Extensions of the Shape Functions with Penalization (SFP) parameterization for composite plies optimization

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Nomenclature

C^l =   linear anisotropic material stiffness matrix of the ply l

C_i^l =   linear anisotropic material stiffness matrix of the candidate orientation i in the physical ply l

n^l =   number of candidate orientations in the ply l

w_i^l =   weighting factor for the candidate orientation i in the ply l

SFP   =   Shape Functions with Penalization

R, S, T=   design variables in the SFP material parameterization

p =   exponent used in the SFP parameterization

I.   Introduction

The problem of selecting suitable fiber orientations in composite structures has been studied for a long time [1-7]. In most aerospace applications, the candidate materials are restricted to the conventional angles with plies oriented at 0°, 45°, -45° and 90°. Sometimes, other orientations are used, such as 30° and 60° [8]. The selection of optimal orientations is by nature a discrete optimization problem. In this paper, the numerical optimization problem is expressed in terms of continuous design variables thanks to a specific parameterization called Shape Functions with Penalization, SFP [9], and a reliable gradient-based optimization method relying on the sequential convex programming approach [10,11] developed for problems involving continuous variables is applied [12]. The analysis of the modeled composite structure is classically carried out with the finite element method. Initially developed for

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the 4 conventional plies 0°, 45°, -45° and 90° in [9], it is shown in this paper that SFP can be used for a different number of candidate orientations. The extension of SFP is demonstrated for the selection of the optimal local fiber orientations in a non homogeneous membrane, for a maximum of 8 candidate orientations.

II. The SFP material parameterization method

A. Continuous material parameterization for composite plies

The SFP method proposed in [9] is an alternative to the DMO (Discrete Material Optimization) approach developed in [13]. Both approaches are an extension of the multi-phase topology optimization of [14]. Here, SFP is used to select composite plies in a set of candidate orientations, in a formulation including continuous design variables. When applied to a composite ply noted l, it consists in writing the linear elastic anisotropic material stiffness matrix $\mathbf{C}^l$ as a weighted sum over the stiffness of some candidate materials $\mathbf{C}_i^l$:

$$\mathbf{C}^l = \sum_{i=1}^{n^l} w_i^l \mathbf{C}_i^l \tag{1}$$

with

$$\sum_{i=1}^{n^l} w_i^l = 1 \tag{2}$$

and

$$0 \le w_i^l \le 1 , \quad i = 1,...,n^l \tag{3}$$

where n^l is the number of candidate orientations in the ply l, and w_i^l are weighting factors. While DMO can be used for any number of candidate materials, SFP was derived in [9] for 4 candidate orientations, i.e. 0°, 45°, -45° and 90°. In this paper, the SFP method is extended to different numbers of candidate orientations. The cases of 3 and 8 candidate orientations are described. Eventually, the SFP method could be extended to 5, 6, 7 and even more orientations, based on the same principle.

B. SFP for 4 candidate materials

As proposed in [9], the shape functions of the 4-nodes quadrangular element can be used as weights in Eq.(1), as depicted in Fig. 1. These specific functions satisfy the conditions of Eq.(2) and Eq.(3). The 4 weights of the Shape Functions (SF) parameterization are written in Eq. (4). Two design variables are enough to identify each of the 4

vertices. These variables, termed R and S, are the coordinates of the reference quadrangle, classically used for the integration of the stiffness matrix and the force vector in the finite element method. They identify each vertex of the quadrangle, corresponding to each candidate material (Fig. 1). The shape functions in (4) are bilinear in terms of R and S. As a result, SF does not penalize the intermediate values of the design variables, and a mixture of the 4 candidate plies can sometimes be observed at the solution. The Shape Functions with Penalization scheme, termed SFP, is then proposed. It is written in (5). The intermediate values of the design variables are now penalized in a scheme which is similar to the SIMP law used in topology optimization. The condition in Eq.(2) is no longer satisfied for the intermediate values of the design variables appearing during the iterative process. However, numerical tests demonstrate that this is not an issue, and that this doesn't penalize the convergence towards a solution when the value of the exponent p in Eq. (5) is large enough. The shape of the weighting coefficients w_i in (5) is illustrated in Figure 2.

