,999	$\leq 1$
	Track width [mm]	0,991	$\leq 1$
	Volume constraint	0,25	0,25

OOfelie Graph



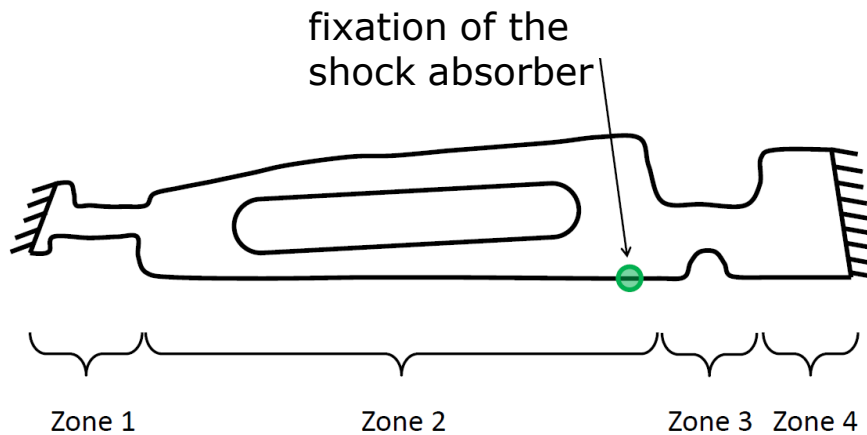
Node for the fixation of the shock absorber

OOfelie Graph

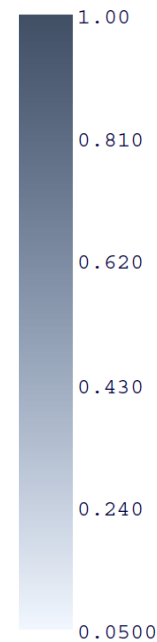


## Step 2: Result analysis

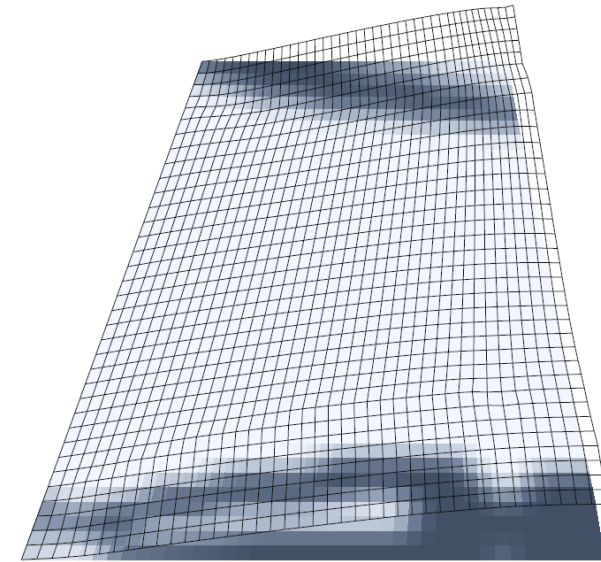
- This mechanism satisfies the basic requirements of a suspension.
- The upper part is a beam that bends through its length.
- The bottom part is



