Université de Liège

Aerospace & Mechanical Engineering

ACOMEN Conference

November 14-17, 2011, Liège, Belgium

Modelling of frictional unilateral contact in automotive differentials

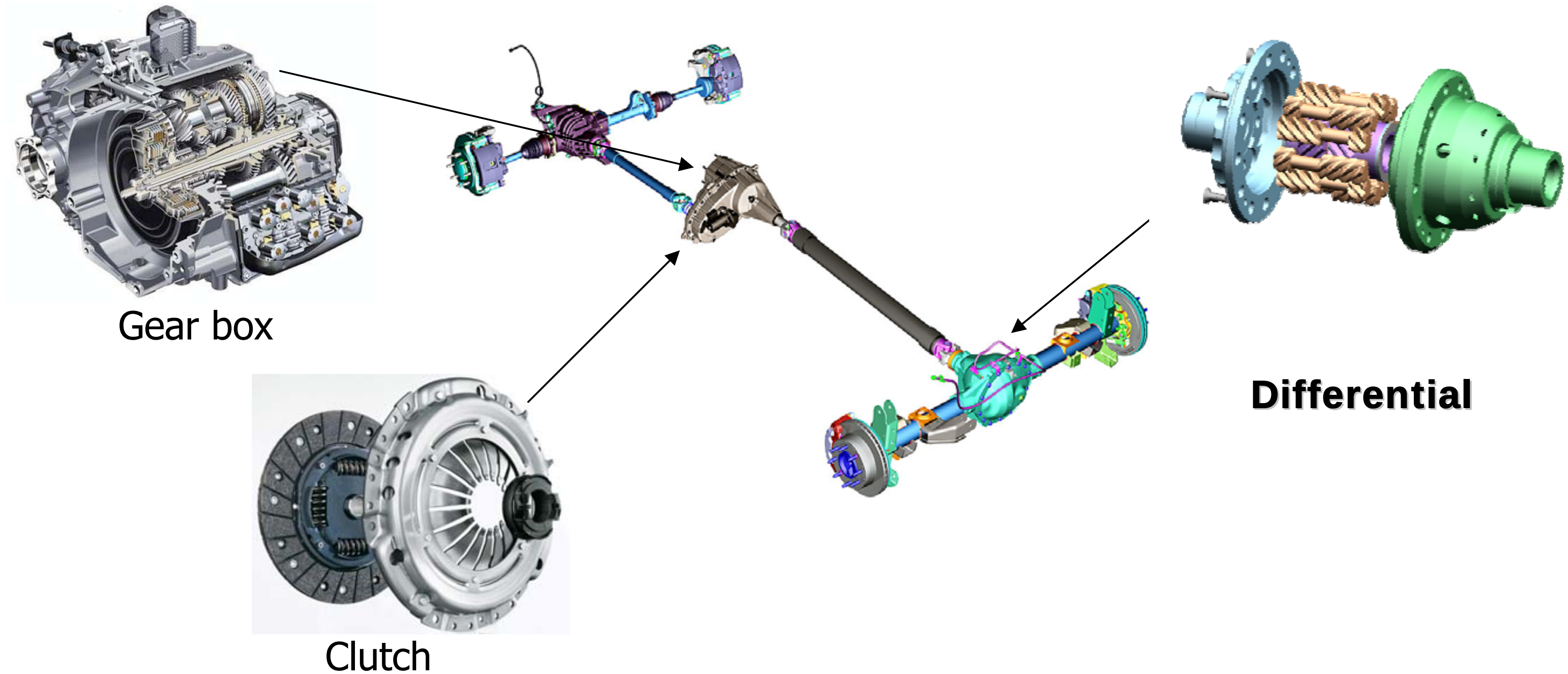
Virlez Geoffrey
PhD student – Research Fellow FRIA

University of Liège, Belgium
Automotive Engineering Department

Ingénierie des Véhicules terrestres

November 2011

Driveline modelling



Complex phenomena involved: backlash, stick-slip, contact, discontinuity, hysteresis, non linearity

➔ Numerical problems

In this work, TORSEN differential modelling and focus on contact formulations

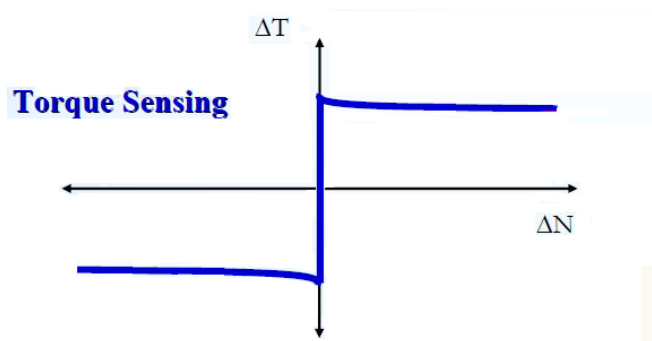
Outline

- Description of the application : TORSEN differential
- Continuous impact modelling
  - Formulation
  - Numerical results for benchmark and differential model
- Squeeze film modelling
  - Formulation
  - Numerical results for differential model
- Differential in full vehicule model
- Conclusion

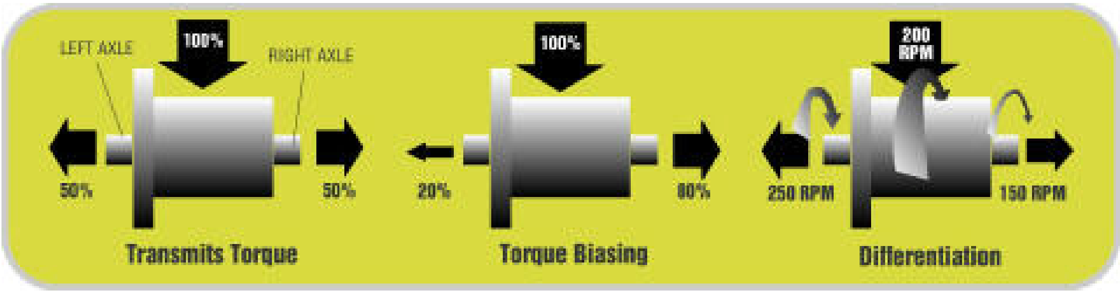
Université de Liège

TORSEN differential

- Limited slip differential
  - Allow a variable torque distribution between the output shafts ➔ avoid spinning when ground adherence is not sufficient on one driving wheel
- Torque transfer before differentiation (torque sensing)
- Full mechanical system