$$w_i^{SF} = \frac{1}{4}(1 \pm R)(1 \pm S) \qquad \text{for SF}$$

that is

$$w_1 = \frac{1}{4}(1-R)(1-S) \qquad\qquad w_2 = \frac{1}{4}(1+R)(1-S)$$

$$w_3 = \frac{1}{4}(1+R)(1+S) \qquad\qquad w_4 = \frac{1}{4}(1-R)(1+S) \qquad (4)$$

$$w_i^{SFP} = \left(w_i\right)^p \qquad\qquad \text{for SFP} \qquad (5)$$
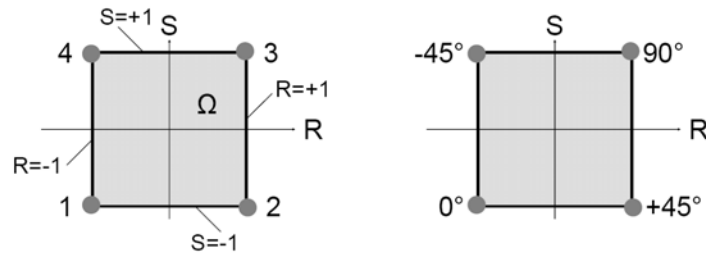

Fig. 1 SFP for 4 candidate materials

The advantage of SFP compared to DMO is that only 2 design variables are sufficient to select the optimal plies in a set of 4 candidate orientations. With DMO, 4 design variables are needed (see [13]).
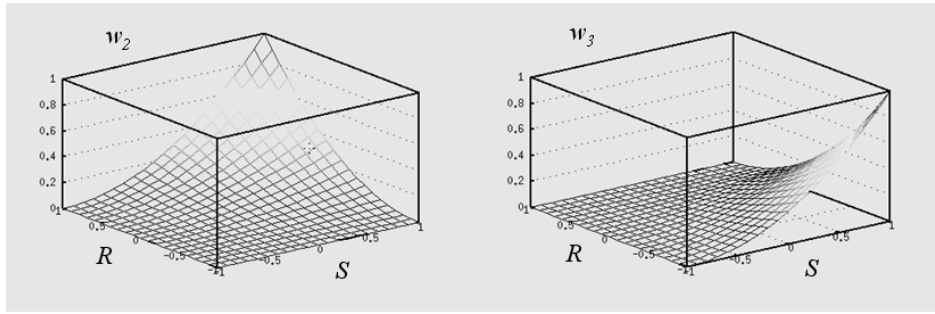
Fig. 2 Weighting coefficients in SFP for 4 candidate materials

C. SFP for 3 candidate materials

A SFP formulation is proposed in [15] for 3 candidate sub-laminates, i.e. $[0_2]$, $[\pm45]$ and $[90_2]$. In that case, the 3-nodes triangular element is used to define the weighting factors (Fig. 3). In order to avoid any biased solution, a double thickness value is associated to the candidate plies with fibers oriented at 0° and 90°, while a unit thickness is assigned for the 45° and for the -45° plies. The resulting weights w_i are given in Eq.(6).

$$
\begin{aligned}
w_1 &= (1 - R - S)^p \\
w_2 &= (R)^p \\
w_3 &= (S)^p
\end{aligned}
$$

(6)

In order to limit the possible values of R and S to meaningful solutions, the additional constraint of Eq.(7) must be considered in the problem.
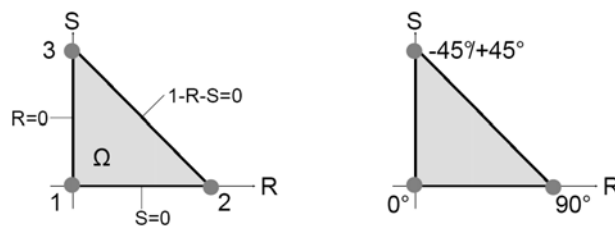
$$1 - R - S \geq 0 \tag{7}$$

Fig. 3 SFP for 3 candidate materials

D. SFP for 8 candidate materials

The same principle can be applied to 8 candidate materials. In that case, the shape functions of a hexahedral element are used to define the weighting factors, with an exponent p to avoid the mixtures of candidate materials at the

solution. Here, 3 design variables are enough to select the optimal ply in a set of 8 candidates, what significantly decreases the size of the optimization problem compared to DMO, which would require 8 design variables. The first weighting factor w_1 is written as follows:

$$w_1 = \left[\frac{1}{8} (1-R)(1-S)(1-T) \right]^p \tag{8}$$

This kind of parameterization can be used when the set of candidate material is {-60°,-45°,-30°,0°,30°,45°,60°,90°}.