Density



OOfelie Graph



Zone 1

Zone 2

Zone 3

Zone 4

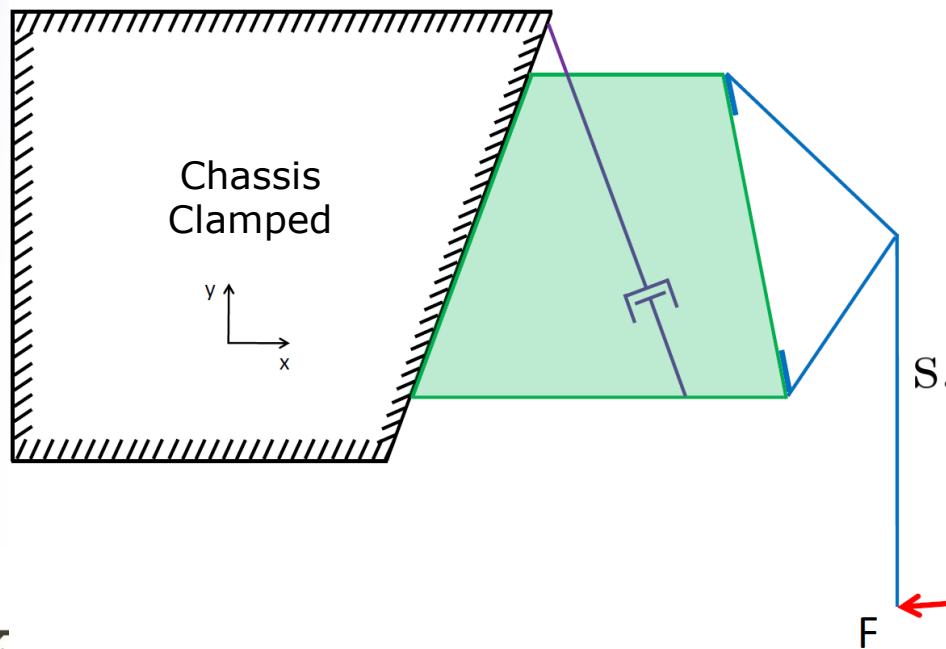
Flexibility





## Step 3: Lateral rigidity

- The mechanism must be stiff enough to be able to support lateral loadings (Cornering,...).
- This lateral stiffness must be much more important than the vertical stiffness (about 20X).
- Introduced as a constraint.



$$\min_{\mathbf{u}} \frac{\text{Compliance}}{\text{Flexibility}}$$

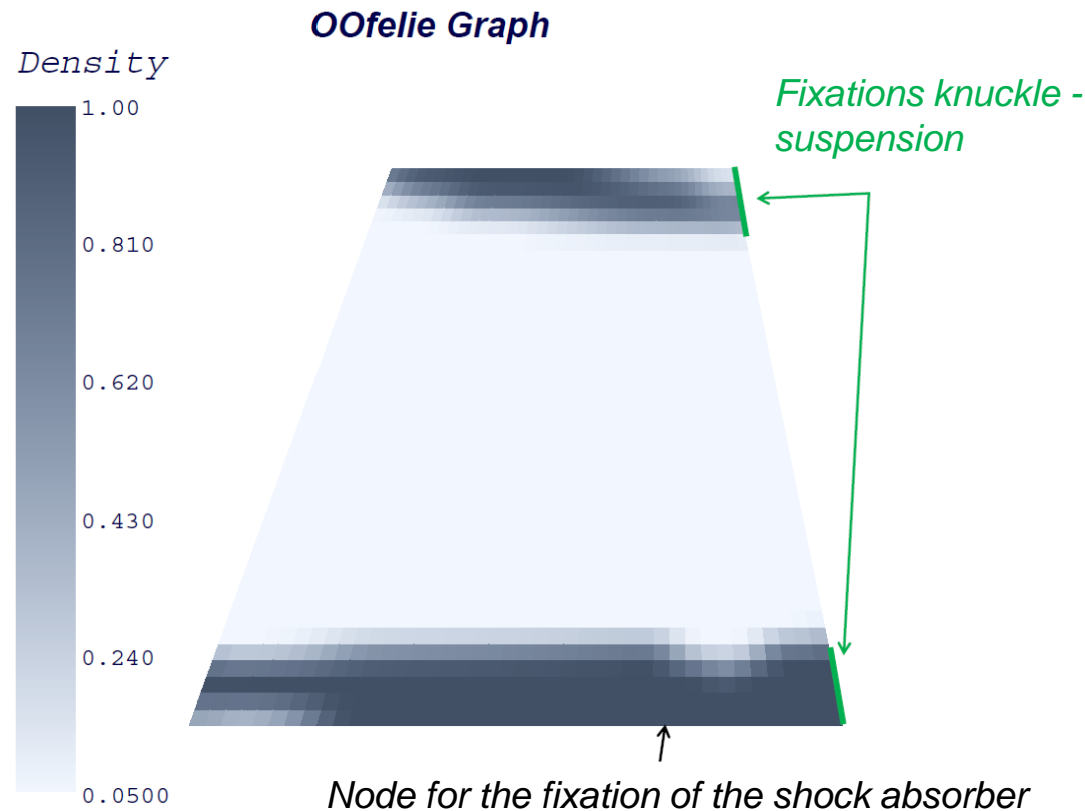
$$\text{s.t.} \left\{ \begin{array}{l} \cdot \text{Max Displ} \leq 80\text{mm} \\ \cdot \text{Camber angle} \leq 1\text{deg} \\ \cdot \text{Track width} \leq 1\text{mm} \\ \cdot \text{Lat. Displ.} \leq 5\text{mm} \\ \cdot \text{Volume} \leq 0.25 \end{array} \right.$$