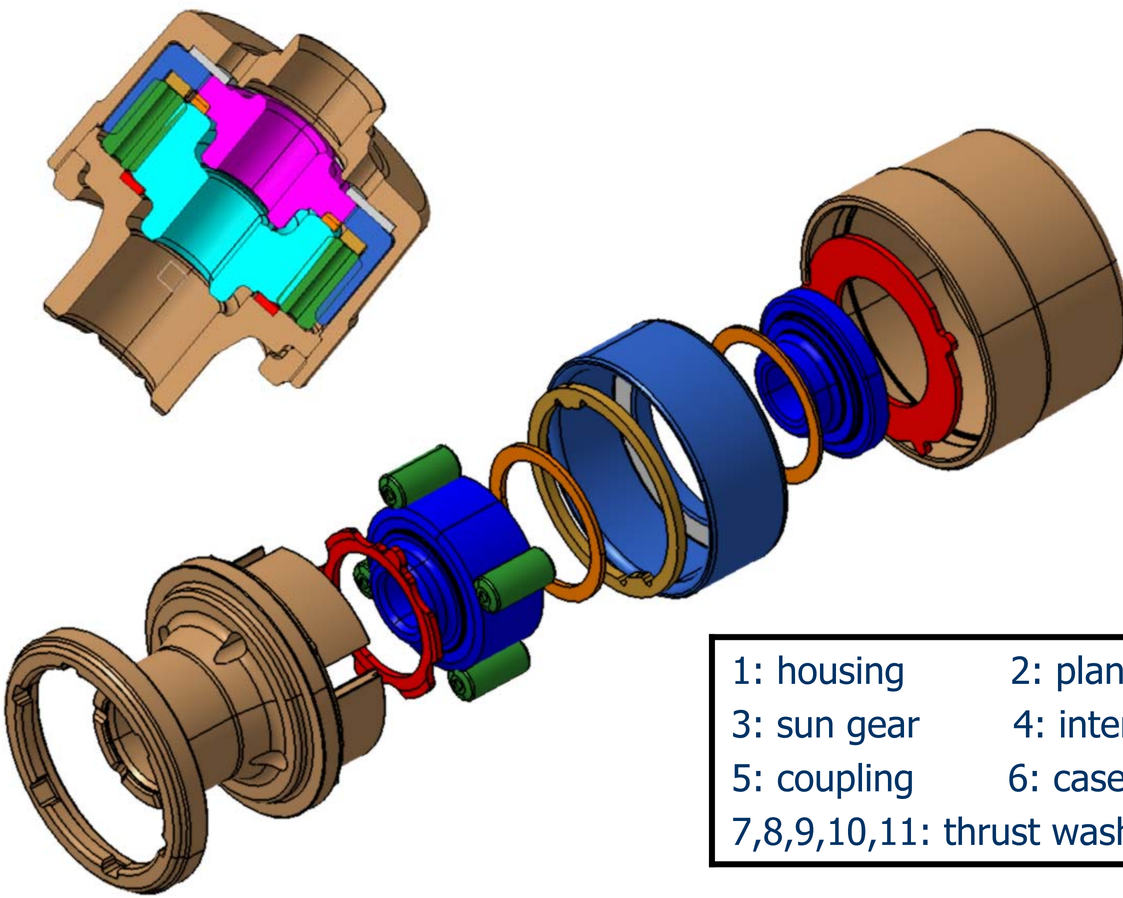


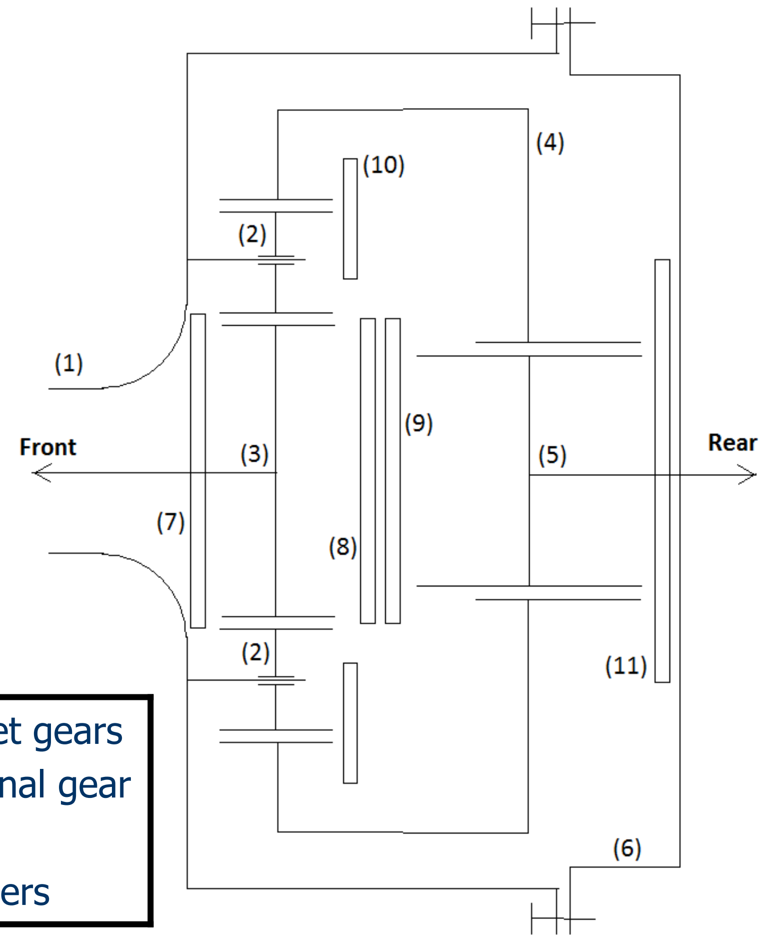
JTEKT



Type C Torsen

- Central differential
- Housing, helical gear pairs and thrust washers
- Locking due to relative friction between gears & washers
- 4 working modes



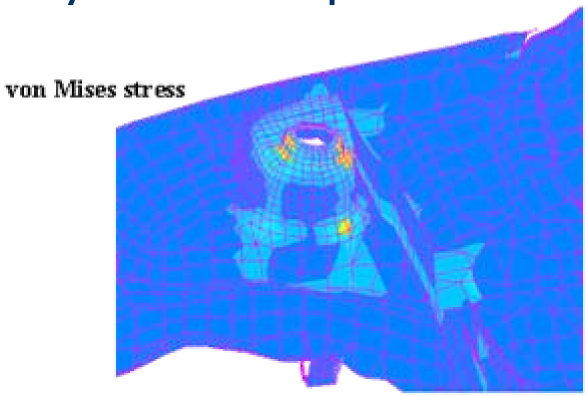


1: housing
2: planet gears
3: sun gear
4: internal gear
5: coupling
6: case
7,8,9,10,11: thrust washers

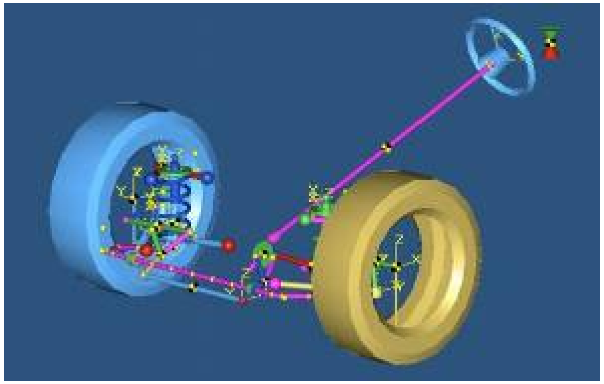
Université de Liège

Evolution of vitual prototyping

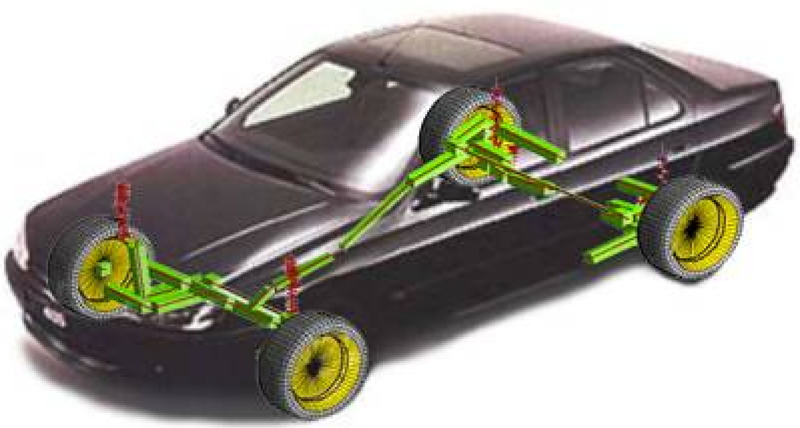
- Finite Element: structural analysis of components



- Multibody system: mechanism of rigid bodies



- Flexible Multibody systems:
  System approach (MBS)
  & structural dynamics (FEM)



FE coordinates

- Many interactions between transmission components are due to flexibility

  ➔ nonlinear finite element method based on the absolute nodal coordinates

SAMTECH

- Software: Samcef Field/MECANO
- Rigid ***and*** flexible bodies
- Parametrization of rotations with the cartesian rotation vector + updated Lagrangian approach