E. SFP for other numbers of candidate material

The parameterization for the 8 candidate orientations can be altered in order to propose solutions for less than 8 candidate materials. The idea is to assign the same candidate orientation to more than one vertex of the hexahedral element. The approach is demonstrated for the set of {-45°,0°,30°,45°,90°} candidate orientations.

III.   Numerical procedure

Our own implementation of MMA [11,12] available in the BOSS Quattro optimization tool box [16] is used, together with the SAMCEF finite element code for the structural analyses [17]. The optimization problems consists to maximize the global in-plane structural stiffness of a non homogeneous composite membrane, in a linear static analysis. Since a gradient-based optimizer is used, the derivatives must be available. The sensitivities are here computed by finite differences.

IV.   Application

Simplified problems are solved in this paper, since they are sufficient to demonstrate the capabilities of the developed approach. The structures therefore include only one ply over their thickness, and a simple objective function is minimized. Composite design is much more complicated when practical (industrial) problems are addressed. The extension of the design process to more realistic test cases, taking into account design rules on the stacking sequence and manufacturing constraints, is not direct at all, and will be the topic of future researches.

In a first application, the structure illustrated in Fig. 4 is studied. It is divided into 16 regions of independent fibers orientations. In each region, we are looking for the best ply amongst the candidate orientations, when the compliance is minimized with respect to a static load. The orientations can vary from region to region at the solution, leading to a non homogeneous ply. The structure is clamped on its left side, and is submitted to a vertical concentrated load at its lower right corner. The material is C12K/R6376 Graphite/epoxy prepreg (see [9]).
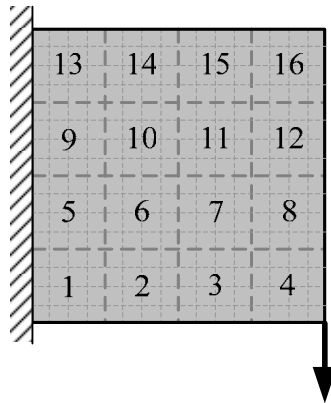
Fig. 4 Non homogeneous membrane test case

The solutions obtained for 3, 4 and 8 candidate orientations are presented in Fig. 5. The solution for 4 candidate orientations has already been published in [9]. It is now compared to the solutions for 3 and 8 candidate orientations. For 3 and 4 candidate orientations, the problem includes 32 design variables. For 8 candidate orientations, 48 design variables are enough. With DMO, the problem would have included 48, 64 and 128 design variables, respectively, for 3, 4 and 8 candidate orientations in each region.
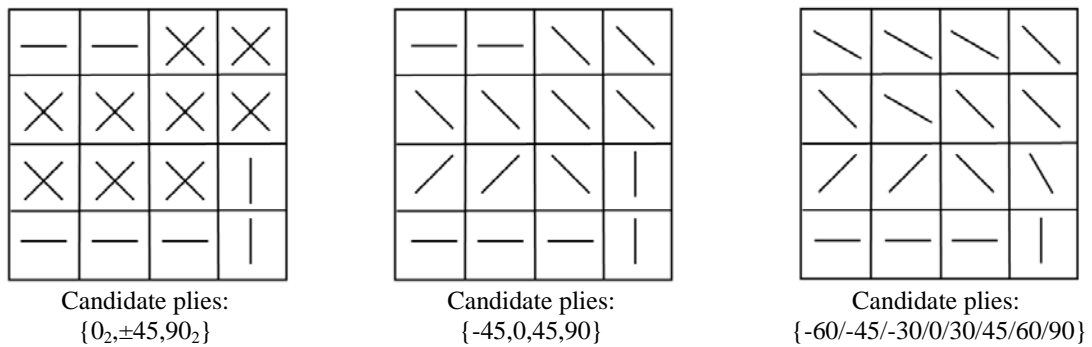

| Candidate plies: $\{0_2, \pm45, 90_2\}$ | Candidate plies: $\{-45, 0, 45, 90\}$ | Candidate plies: $\{-60/-45/-30/0/30/45/60/90\}$ |