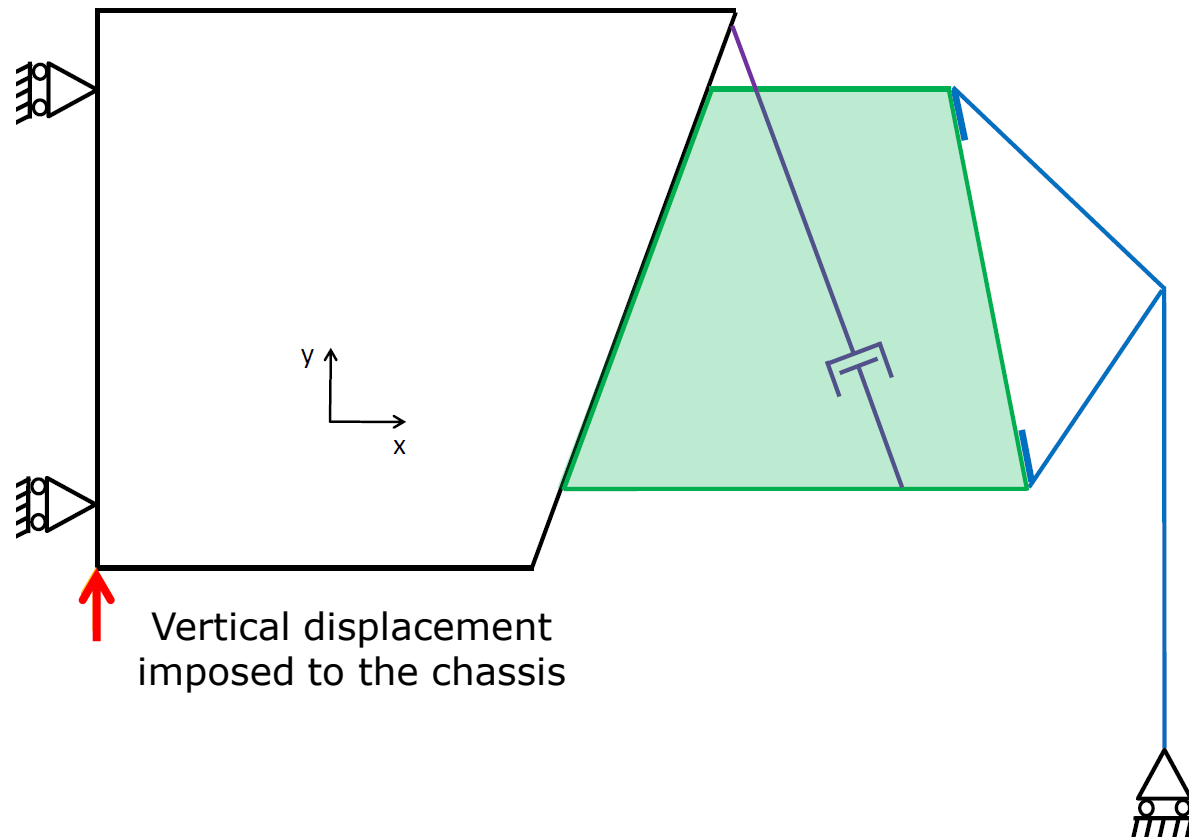
## Step 3: Results analysis

- As the bottom part works in compression, this load case modifies this part by removing the hole.
- Except this modification, the mechanism is quite similar as the one in step 2.