- Equations of motion

Penalty factor

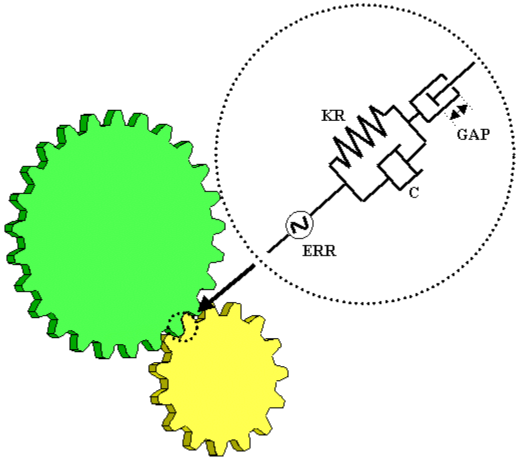
Scaling factor

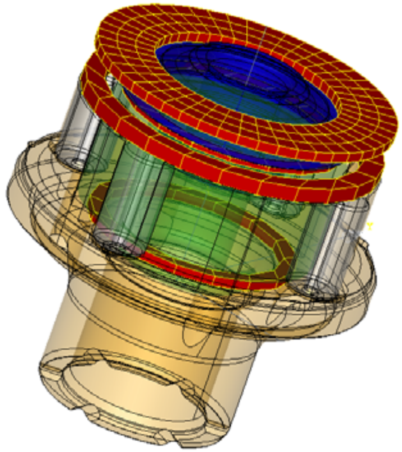
$$
\begin{aligned}
M(q)\ddot{q} + g^{gyr}(q,\dot{q}) + g^{int}(q) + \Phi_q^T(p\Phi + k\lambda) &= g^{ext}(t) \\
k\,\Phi(q,\dot{q},t) &= 0
\end{aligned}
$$

Constraints: joints, rigidity

TORSEN differential modelling

- Gear pair element:
  - Global kinematic joint defined between 2 nodes: one on each gear wheels (rigid body)
  - Spring, damper, backlash, load transmission error, friction,…
- Contact condition:
  - Flexible/flexible or rigid/flexible
  - Augmented lagrangian or penalty method
- TORSEN model globally validated with experimental data
- Several drawback due to the contact formulation ➔ developpement of new contact model (rigid-rigid)





Continuous impact modelling

- Restitution coefficient:
  - summarizes the kinetic energy loss
  - depends on shapes and material properties of colliding bodies and their relative velocity
  - roughly estimated by experince, determined by costly experiments or multi-scale simulations

- Contact force law

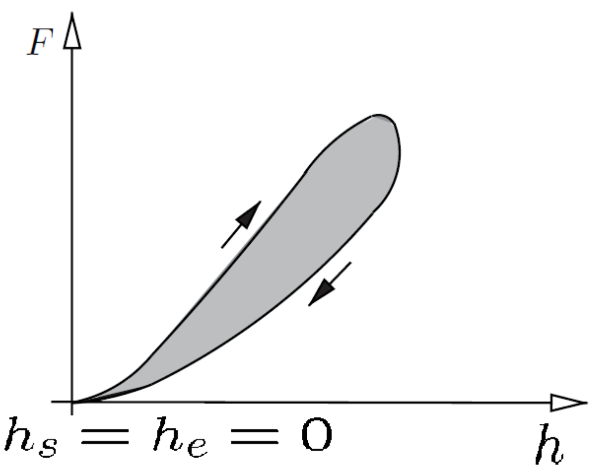
$$F(h,\dot{h}) = k\ h^n + c\ h^n\ \dot{h}$$

$$c = \frac{3(1-e^2)}{4}\frac{k}{\dot{h}_s}$$

Restitution coefficient

$$(0 \leq e_i \leq 1)$$

Total energy loss ↙ ↘ No energy loss



$h_s = h_e = 0$

$\dot{h}_s > 0$

$\dot{h}_e < 0$

(R. Seifried)

Continuous impact modelling

- Contact force law

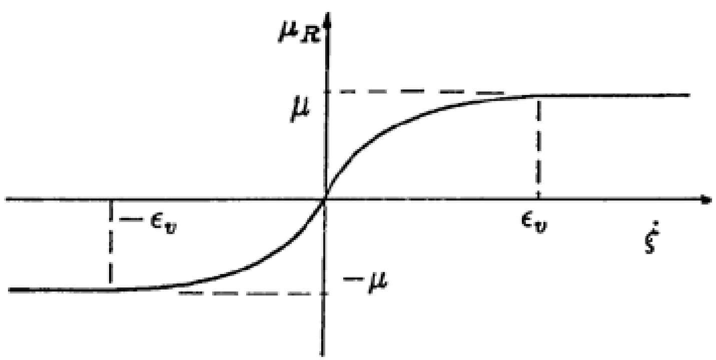
$$F(h,\dot{h}) = k\ h^n + c\ h^n\ \dot{h}$$

- Friction Torque

$$M = 2\pi\ \mu_R\ \frac{F(h,\dot{h})}{S}\ \frac{r_{ext}^3 - r_{int}^3}{3}$$

Regularization to avoid discontinuities

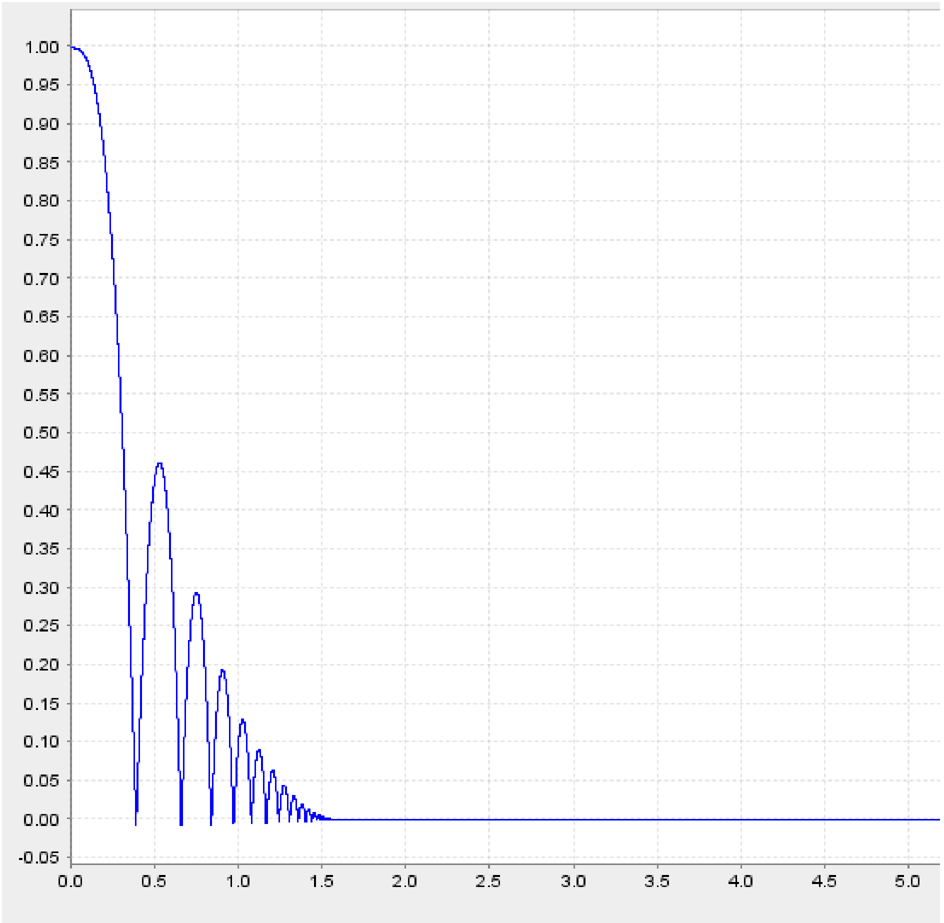
$$\mu_R(\dot{\xi}) = \begin{cases} \mu(2 - \frac{|\dot{\xi}|}{\epsilon_v})\frac{\dot{\xi}}{\epsilon_v} & |\dot{\xi}| < \epsilon_v \\ \mu\frac{\dot{\xi}}{|\dot{\xi}|} & |\dot{\xi}| \geq \epsilon_v \end{cases}$$



Benchmark

- Bouncing ball

Height (m)