Fig. 5 Resulting fiber orientations for 3, 4 and 8 candidate plies

For the case of 3 candidate materials, a large value of the exponent p must be used, since a mixture of candidate orientations can be observed at the solution. As reported in Table 1, p must be larger than or equal to 5 in that case to identify an interpretable solution with a unique fiber orientation in each region. The number of iterations reported in Table 1 provides an idea about the computational cost needed to reach an optimum. For a comparison, some results obtained with DMO are reported in Table 2, where DMO4 and DMO5 are two variants of DMO, as

explained in [9,13]. As is the case for SFP, DMO works with en exponent p, which is chosen equal to 5 in the application. The optimal ply distributions obtained with SFP and DMO for 3 candidate materials are illustrated in Figure 6. These solutions are in addition compared to the direction of the principal stress for an isotropic elastic material.

Table 1. Results with SFP

3 candidate materials	3 candidate materials	4 candidate materials	8 candidate materials
p = 3	*p = 5*	*p = 3*	*p = 3*
Mixture at the solution	*No mixture at the solution*	*No mixture at the solution*	*No mixture at the solution*
8 iterations	*13 iterations*	*4 iterations*	*15 iterations*

Table 2. Results with DMO

3 candidate materials		4 candidate materials		8 candidate materials
DMO4, p = 5	*DMO5, p = 5*	*DMO4, p = 5*	*DMO5, p = 5*	*DMO4, p = 5*
Mixture at the solution	*No mixture at the solution*	*Mixture at the solution*	*No mixture at the solution*	*Mixture at the solution*
23 iterations	*5 iterations*	*18 iterations*	*5 iterations*	*33 iterations*

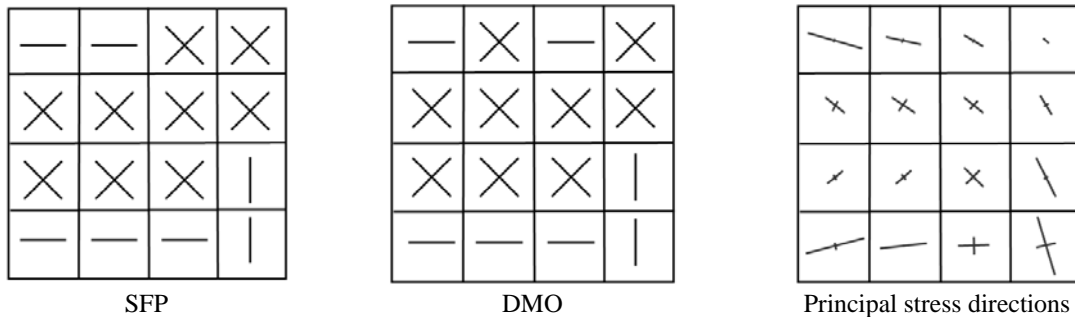
SFP   DMO   Principal stress directions

Fig. 6 Solution obtained with DMO, SFP and the principal stress directions

The iteration history for SFP with eight candidate materials is illustrated in Figure 7, where the evolution of the objective function as well as the variation of the design variables associated to regions 1 and 16 of Figure 4 are reported.
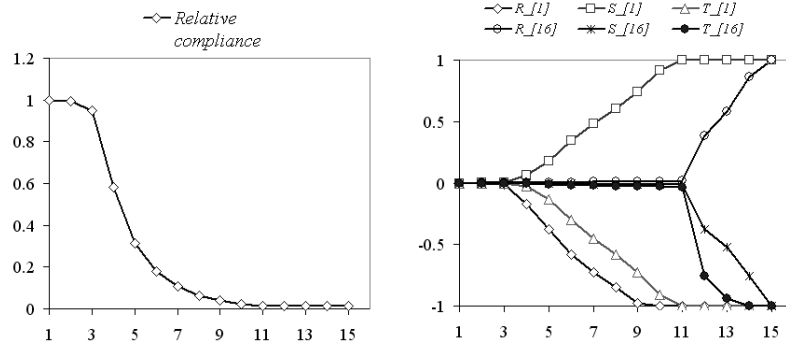
Fig. 7 Iteration history for SFP with 8 candidate orientations, in the problem of Fig. 4