## Step 4: Bounce movement

- When the vehicle has bounce movement, the camber angle and the track width vary.
- Limitations on these variations are imposed.



## Step 4: Results analysis

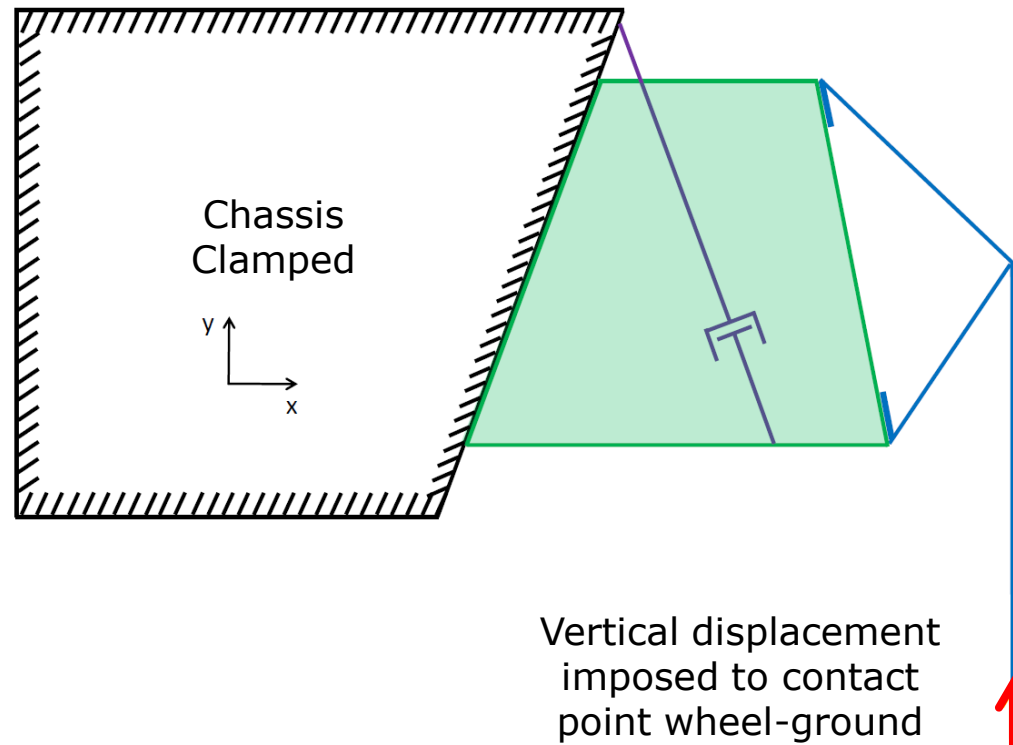
- The restrictions imposed during the bounce movement are already satisfied by the mechanism → No modification.

	« Variable »	Value obtained	Target value
Load case 1	Displacement amplification in the shock absorber	0.901	/
	Vertical displacement of the contact point wheel-ground [mm]	79,864	$\leq 80$
	Camber angle [°]	0,532	$\leq 1$
	Track width [mm]	0,994	$\leq 1$
Load case 3	Lateral displacement of the contact point wheel-ground [mm]	4,997	$\leq 5$
Load case 4	Bounce : Track width [mm]	0,0124	$\leq 0,02$
	Bounce : Camber angle [°]	0,0064	$\leq 0,015$
	Volume constraint	0,25	0,25

Bounce: The values obtained are for a bounce displacement of 1mm.