Time (s)

e=0.8

n=1,5

k=1e10 N/m

h_0=1 m

m=0,85 kg

a=10 m/s²

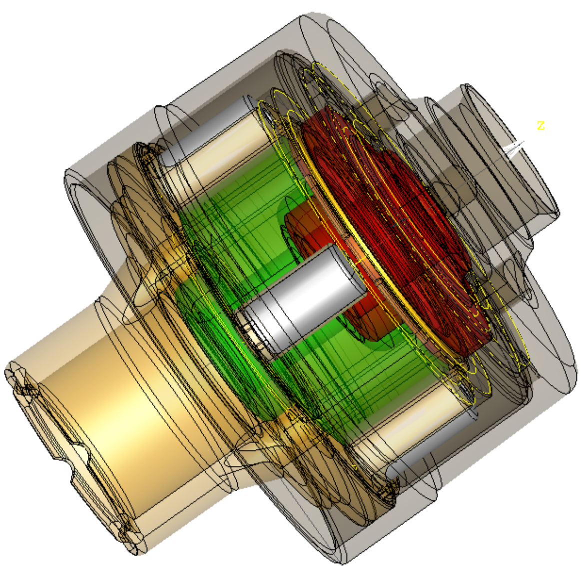
Aerospace & Mechanical Engineering

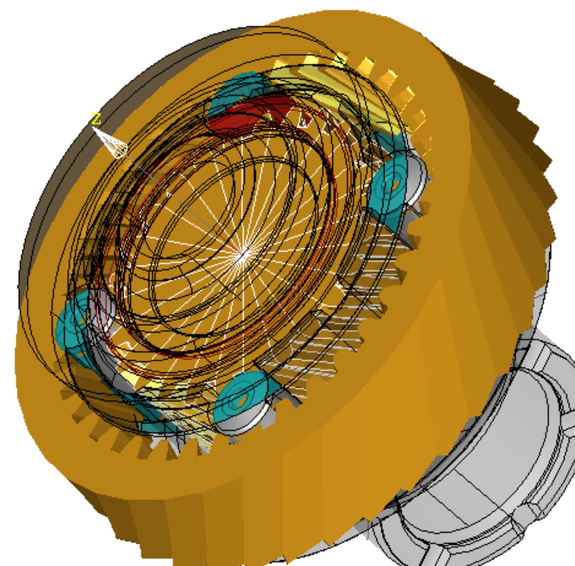
Université de Liège

TORSEN differential model

Assumptions: - kinematic joint between Planet gears and housing modeled as hinge joints
- two contact conditions neglected

- 15 rigid bodies, 878 dof
- Constraints :
  - 8 gear pairs
  - 5 contact relations
  - 4 hinges
  - 1 screw joint