Fig. 8 illustrates two results obtained with 5 candidate materials, {-45°,0°,30°,45°,90°}, with two different sets of candidate orientations, with either the 30° or the 45° plies assigned to 4 vertices of the hexahedral element, the other candidate orientations taking the values 0°, 90° and -45°. This is then a degenerated case of the 8 candidate plies, with the sets {90°,45°,0°,-45°,30°,30°,30°,30°} and {90°,30°,0°,-45°,45°,45°,45°,45°}. With these different definitions of the material parameterization, the design domain is different, and so is the obtained solution. Moreover, the design domain in SFP is not convex, as was already reported for the DMO approach [13], and only local optima can be identified. Anyway, even if different, the solutions of Fig. 8 are physically meaningful.
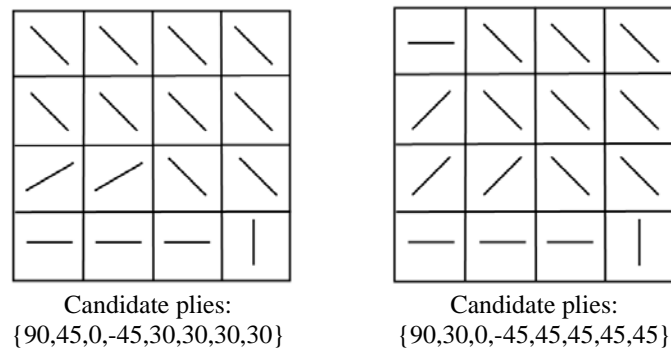

Candidate plies:
{90,45,0,-45,30,30,30,30}

Candidate plies:
{90,30,0,-45,45,45,45,45}

Fig. 8 Resulting fiber orientations for 5 candidate plies

In a second application, a MBB-like structure made of one ply is studied. Because of the symmetry, only one half of the structure is modeled. It is divided in 48 regions of independent fiber orientation, and the set of conventional plies oriented at 0°, 45°, -45° and 90° is used. The number of design variables is equal to 96 for SFP, and 192 for DMO. Six and five iterations are necessary, respectively for DMO5 (with $p = 5$) and SFP (with $p = 2$),

to reach the solutions illustrated in Fig. 9. Unexpected orientations in the vicinity of the load lead to a higher value of the optimal compliance for DMO.
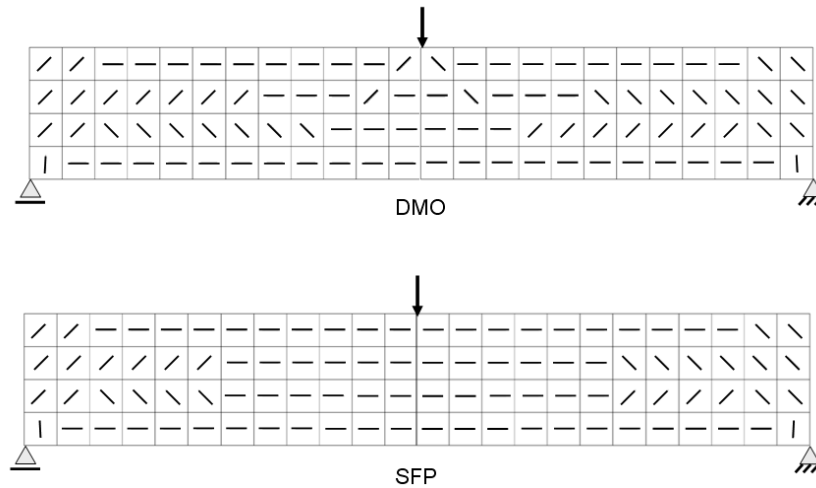

Fig. 9 Resulting fiber orientations for 4 candidate plies with DMO and SFP for the MMB-like structure

V. Conclusion

It is shown in this paper that the SFP parameterization method, which uses weighting factors based on the shape functions classically used in finite elements, can be used for the selection of 3, 4 and 8 candidate fiber orientations. The approach is also extended to less than 8 candidate materials. Compared to DMO, SFP requires a smaller number of design variables, what is interesting for large scale optimization problems.

Acknowledgments

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