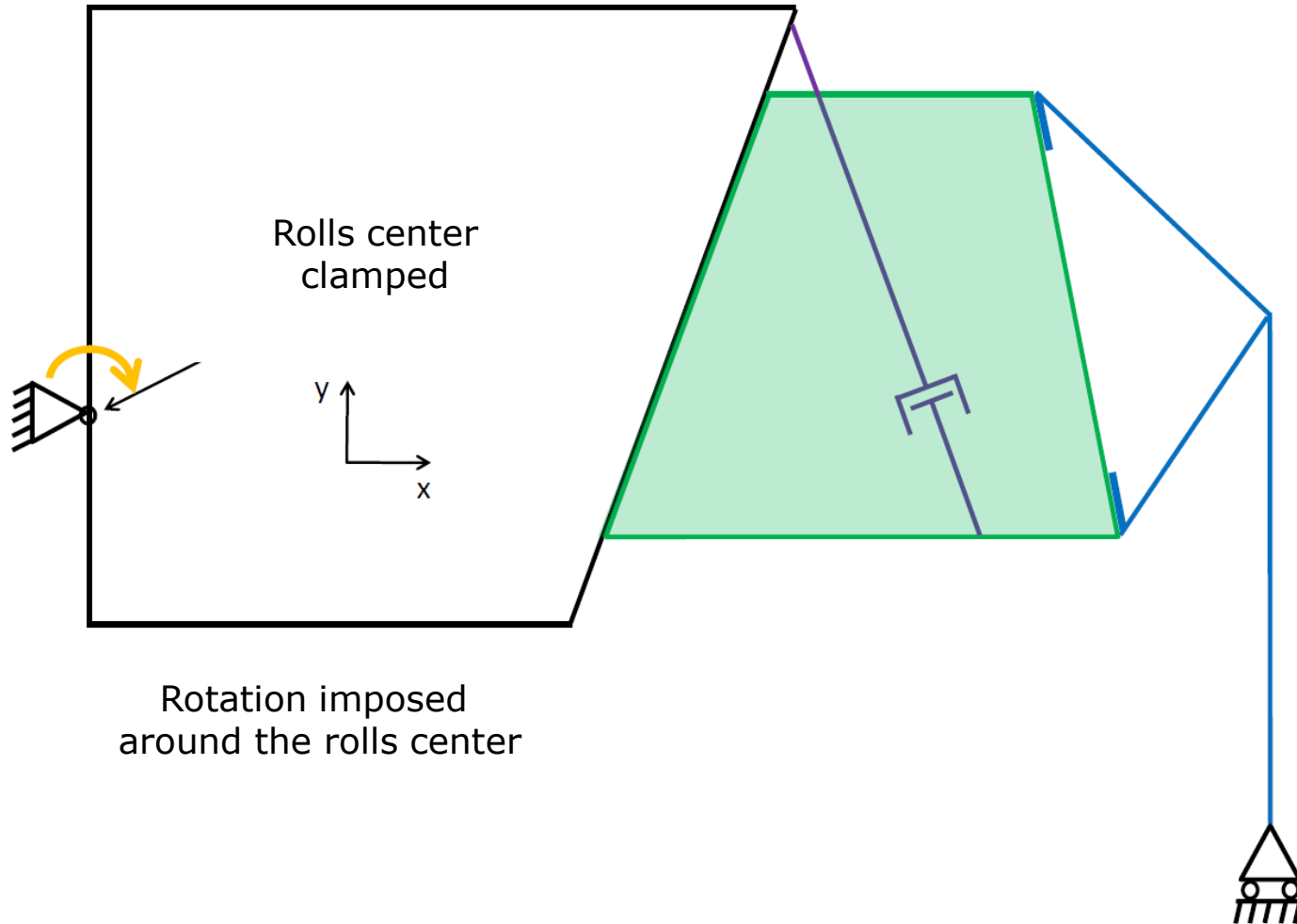
# Step 5: Roll movement – Roll center height

- When the vehicle has roll movement, the camber angle and the track width vary.
- The first step is to identify the roll center.
  - The roll center depends on the suspension configuration.
  - Must be computed at each iteration.
  - Different methods exist.
- The roll center height influences the dynamic behavior.
- Restriction:  
Roll center height must be between 50 et 200 mm.