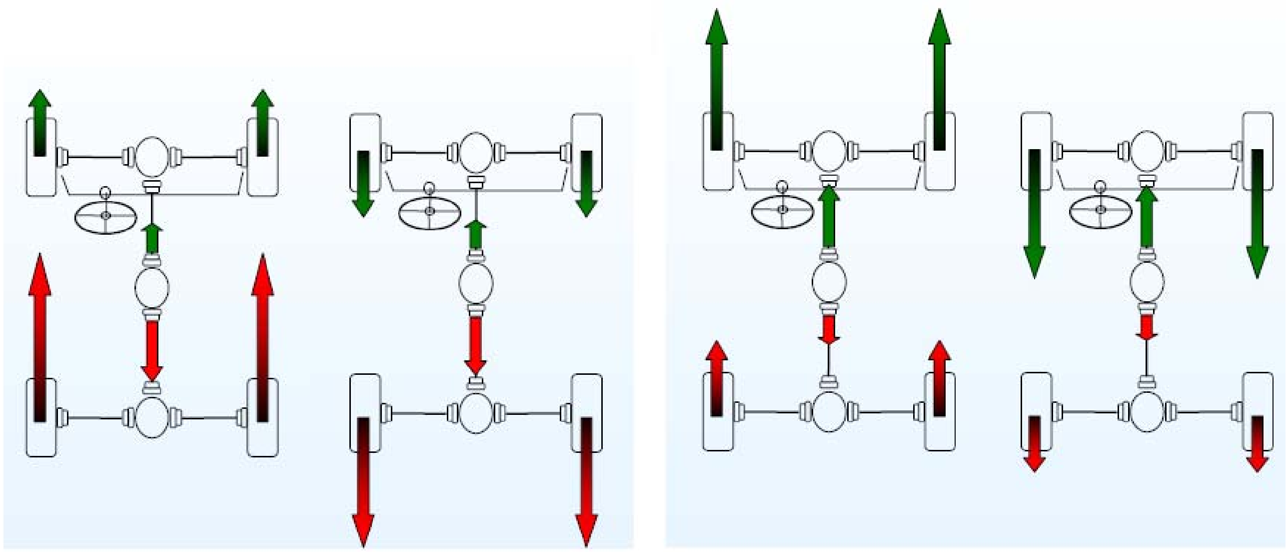


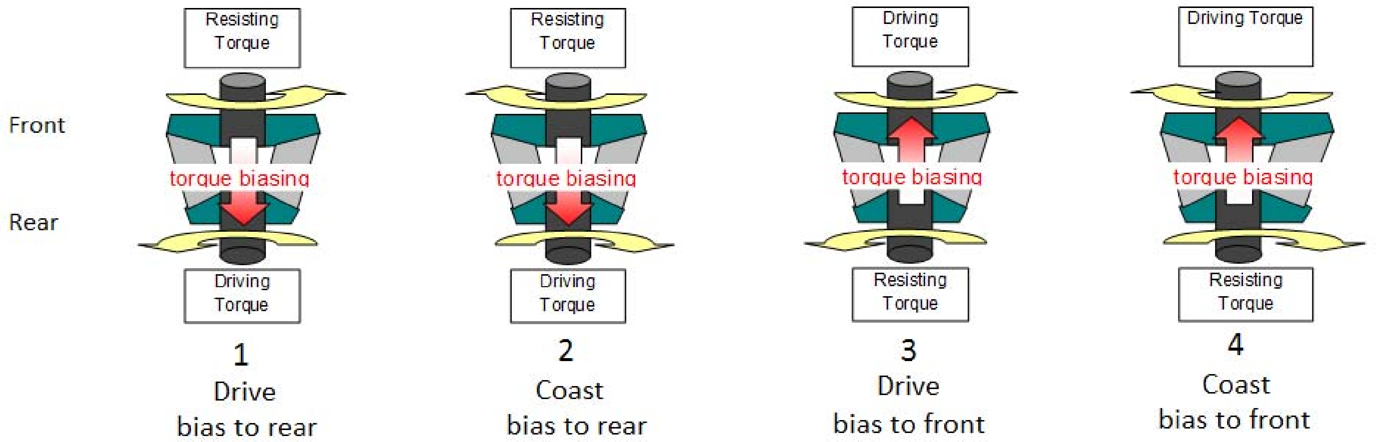


TDR computation for the 4 locking modes

- TDR : Torque Distribution Ratio

$$TDR = \frac{T_1}{T_2}$$



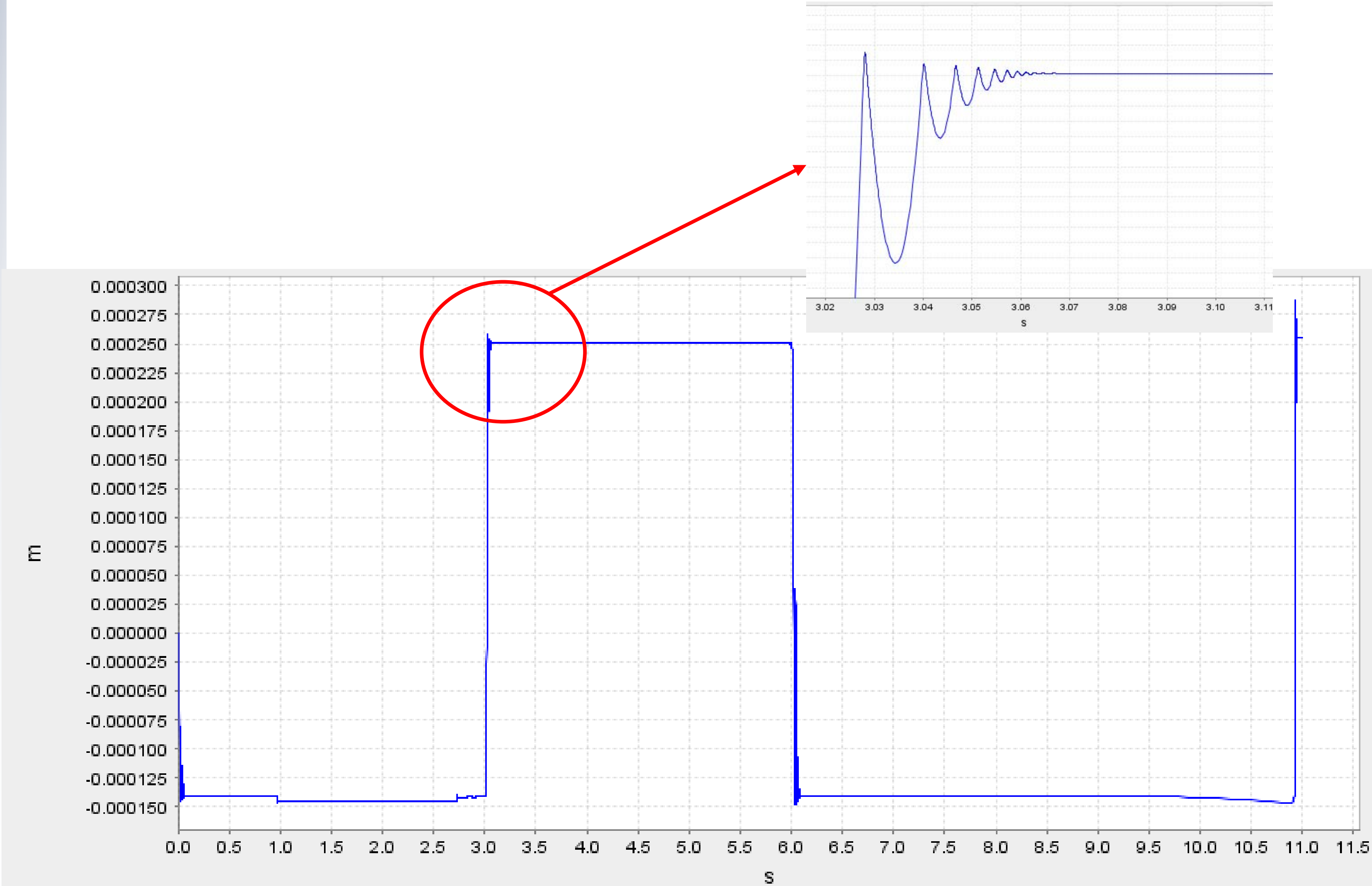


Configuration on vehicle

Aerospace & Mechanical Engineering

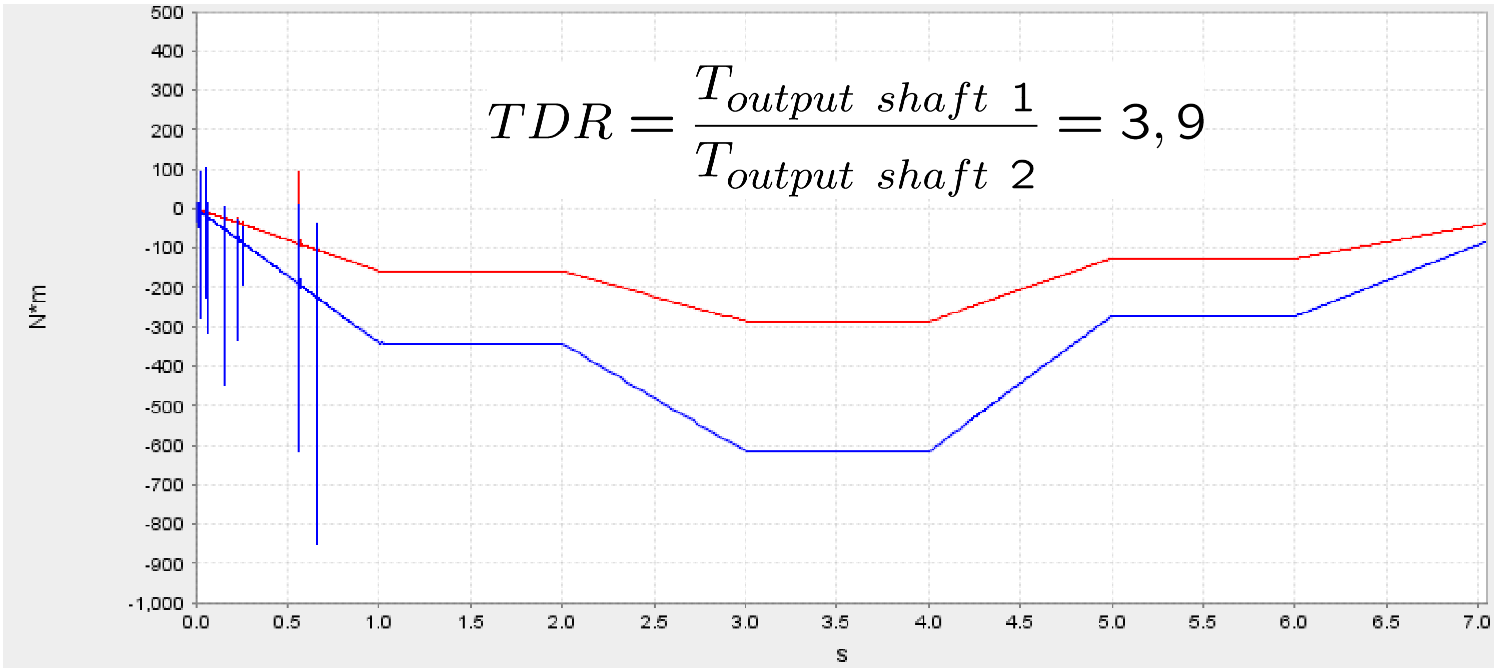
Axial displacement of gear wheel

Aerospace & Mechanical Engineering



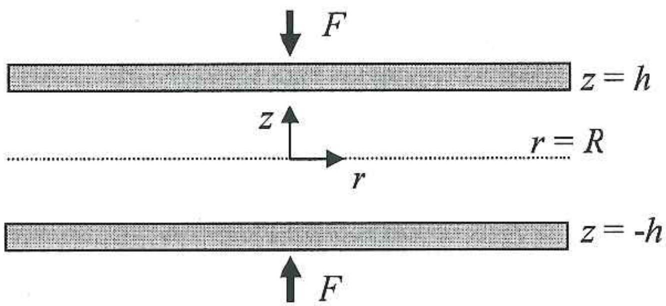
TORSEN differential

- Mode: Drive to rear



$$TDR = \frac{T_{output\ \ shaft\ \ 1}}{T_{output\ \ shaft\ \ 2}} = 3,9$$

mode	1 (Drive, rear)	2 (Coast, rear)	3 (Drive, front)	4 (Coast, front)
TDR simulation	3.9	2.94	1.56	1.65
TDR experimental	4.02	2.82	1.57	1.62



- Assumptions:
  - 2 plates (2h<<R)
  - Newtonian fluid
  - Isothermic fluid in steady state
  - No sliding on walls
  - Axisymmetric (cylindrical coordinates)

- Continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(r\ v_r) + \frac{\partial v_z}{\partial z} = 0$$

- Momentum equation (radial component): inertia terms, elongation gradient and $\frac{v_r}{r^2}$ neglected)

$$\frac{\partial p}{\partial r} = \mu\frac{\partial^2 v_r}{\partial z^2}$$

Squeeze film modelling

- Boundary and initial conditions

$$v_r = 0,\ z = h \qquad \frac{\partial v_r}{\partial z} = 0,\ z = 0$$

$$p = p_{atm},\ r = R \qquad h = h_0,\ t = 0$$

- Momentum equaion ➔ velocity profile

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2} \quad \xrightarrow{\int\int dz^2} \quad v_r = -\frac{h^2}{2\mu}\frac{\partial p}{\partial r}\left[1 - \left(\frac{z}{h}\right)^2\right]$$

- Continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(r\ v_r) + \frac{\partial v_z}{\partial z} = 0 \quad \xrightarrow{\int\int dr dz} \quad -\dot{h}\pi r^2 = 2\pi r \int_0^h v_r dz$$

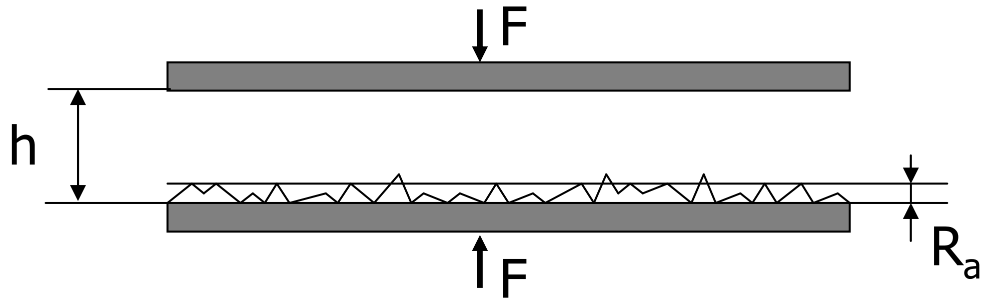
- Pressure profile and force applied on plates

$$p - p_{atm} = \frac{3\mu \dot{h} r}{2h^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right) \qquad F(h, \dot{h}) = \frac{3\mu \dot{h}}{2h^3}\left(-\frac{R^4}{4}\right)$$

Squeeze film modelling

- Penalty method to model metal-metal contact

$$F(h,\dot{h}) = \begin{cases} \frac{A^2\mu\dot{h}}{2h^3} & if \quad h > R_a \\ \frac{A^2\mu\dot{h}}{2h^3} + K(h-R_a) & if \quad h < R_a \end{cases}$$



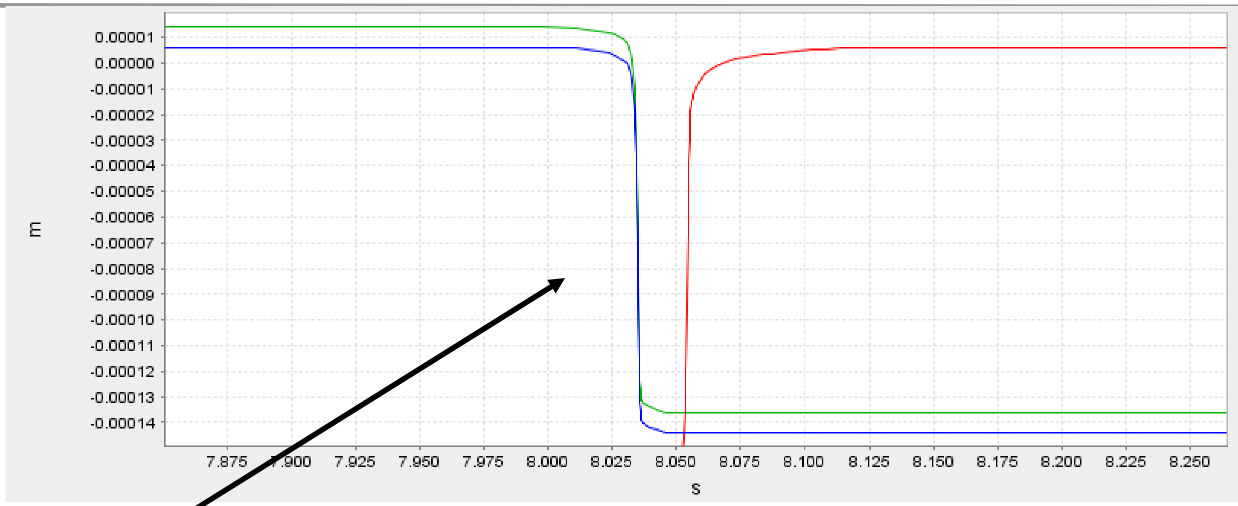
- Friction torque (metal-metal contact)

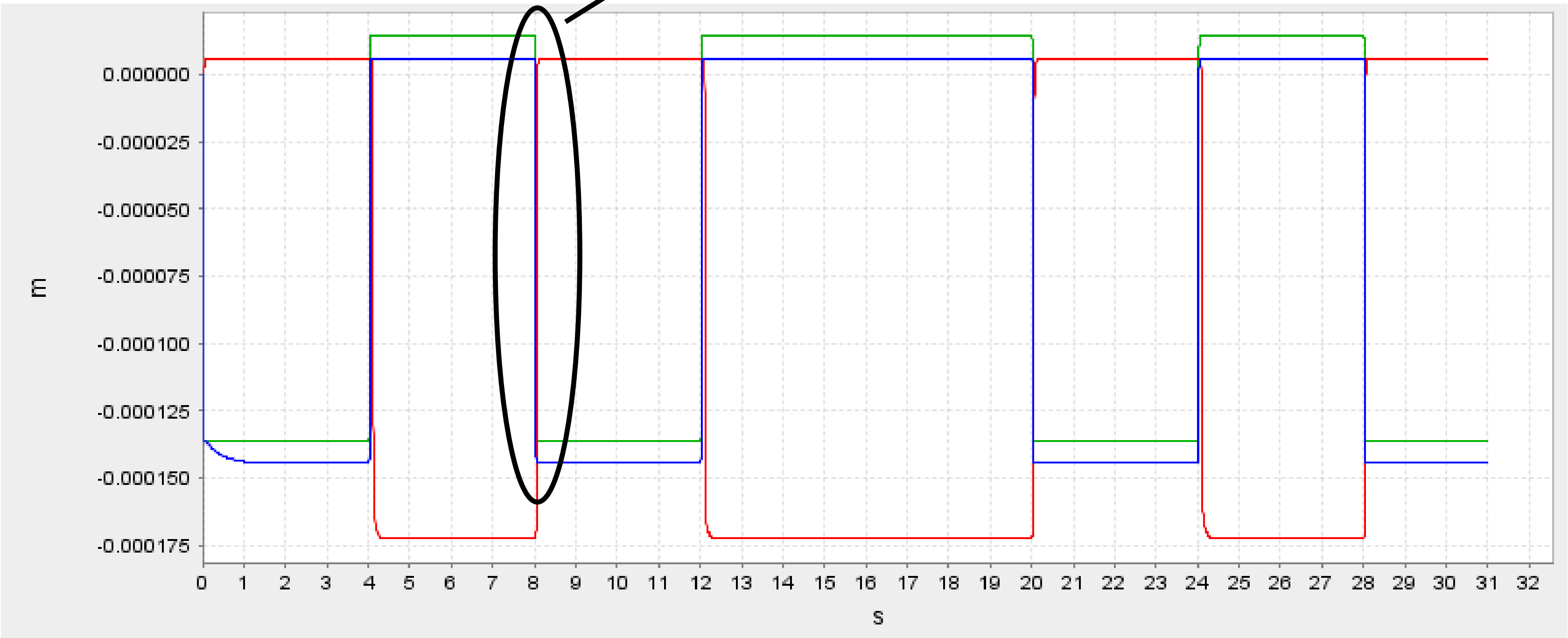
$$M_{fr} = \int_A \mu_R \; p \; r \; \mathrm{d}S = 2\pi \; \mu_R \; p \; \frac{r_{ext}^3 - r_{int}^3}{3}$$