## Step 5: Load case

### ■ Roll movement simulation



# Step 5: Results analysis

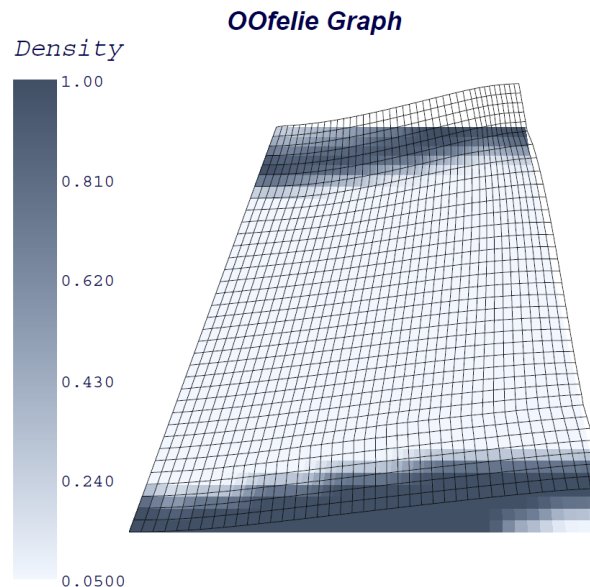
	« Variable »	Value obtained	Target value
Load case 1	Displacement amplification in the shock absorber	0.731	/
	Vertical displacement of the contact point wheel-ground [mm]	77.889 $[10^{-1}]$	$\leq 80$
	Camber angle $[^{\circ}]$	1,635 $[10^{-1}]$	$\leq 1$
	Track width [mm]	1,514 $[10^{-3}]$	$\leq 1$
Load case 3	Lateral displacement of the contact point wheel-ground [mm]	5,257 $[10^{-4}]$	$\leq 5$
Load case 4	Bounce : Track width [mm]	0,0194 $[10^2]$	$\leq 0,02$
	Bounce : Camber angle $[^{\circ}]$	0,0202 $[10^2]$	$\leq 0,015$
Load case 5	Roll center height [mm]	102.838 $[10^{-3}]$	$50 \leq val \leq 200$
Load case 6	Roll : Track width [mm]	1,557 $[10^{-4}]$	$\leq 2$
	Roll : Camber angle $[^{\circ}]$	0,736 $\theta_{roulis}$ $[10^{-4}]$	$\leq 0,7 \theta_{roulis}$
	Volume constraint	0,25 $[2.510^{-4}]$	0,25

Roll: The values obtained are for a roll movement  $1^{\circ}$ .



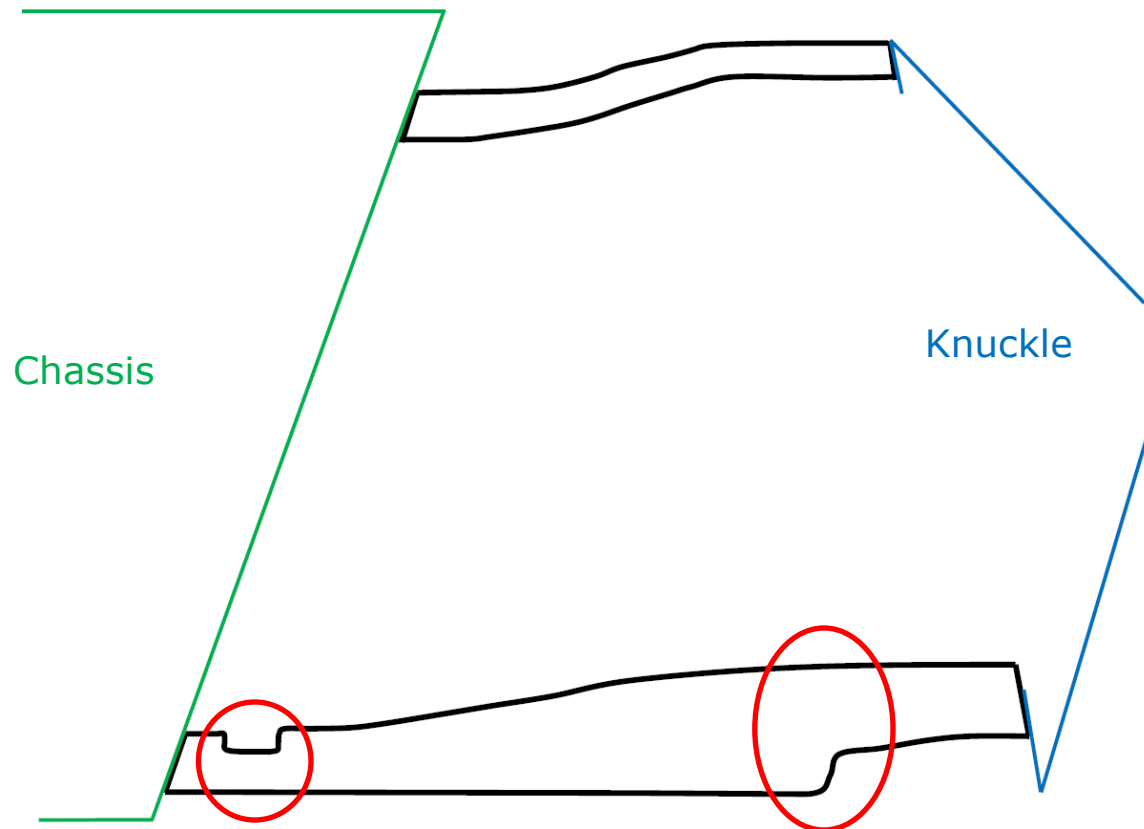
## Step 5: Results analysis

- Impossible to satisfy all the criteria when the roll criteria are introduced (too restrictive constraints)
  - Relaxation factors allow violating the constraints when there is no solution.
  - More this factor is small, more the violation of the constraint is harmful for the optimization process.
  - With these factors, one can choose to give more importance to some criteria.



# Resulting mechanism

- In this mechanism, no joint!
- The mobility comes from the flexibility of the different members. Reduction of the thickness at different positions allows having the right displacement.