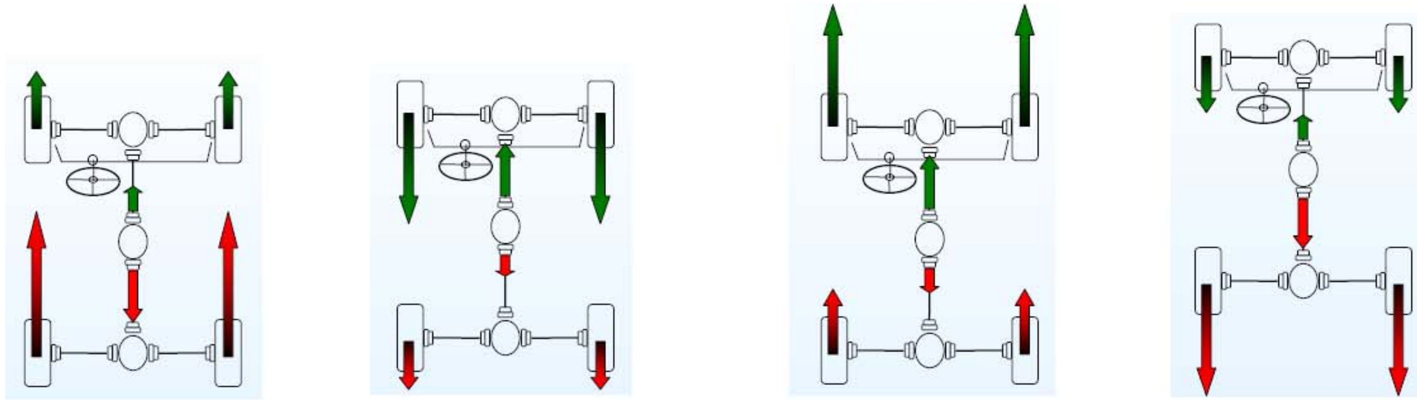
- Viscous resistance

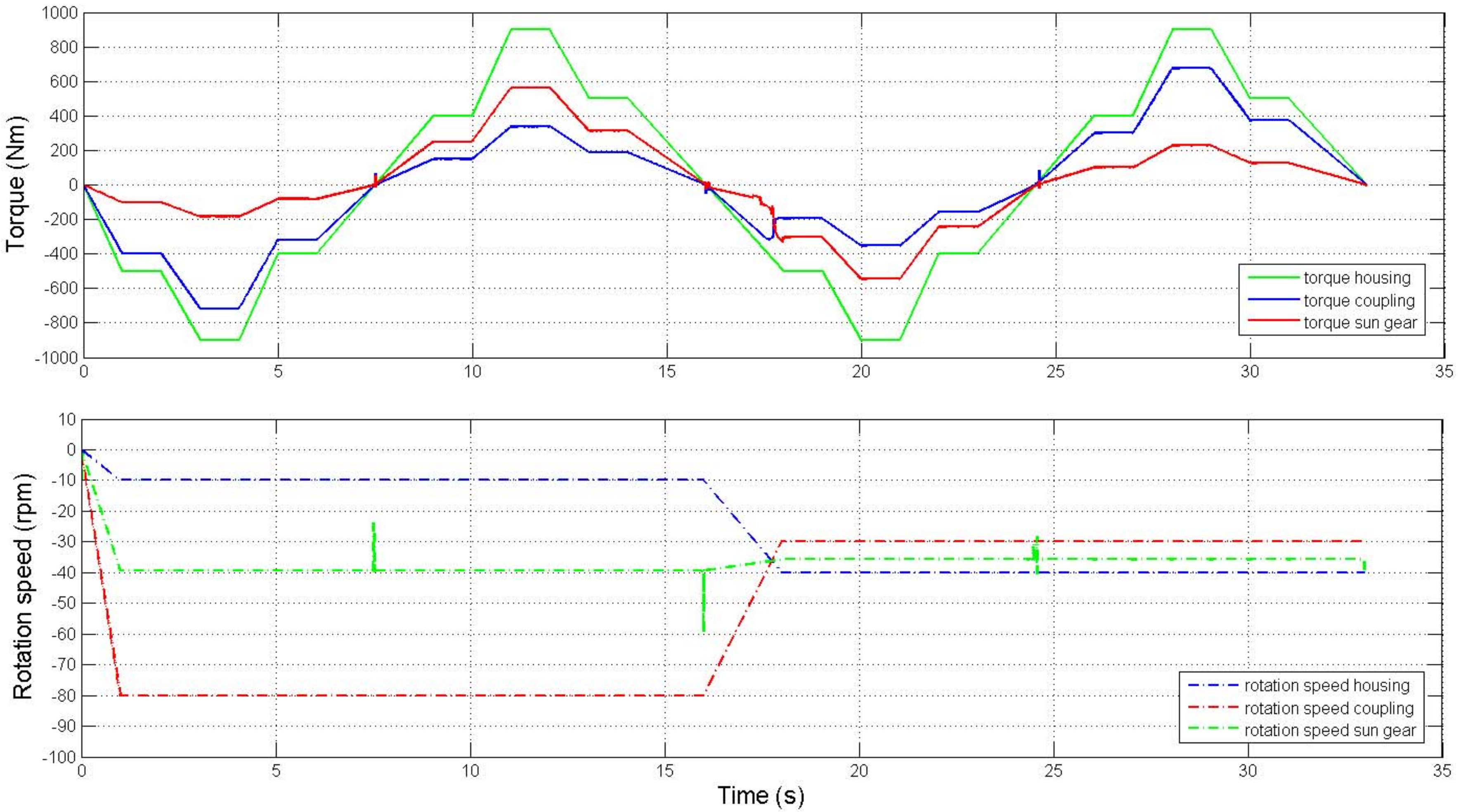
$$M_v = \int_A \frac{\mu \; \omega \; r^2}{h} \mathrm{d}S = 2\pi \; \frac{\mu \; \omega}{h} \; \frac{r_{ext}^4 - r_{int}^4}{4}$$

Axial displacements of gear wheels







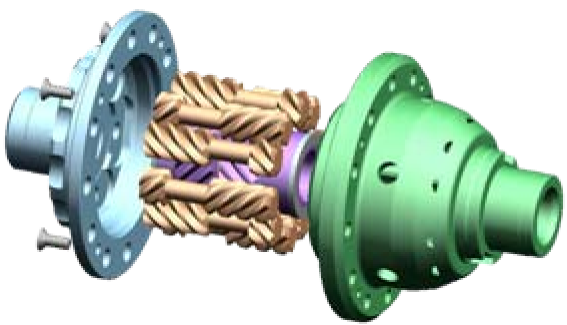

Torque (Nm)
torque housing
torque coupling
torque sun gear
Rotation speed (rpm)
rotation speed housing
rotation speed coupling
rotation speed sun gear
Time (s)