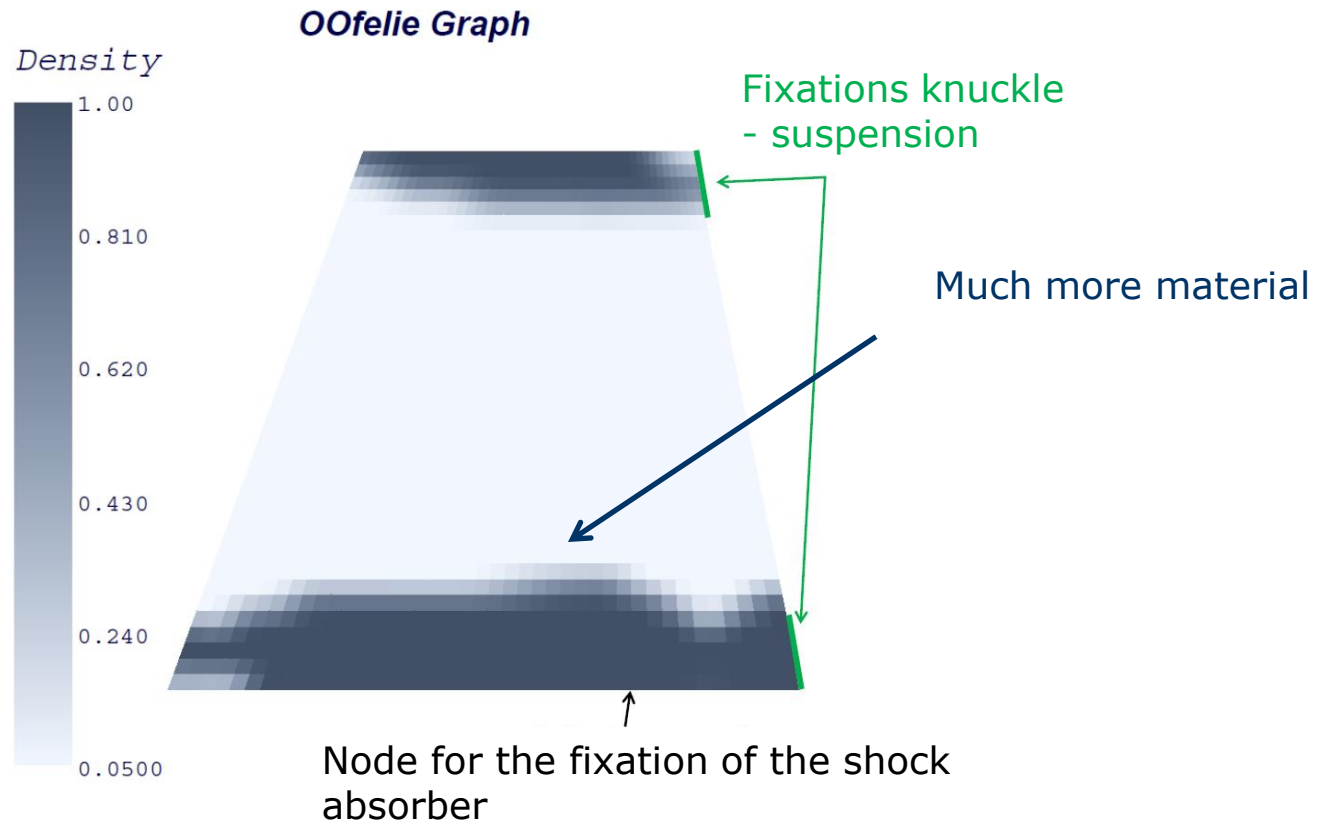


# Numerical applications



# Improvement of the lateral stiffness

- One can want to increase the lateral rigidity.
- Keeping a restriction on the volume of  $V < 0.25$ , a lot of constraints are violated.
- Increasing the volume restriction to 0.3, we get



# Improvement of the lateral stiffness

	« Variable »	Value obtained	Target value
Load case 1	Displacement amplification in the shock absorber	0.736	/
	Vertical displacement of the contact point wheel-ground [mm]	75.254	$\leq 80$
	Camber angle [°]	1.158	$\leq 1$
	Track width [mm]	1.149	$\leq 1$
Load case 3	Lateral displacement of the contact point wheel-ground [mm]	4.060	$\leq 4$
Load case 4	Bounce : Track width [mm]	0.0153	$\leq 0,02$
	Bounce : Camber angle [°]	0.0148	$\leq 0,015$
Load case 5	Roll center height [mm]	106.402	$50 \leq val \leq 200$
Load case 6	Roll : Track width [mm]	1.670	$\leq 2$
	Roll : Camber angle [°]	$0,805 \theta_{roulis}$	$\leq 0,7 \theta_{roulis}$
	Volume constraint	0.3	0.3

Very small violations!

# Conclusions & Perspectives



# Conclusions

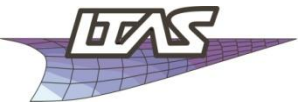
- A robust and efficient method to design a compliant suspension in 5 steps.
- When the constraints on the different criteria are too restrictive, necessity to do compromise.
- This study has been realized with load cases corresponding to a compact car (C-segment) but it can be extended to other vehicles.



- The study has been carried out in the front plane → extension to the real 3D case.
- Linear analysis has been used to develop the method. But as there are large displacements, a non-linear analysis should be used to go one step further in the design.
- Restriction on the level of admissible stresses
- Impose criteria to get a feasible shape for manufacturing.
- The load cases are static but it would be more interesting and more accurate to work with dynamic loading.



*THANK YOU VERY MUCH  
FOR YOUR ATTENTION*



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