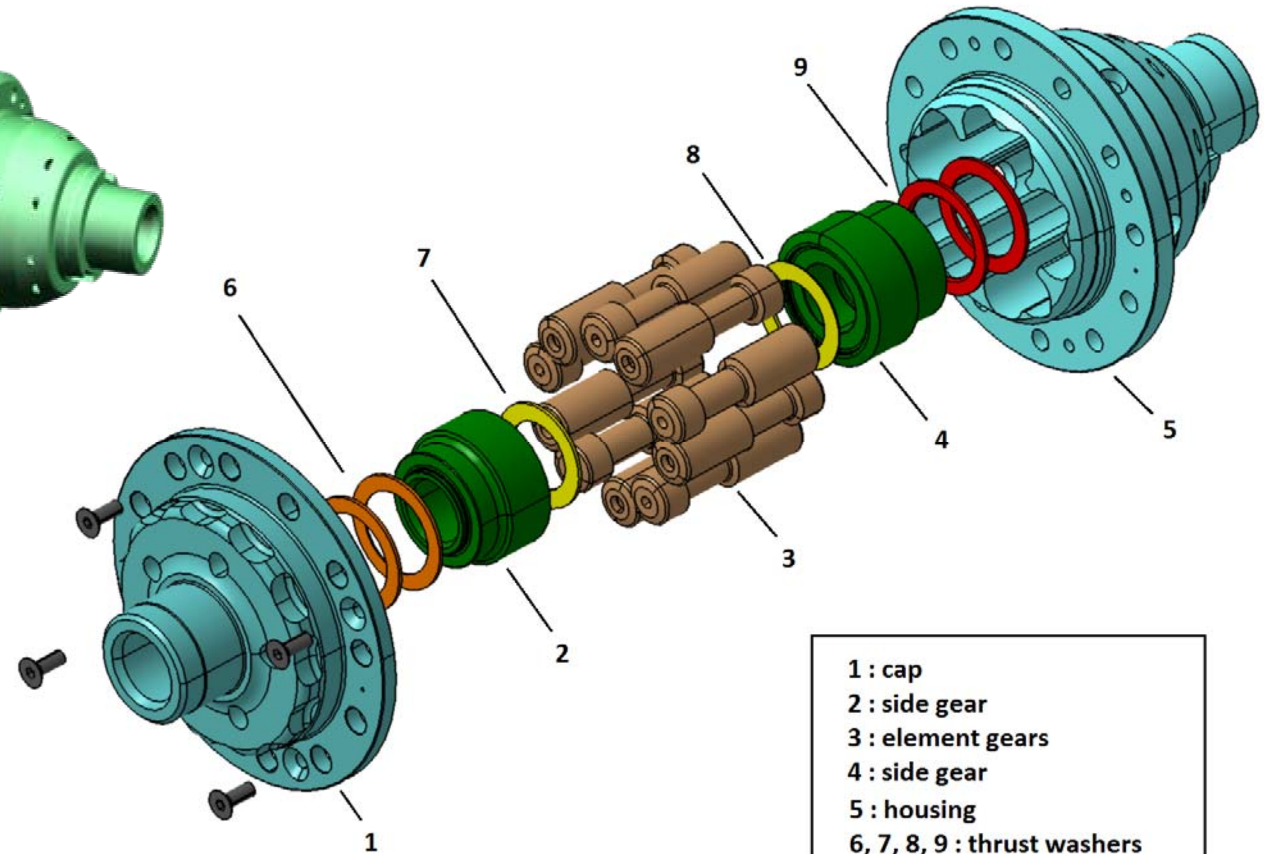
Aerospace & Mechanical Engineering

Université de Liège

Type B Torsen

- Front or rear differential
- Housing, helical gear pairs and thrust washers (no ring gear)
- Locking due to relative friction between gears & washers
- 4 working modes

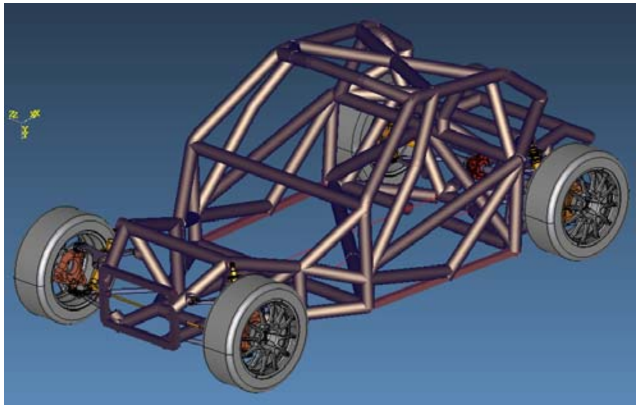




1 : cap
2 : side gear
3 : element gears
4 : side gear
5 : housing
6, 7, 8, 9 : thrust washers

Differential in vehicle model

- Rear differential included in full vehicle model with:
  - rigid driveshafts
  - flexible chassi (beams)
  - suspensions fixed on chassi with bushings
  - tyre models (Pacejka)



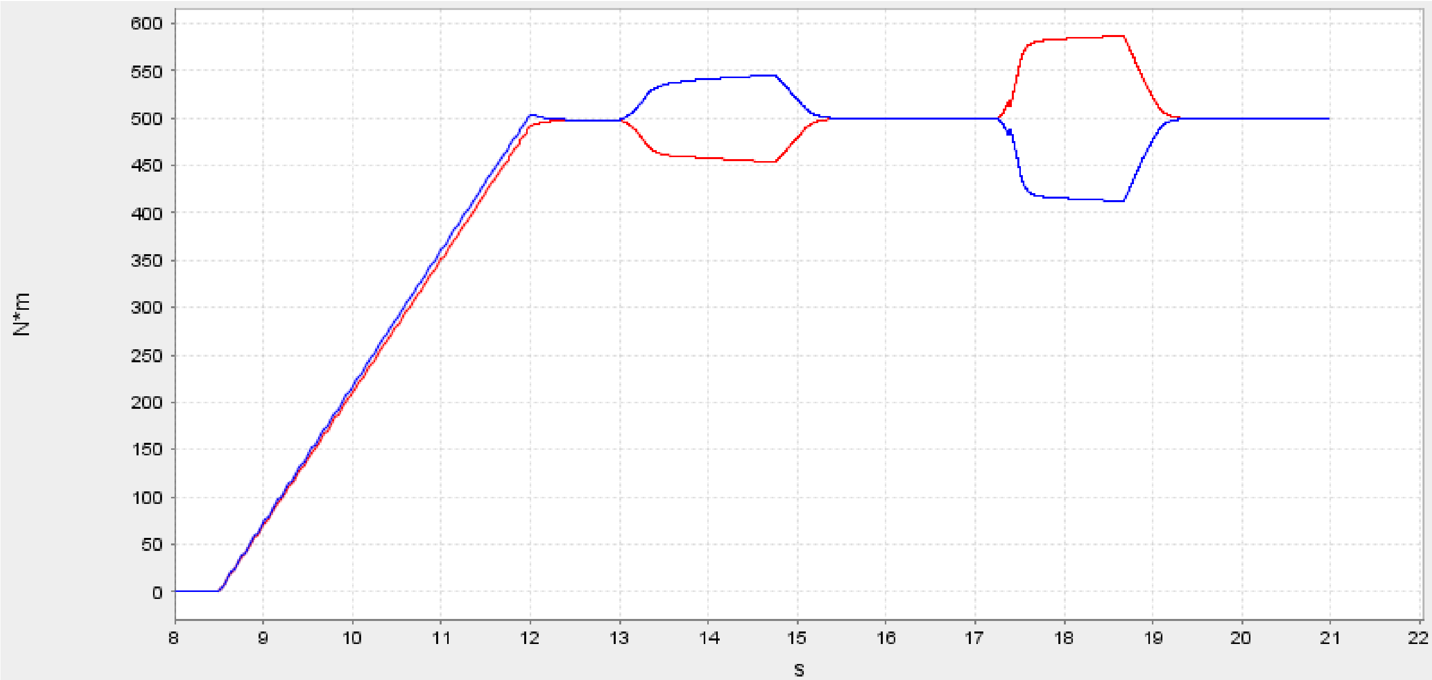
- TORSEN Type B
  - 20 rigid bodies
  - 20 gear pairs
  - 26 contacts
  - 10 cylindric joints
- 12730 dof
- Computational time ≈ 30 min.

Université de Liège

Vehicle model

- Torque transfer of TORSEN differential when a vehicle accelerates on a slippery surface

Torque on right and left rear wheel



Conclusion

- Global continuous contact formulations between rigid bodies:
    - ➢Continuous impact modelling
    - ➢Squeeze film + penalty method
- Validation of Torsen differential model with experimental data
- Include in a full vehicle model

- Outlook :
    - Stick-slip
    - Flexible driveshafts
    - Modelling of other automotive transmission components

Université de Liège

Questions / Answers

Thank you for your attention !

Modelling of frictional unilateral contact in automotive differentials

Aerospace & Mechanical Engineering

Université de Liège

References

R. Seifried, W. Schiehlen, and P. Eberhard. The role of the coefficient of restitution on impact problems in multi-body dynamics. *Proc. IMechE, Part K:J. Multi-body Dynamics*, 224:279–306, 2010.

Hamid Lankarani and Parviz Nikravesh. Continuous contact force models for impact analysis in multibody analysis. *Nonlinear Dynamics*, 5:193–207, 1994.

M. Géradin and A. Cardona. *Flexible multibody dynamics*. John Wiley & Sons, 2001.